

Design and Simulation of a PID Controller for Motion Control Systems

Zakariyya Hassan Abdullahi^{1,a}, Bashir Ahmed Danzomo^{1,b}, Zainab Suleiman Abdullahi^{2,c}

¹ Department of Computer Engineering Hussaini Adamu Federal Polytechnic, Kazaure, Jigawa State Nigeria

² Department Mechanical Engineering, Hussaini Adamu Federal Polytechnic, Kazaure, Jigawa State Nigeria

^ahassanzakariyya8@gmail.com, ^bzeeshazahm05@gmail.com, ^cbdzomo@gmail.com

Abstract. Motion control system plays important role in many industrial applications among which are in robot system, missile launching, positioning systems etc. However, the performance requirement for these applications in terms of high accuracy, high speed, insignificant or no overshoot and robustness have generated continuous challenges in the field of motion control system design and implementation. To compensate this challenge, a PID controller was design using mathematical model of a DC motor based on classical root-locus approach. The reason for adopting root locus design is to remodel the closed-loop response by putting the closed-loop poles of the system at desired points. Adding poles and zeros to the initial open-loop transfer function through the controller provide a way to transform the root locus in order to place the closed-loop poles at the required points. This process can also be used for discrete-time models. The Advantages of root locus over other methods is that, it gives the better way of pinpointing the parameters and can easily predict the fulfilment of the whole system. The controller performance was simulated using MATLAB code and a reasonable degree of accuracy was obtained. Implementation of the proposed model was conducted using-Simulink and the result obtained shows that the PID controller met the transient performance specifications with both settling time and overshoot less than 0.1s and 5% respectively. In terms of steady state error, the PID controller gave good response for both step input and ramp.

1. Introduction

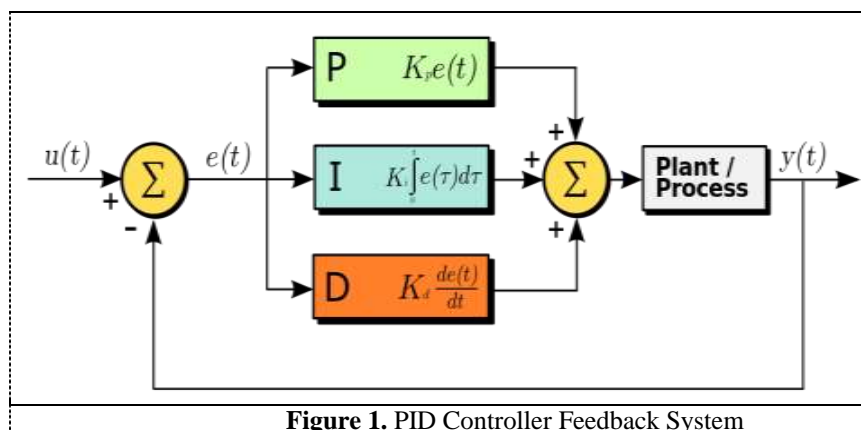
Tool Motion control is a sub- discipline of automation with which the position or velocity of machine are regulated by using devices like hydraulic pump and electric motor, a servo. Motion control system plays significant role in many industrial applications among which are in robot system, missile launching, positioning systems etc. However, the performance requirement for these applications in terms of high accuracy, high speed, insignificant or no overshoot and robustness have generated continuous challenges in the field of motion control system design and implementation. The mechatronic approach to system design offers a solution to this problem by applying con-current engineering instead of the traditional sequential approach. In light of these, various techniques starting from classical to modern design approach have been developed and employed by researchers and industrialists in designing appropriate controller to meet the above demands [1] and [2].



Nowadays, modern control systems, like semiconductor manufacturing instruments, transportation, X-Y driving devices as well as robots have been important systems which normal require high speed and high efficiency linear motions [4]. These systems splitted into two components; which include the mechanical parts with servo drive systems and the servo controllers that regulate the multi axis agitation of the mechanical parts. In general, the system has consisted of an X-Y table, where each every motion axis can be driven by individual actuator arrangements, like DC or AC motors [5]. DC motors has been prominent in the industry because of their uncomplicated control manner. the armature voltage to differ the motors speed and because the motor carbon brush alongside the convertor were mechanical units, they tend to produce sparks and create ravage when the motor is operating, this is another dominant drawback of the DC Motors additionally, DC motor presents risks to the environment and have a short life cycle maintenance of DC motors is also costly, AC motors are normally classified into three groups, namely; the synchronous motors, the induction motor as well as the reluctance motor. Both the rotor and stator of an AC motor are the sole contact bearing units. The rotation of the motor is initiated by the stators magnetic field .this requires some control techniques' unlike the magnetic field guide control to regulate different moments. With the advent of semi conductor control devices, the estimation needed for AC motor regulation can be achieved with greater ease. Due to this advantage, AC motors becomes favourites this days [6].The movement mechanisms using motor mechanisms normally regulated to unknown dynamics linked interferences disturbances and friction among the relevant units, which usually damage the systems performance [3]. In an attempt to enhance the tracking performance in machining cutting, previous literatures have been presented in [9, 10].The versatility of numerical machines with output time delay and the dimensional error are presented in [9]. Frequency feedback and Phase margin analysis are operated for controller design to actuate regions of system steadiness on the other hand, cross coupled dynamics, and uncertainties among relevant axis components were not examined in the stability investigation. Additionally, the shift region nonlinearity as a result of mechanical dynamics was compensated by using adoptive fuzzy controller .with the determination of the stiff region nonlinearity [9] the major aims of this investigation is to examine the application of PID controller in the control of servo motor motion control system.

2. The PID Controller

A PID controller calculates an error value as the difference between a measured variable and a desired set point. The controller (shown in Figure 1) attempts to minimize the error by adjusting the process control inputs. In the absence of knowledge of the underlying process, a PID controller has historically been considered to be the best controller Bennett [8].



The PID algorithm is the most popular feedback controller used within the process industries. It has been successfully used for over 50 years [5] and [6]. It is a robust easily understood algorithm that can provide excellent control performance despite the varied dynamic characteristics of a given system or process plant. The calculation algorithms given in (1) involved three separate constant parameters; P, I, and D. These parameters are interpretable in terms of time; where P relied on the present error, I on the buildup of past errors, and D is an indicator of impending errors, based on ongoing rate of change formula – where, K_p is the proper.

$$u(t) = MV(t) = K_p e(t) + K_i \int_0^t e(\beta) d\beta + K_d \frac{d}{dt} e(t) \quad [1]$$

Where, K_p represents the proportional gain, K_i represents the value integral gain, K_d is the derivative gain, e represents the error, t the period of the sudden time (the current and β are the parameters of integration (takes value from 0 - t). Equivalently, the transfer function in the Laplace domain $L(s)$ of the PID controller is;

$$L(t) = K_p + K_i / s + K_d s \quad [2]$$

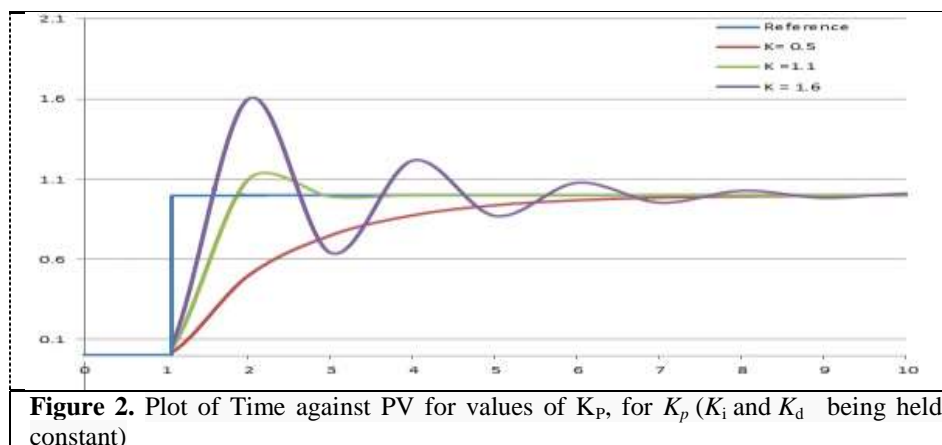
Where s is the complex frequency

2.1. The proportional term, K_p

The proportional term produces an output value that is proportional to the current error value. The proportional response can be adjusted by multiplying the error by a constant K_p , called the proportional gain constant given by;

$$P_{out} = K_p e(t) \quad [3]$$

A high proportional gain results in a large change in the output for a given change in the error (Figure 2). If the proportional gain is too high, the system can become unstable. In contrast, a small gain results in a small output response to a large input error, and a less responsive or less sensitive controller. If the proportional gain is too low, the control action may be too small when responding to system disturbances. Tuning theory and industrial practice indicate that the proportional term should contribute the bulk of the output change.



As a PID controller depends on the estimated process parameters, and not particularly on the knowledge of the process at hand. It is also widely applicable; through manipulating these three parameters of the model, the PID system works with specific process variables. The feedback of the

controller can be explain in terms of its responsiveness to an error, the extend which the system pass over a set-point and the intensity of any system oscillation. The practice of the PID does not assured ideal regulation of the system or even its constancy. Some functions may need adopting only one algorithm to bring the correct system control. This is accomplishing by putting the other variables to zero. A PID controller is referred to as a PI, PD, P or I Controller in the absences corresponding regulation function. PI Controller is fairly familiar since derivative function is delicate to measurement noise, whereas lack of an integral term may restrict the system from meeting its target range.

2.2. The proportional term, K_p

The contribution from the integral term is similar to the magnitude of the variation and the period of the same. The integral in a PID Controller is a representation of the affected error variation overtime. Which presents the accumulate offset that may have been fixed earlier , the accumulated error variation hence multiplied by the integral gain (K_i) and included the controller output ,the integral function can be presented as;

$$I_{out} = K_i \int_0^t e(\beta) d\beta \quad [4]$$

The integral Function pushes the motion of the process towards set point and avoids the resultant steady –state error variation that exists with pure preoperational controller (Figure 3).

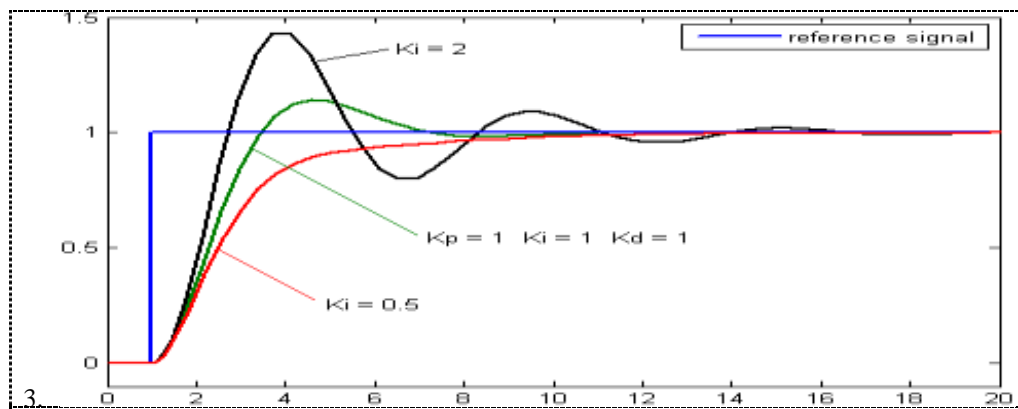


Figure 3. Plot of Time against PV, for Three Values of K_i (K_p and K_d held constant)

2.3 The Derivative term, K_d

The derivative of the process error is estimated by stabling the slope inclination of the error over time as well as multiplying the rate of change by the derivation gain k_p (Figure 4). The size of the contribution of the derivation function to the entire control behaviors is referred to as the derivative gain k_p . The derivative function therefore presented by

$$D_{out} = K_d \frac{d}{dt} e(t) \quad [5]$$

Derivative response for see system performance and enhances the setting time and system balance , optimal derivative is not accidental so that operation of PID Controllers involve an added low pass filtering for the derivation function term to restrain the high frequency gain and noise. Derivation behaviour is sometimes used in reality even by one estimate in just 25% of deployed regulators by virtue of its variable encounter on system balance and other application.

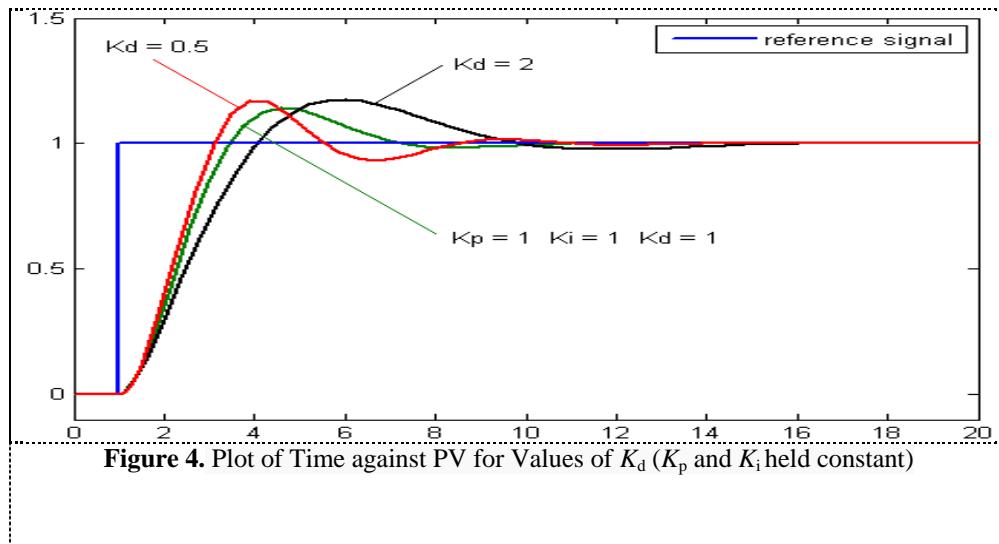


Figure 4. Plot of Time against PV for Values of K_d (K_p and K_i held constant)

4. Materials and Method

4.1. Mathematical Modelling of the Motion Control System

In this research, a Quanser rotary motion system driven by DC motor was considered. Basically, in motion control system, the DC motor acting as power actuator converts electrical energy into rotational mechanical energy as shown schematically in Figure 5. The major motor parameters are; armature resistance R_m , armature inductance L_m , moment of inertia of the motor system J_{eq} , damping friction of the mechanical system B_{eq} and other motor constants in K subscripts. The input to the system is the armature voltage in volts driven by a voltage source. And the measured variable is the angular position of the shaft, θ_m in rad which is a function of the angular velocity of the motor shaft, ω in rad/s, $\dot{\theta} = \omega$

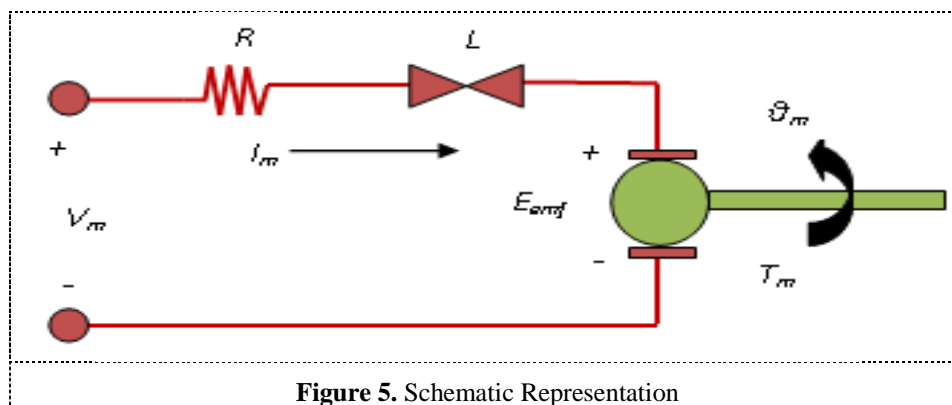


Figure 5. Schematic Representation

Using Kirchoff's law, the voltage equation has been obtained as follows:

$$V_m - R_m I_m - L_m \frac{dI_m}{dt} - E_{emf} = 0$$

[6]

Given that $L_m \ll R_m$, the motor inductance is disregarded, therefore;

$$I_m = \frac{V_m - E_{emf}}{R_m} \quad [7]$$

The Drive emf is proportional to the drive shaft velocity $\dot{\theta}_m$ yields;

$$I_m = \frac{V_m - K_m \dot{\theta}_m}{R_m} \quad [8]$$

By putting Newton's Law to the mechanical part of the system (second law of motion) the torque relation is given by:

$$J_m \ddot{\theta}_m = T_m - \frac{T_l}{\eta_g K_g} \quad [9]$$

Where $\frac{T_l}{\eta_g K_g}$ is the load torque seen through the gears, and η_g is the efficiency of the gear box. Applying the second law of motion at the load of the motor, we have:

$$J_l \ddot{\theta}_l = T_l - B_{eq} \dot{\theta}_l \quad [10]$$

Substituting;

$$J_l \ddot{\theta}_l = \eta_g K_g T_m - \eta_g K_g J_m \ddot{\theta}_m - B_{eq} \dot{\theta}_l \quad [11]$$

Given that $\dot{\theta}_m = K_g \dot{\theta}_l$ and $T_m = \eta_m K_t I_m$ where η_m the motor efficiency, (11) becomes;

$$J_l \ddot{\theta}_l + \eta_g K_g^2 J_m \ddot{\theta}_l + B_{eq} \dot{\theta}_l = \eta_g \eta_m K_g K_t I_m \quad [12]$$

The transfer function (TF) of the system (plant) is presented below as;

$$\frac{\theta_l(s)}{V_m(s)} = \frac{\eta_g \eta_m K_g K_t}{J_{eq} R_m s^2 + (B_{eq} R_m + \eta_g \eta_m K_m K_t K_g^2) s} \quad [13]$$

Where J_{eq} represents the equivalent moment of inertia of the motor system as seen at the output described by the equation.

$$J_{eq} = J_l + \eta_g J_m K_g^2 \quad [14]$$

These system parameters and their values is described in Table 1.

Table 1: System Parameters [5]

| Parameter | Symbol | 0.00767 |
|--|----------|-----------------------|
| Back EMF Constant | K_m | 0.00767 |
| Motor Torque Constant | K_t | 3.87×10^{-7} |
| Motor Moment of Inertia | J_m | 2.0×10^{-3} |
| Equivalent Moment of Inertia at the Load | J_{eq} | 4.0×10^{-3} |
| Equivalent Viscous Damping Coefficient | B_{eq} | 0.9 |
| Gearbox Efficiency | η_g | 0.69 |
| Motor Efficiency | η_m | 14x5=70 |
| System Gear Ratio (For High Gear) | K_g | 2.6 |
| Armature Resistance | R_m | |

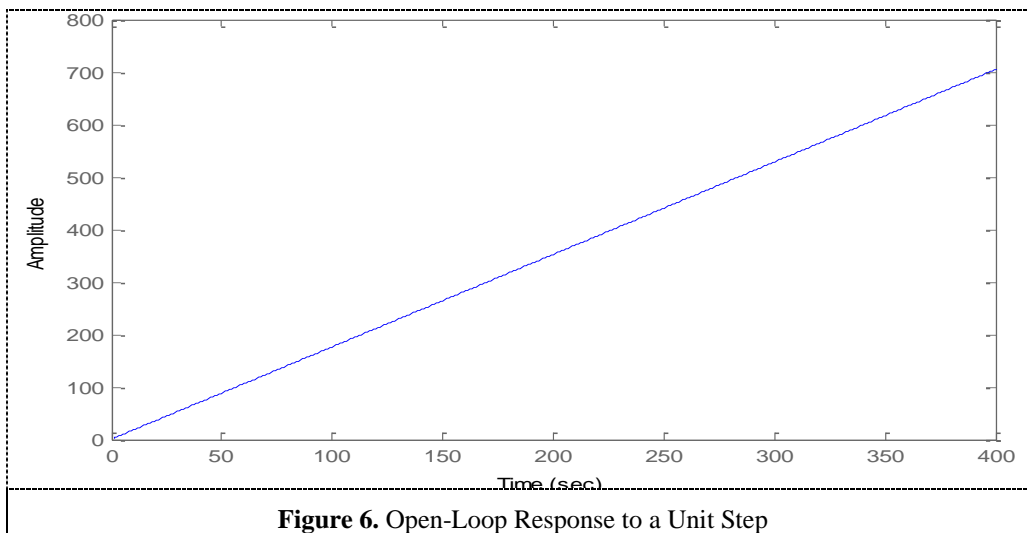
Substituting these values in Table 1 into Equation 13, we have:

$$\frac{\theta_l(s)}{V_m(s)} = \frac{0.1282}{s(0.002s + 0.0728)} = \frac{64.1}{s(s + 36.42)}$$

Therefore the system open loop transfer function is given by:

$$G_p(s) = \frac{\theta_l(s)}{V_m(s)} = \frac{64.1}{s(s + 36.42)} \quad [15]$$

The system's open-loop response to a unit step input was observed using MATLAB code described graphically in Figure 6.

**Figure 6.** Open-Loop Response to a Unit Step

From the response in Figure 6, the open-loop response is unstable and cannot meet any realistic specification, therefore a PID controller with the required settling time, overshoot and velocity error constant has been designed so as to achieve the objective of the research.

4.2. PID Controller Design

The PID controller consists of a proportional plus derivative (PD) compensator cascaded with a proportional plus integral (PI) compensator. The PID controller configuration is of the form:

$$\frac{K_D s^2 + K_P s + K_I}{s} \Rightarrow \frac{K_D (s^2 + as + b)}{s} \quad [16]$$

Where,

$$a = K_P / K_D \text{ and } b = K_I / K_D$$

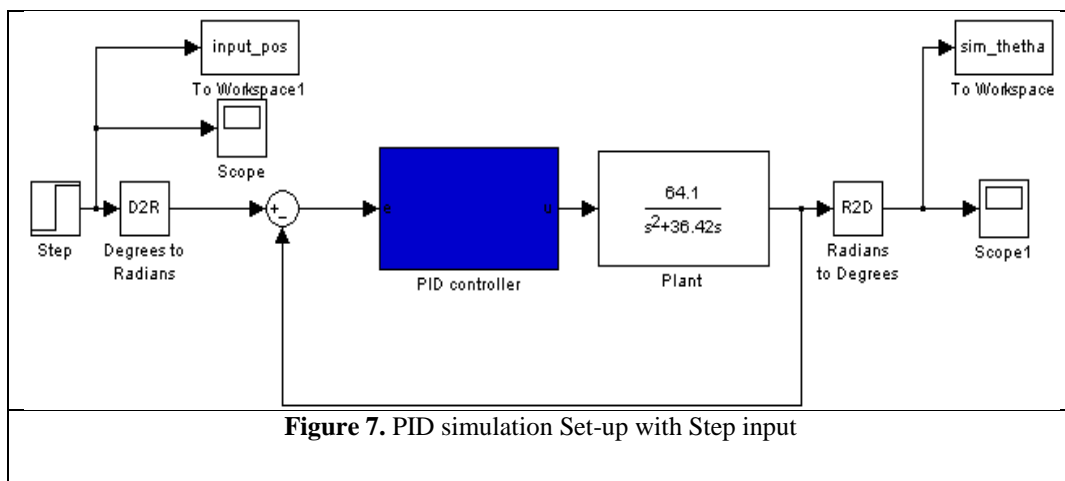
The controller can then be expressed with two zeros and one pole at origin.

$$\frac{K_D (s + z_1)(s + z_2)}{s} \quad [17]$$

The two zeros used to be located anywhere at left side of the root locus plot to achieve the desired response. Considering the root locus plot of figure 3, a pole at origin introduced by the controller doubled the pole at origin, thus introduce additional -140 degree. In line with the objective of this work, a PID controller with the following specifications is to be designed with Settling time, T_s as 0.09s, Overshoot of 2.5% and Velocity error constant, K_v given as 200. Based on these the system response parameters, ω_n and ζ are obtained as 58.38 and ± 0.7613 respectively.

4.3. Simulation of the PID Controller and Result

The proposed PID controller was simulated with the modelled plant in MATLAB –Simulink with 10 degree step and ramp input and with aid of root-locus command the gain which was fine tuned to optimize the system performance as follows. The MATLAB- Simulink set-up for the PID controller is shown in Figure 7 – Figure 8.



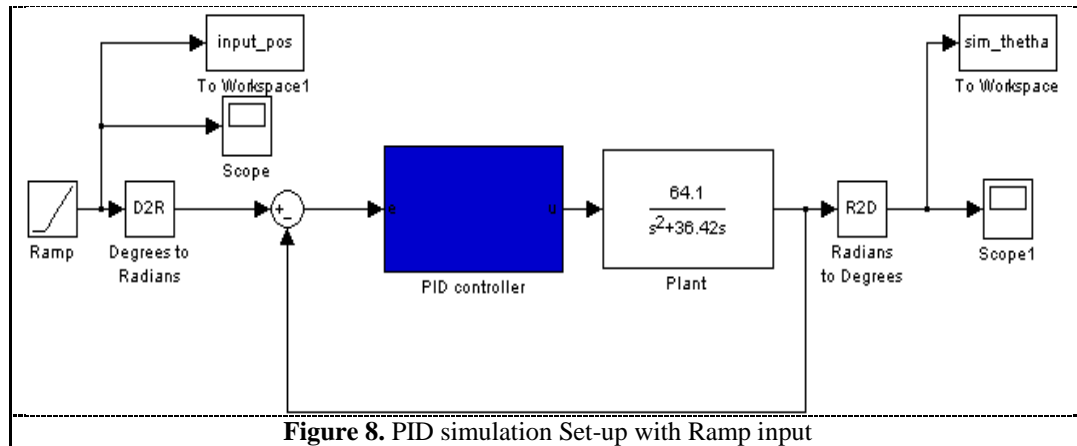


Figure 8. PID simulation Set-up with Ramp input

The response of each set-up for both step input and ramp are given in Figures 9 - 10.

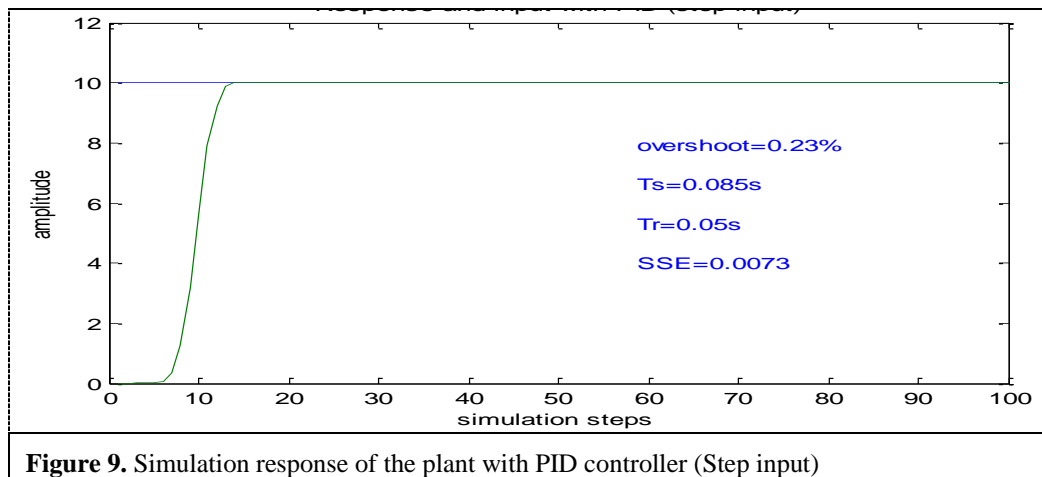


Figure 9. Simulation response of the plant with PID controller (Step input)

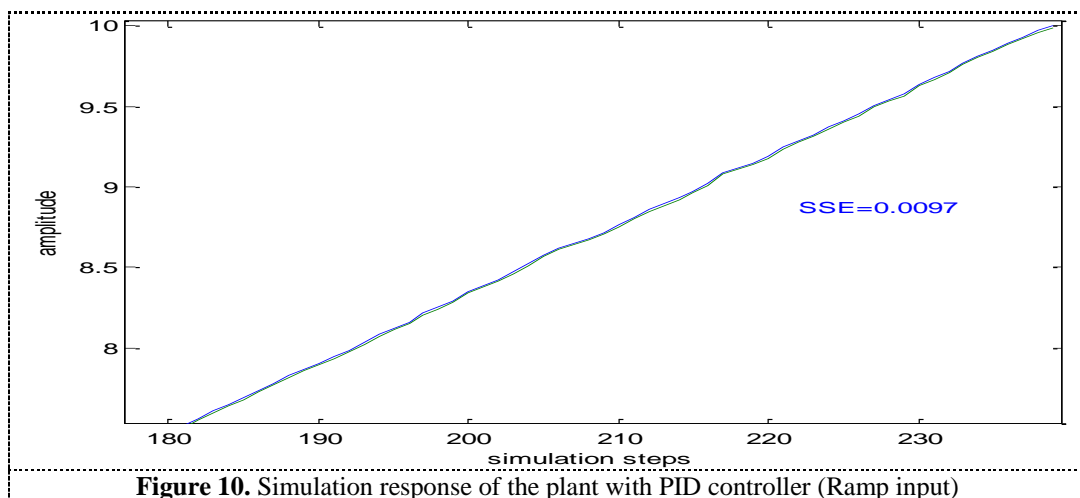


Figure 10. Simulation response of the plant with PID controller (Ramp input)

Table 2 and Table 3 show the overview of the simulation results for the step input and ramp input respectively.

Table 2: Overview of the Simulation Result for Step Input of 10 Degree

| Performance | PID Controller |
|--------------------|----------------|
| Overshoot % | 0.23 |
| Settling Time (s) | 0.085 |
| Rise Time(s) | 0.049 |
| Steady state error | 0.0073 |

Table 3: Overview of the Simulation Result for Ramp Input

| Performance | PID Controller |
|--------------------------|----------------|
| Steady state error (SSE) | 0.0097 |

5. Hardware Implementation

5.1. Experimental Set-Up and Results

The PID controller was implemented experimentally on one of the available motion control system shown in Figure 12, 13 and 14 .Thus the experimental set-up consists of rotary motion system driven by DC motor, Encoder position sensor and Simulink-based controller implementation with Winton compiler. The Simulink based set-up of the experiment for the PID controller is giving in Figure 11.

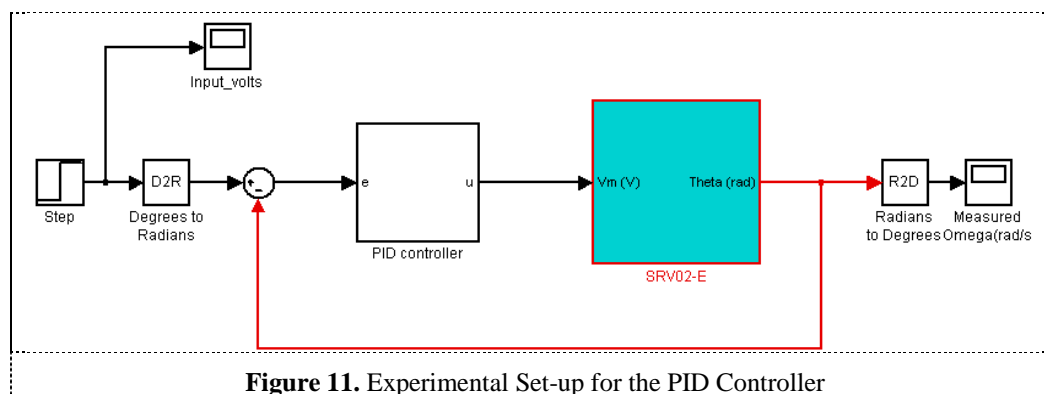


Figure 11. Experimental Set-up for the PID Controller

The experimental response of the plant with PID controller for step input and ramp input is shown in Figure 12 - Figure 15 respectively.

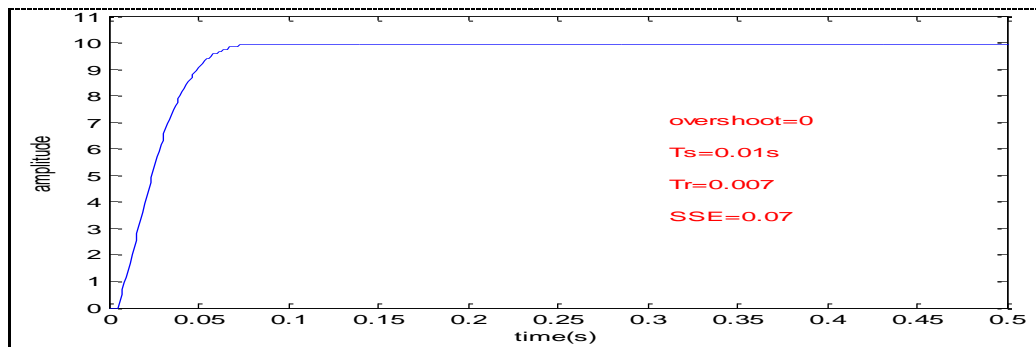


Figure 12. Response obtained by plant to a 10 degree step input with PID

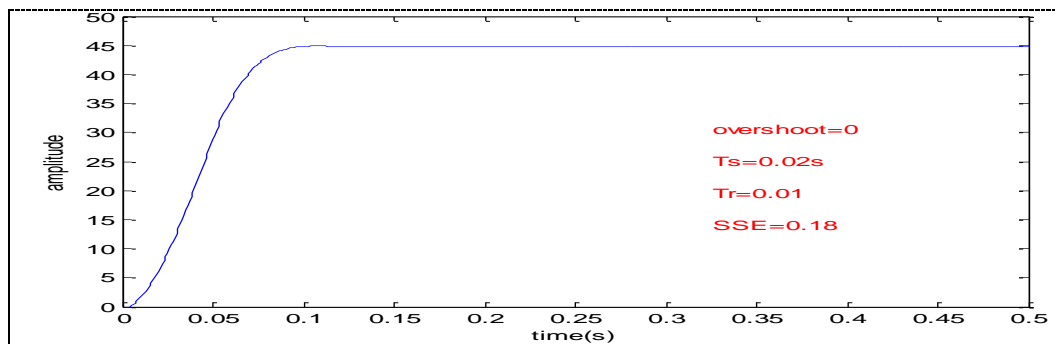


Figure 13. Response obtained by plant to a 45 degree step input with PID

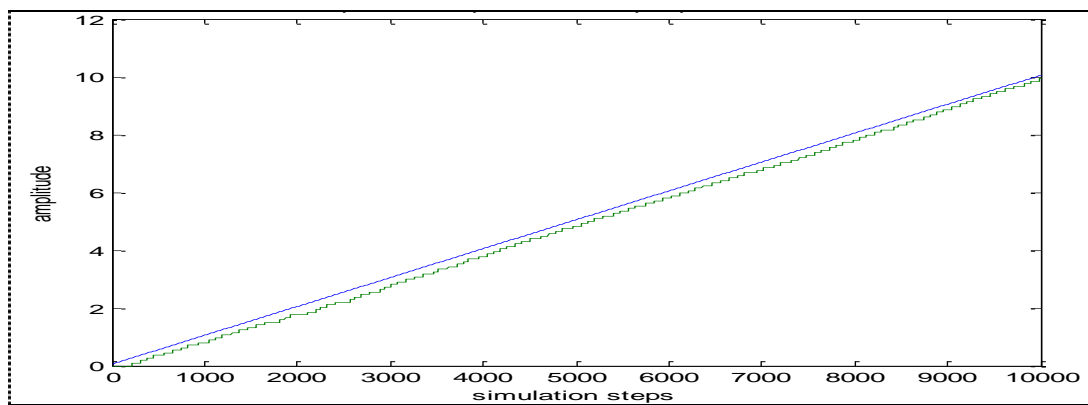


Figure 14. Response of the plant to a Ramp input with PID

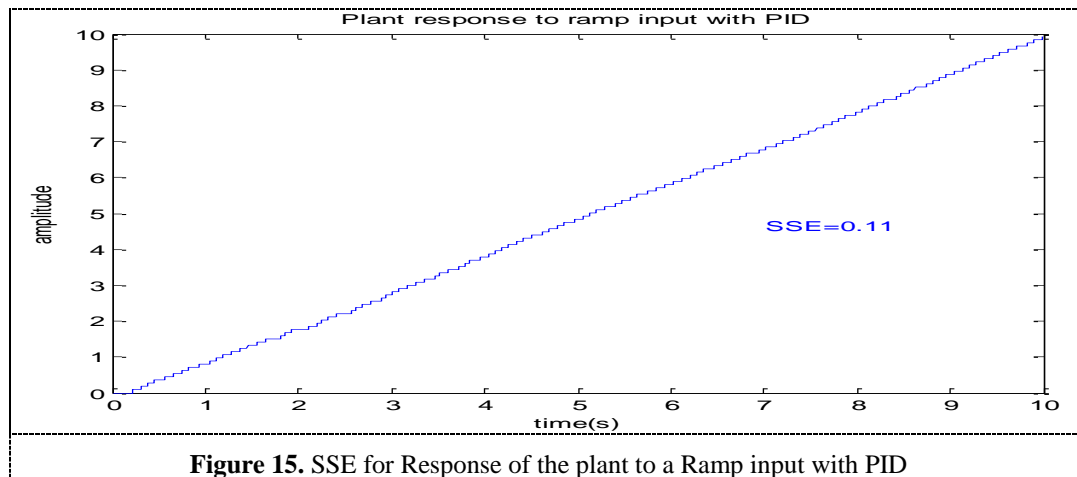


Figure 15. SSE for Response of the plant to a Ramp input with PID

5.2. Fine-Tuning the PID Controller

The controller gains were fine-tuned with the aid of root-locus plot to enhance the experimental response of the system and the resulting response obtained for the PID controller for 10 and 45 degree step input are shown in Figures 16 and 17 while an overview of the experimental results obtained for step and ramp input are shown in Tables 4 and 5.

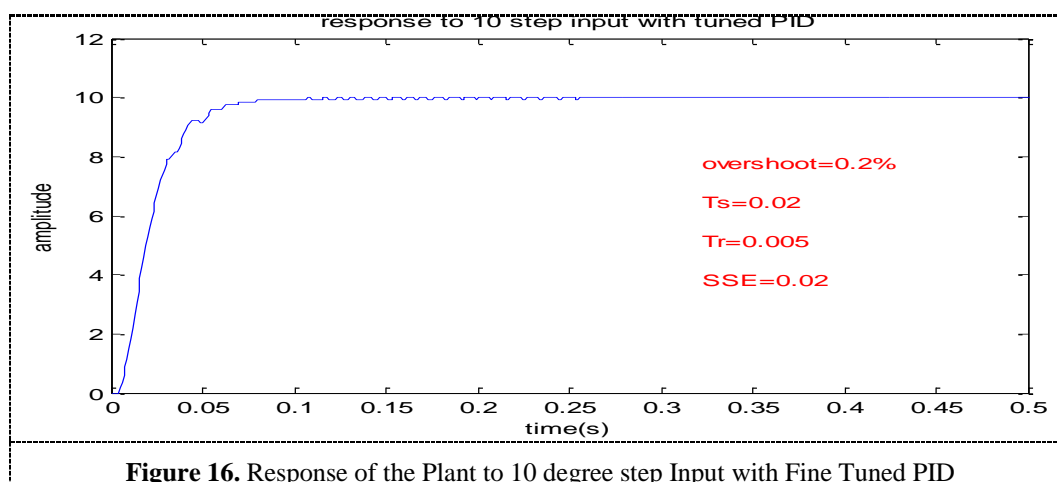


Figure 16. Response of the Plant to 10 degree step Input with Fine Tuned PID

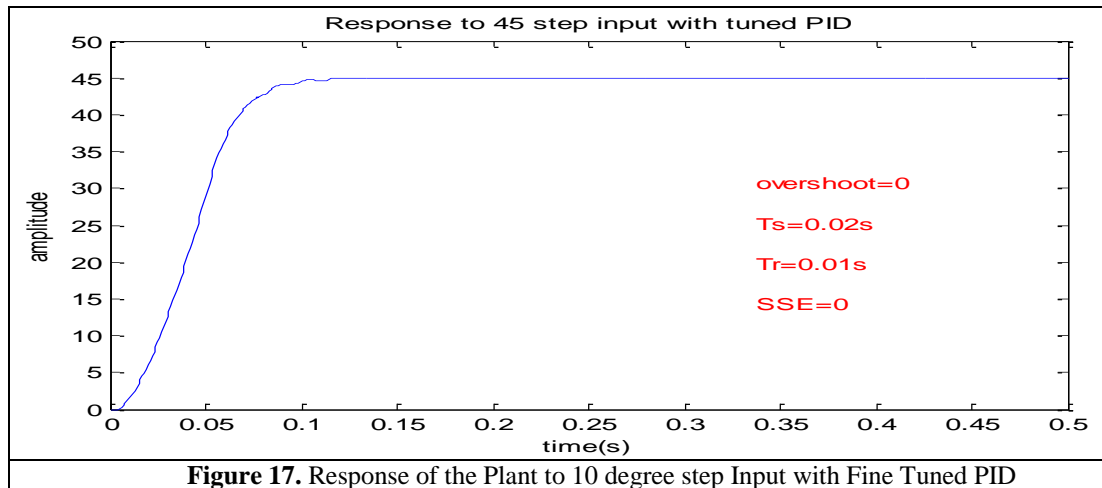


Figure 17. Response of the Plant to 10 degree step Input with Fine Tuned PID

Table 4: Overview of the Experimental Results with Step Input

| Performance | PID (10 Degree Step) | PID (45 Degree Step) | PID Tuned (10 Degree Step) | PID Tuned (45 Degree Step) |
|---------------------------|----------------------------|----------------------------|-------------------------------|-------------------------------|
| Overshoot (%) | 0 | 0 | 0.2 | 0 |
| Settling Time (s) | 0.01 | 0.02 | 0.02 | 0.02 |
| Rise Time (s) | 0.007 | 0.01 | 0.005 | 0.01 |
| Steady State Error | 0.07 | 0.18 | 0.02 | 0 |

Table 5: Overview of the Experimental Results with Ramp Input

| Performance | PID |
|--------------------|------|
| Steady State Error | 0.11 |

5.3. Validation of the Result

The experimental result was validated by comparing result obtained for Lead-lag controller implementation with motion rotary system in Ismail (2008). From the summary of the comparative analysis conducted on the two controllers as shown in Table 6 and Table 7, the simulation results obtained for both controllers met the desired system specifications with PID given a better response in terms of overshoot and steady state error. However Lead-Lag compensator is seen to perform better in respect of transient response of settling and rise time. There are marginal differences between the simulation performance of the designed controller and experimental performance. An over damped responses were recorded for both controllers in the experiment. These could be attributed to the fact that the mathematical model used to design the controllers did not capture the inherent dynamic characteristics of the system which includes the nonlinearity effect of such factor as frictions.

Table 6: Comparison between the PID Controller and Lead-Lag Controller With Step Input

| | PID CONTROLLER INPUT | | RESULT WITH STEP | | LEAD-LAG CONTROLLER (Isma'il, 2008) | |
|-------------------|---------------------------------|---------------------------------|---------------------------------------|----------------------------------|---|---|
| PERFORMANCE | PID (With 10 Degree Step Input) | PID (With 45 Degree Step Input) | PID Tuned (With 10 Degree Step Input) | PID Tuned (45 Degree Step Input) | Lead-Lag Controller (With 10 Degree Step Input) | Lead-Lag Controller (With 45 Degree Step Input) |
| Overshoot (%) | 0 | 0 | 0.2 | 0 | 0 | 0 |
| Settling Time (s) | 0.01 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 |
| Rise Time (s) | 0.007 | 0.01 | 0.005 | 0.01 | 0.003 | 0.01 |
| SSE | 0.07 | 0.18 | 0.02 | 0 | 0.24 | 0.18 |

Table 7: Comparison between the PID Controller and Lead-Lag Controller with Ramp Input

| Performance | Lead-Lag Controller (Isma'il, 2008) | PID Controller |
|--------------------|-------------------------------------|----------------|
| Steady state error | 0.13 | 0.11 |

6. Conclusion

The present investigation has demonstrated the design of a PID controller using the classical root-locus approach. The simulation as well as experimental investigation use of this approach in handling simple positioning control system to a reasonable degree of accuracy for relatively simple situation where the mathematical model of the system is available. From the experimental investigation, the PID controller performs better for both step and ramp inputs with settling time and overshoot less than 0.1s and 5% respectively.

References

- [1] Richard C D and Robert H B, 2001, Pren Hal Int'l
- [2] Katsuhiko O, 2002, Pear. Educ int'l
- [3] Quans Rot Motion Syst Lab Manual. 2007
- [4] Liu T H, Lee Y. C. Lee and Chang Y H 2004 IEEE Aero El Syst
- [5] Lin F J and Shen P H, 2006, IEEE Ind Electron M
- [6] Fuyu, D Xing'an C and Jiasheng C, 2005
- [7] Groovier M P, Fund of Mod Manuf Mater
- [8] Lim H, Seo, J W, Choi, C H, 2000
- [9] Park E C, Lim H and Choi C H, 2003 IEEE Contr Syst Mag
- [10] Gieras F and Piech Z J, 2005 CRC Pres
- [11] Chong K H, GCY and L Y, 2005, IEEE Contr Syst Mag