

# Analysis of Nonlinear Dynamics in Linear Compressors Driven by Linear Motors

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**Abstract.** The analysis of dynamic characteristics of the mechatronics system is of great significance for the linear motor design and control. Steady-state nonlinear response characteristics of a linear compressor are investigated theoretically based on the linearized and nonlinear models. First, the influence factors considering the nonlinear gas force load were analyzed. Then, a simple linearized model was set up to analyze the influence on the stroke and resonance frequency. Finally, the nonlinear model was set up to analyze the effects of piston mass, spring stiffness, driving force as an example of design parameter variation. The simulating results show that the stroke can be obtained by adjusting the excitation amplitude, frequency and other adjustments, the equilibrium position can be adjusted by adjusting the DC input, and to make the more efficient operation, the operating frequency must always equal to the resonance frequency.

## 1. Introduction

Domestic and industrial applications, such as compressors, valves, pumps, vibrators, and so on, involve linear reciprocating motion. Usually, such motion is achieved by using rotary motors and rotary-to-linear mechanical converters. However, the overall efficiency of the system is relatively low, due to the inherent low efficiency of induction motors and the mechanical friction which is associated with the converters. A reciprocating compressor driven by a linear motor<sup>[1]</sup> eliminates the side force on the cylinder wall caused by the crank shaft and therefore not only significantly reduces the frictional loss but also provides a simple means for modulating the refrigerator load according to the demand and results in additional energy saving. Linear oscillating motors are inherently more potential in energy efficiency than other types for light duty compressors. This is why research has focused on this topic for the past 30 years. The major concern in previous studies<sup>[2]</sup> was that system performance under stable operations and, system instability problems, which might arise in practice, were not well understood probably because of complexities in the dynamic modeling of the system.

Configuration of a linear compressor under study is simple: a circular cylinder fixed onto a stator is supported by ground springs and a piston fixed onto a mover is suspended by piston springs. Piston movement, however, is determined in a rather complicated way by interaction of mechanical parts with electromagnetic and thermodynamic subsystems. The gas force in compression chamber possesses inherently nonlinear characteristics with respect to the piston motion and, hence, its dynamic components are coupled to static components. Furthermore, motion of the piston is not mechanically constrained. Therefore, piston motion becomes uncontrollable under some operating conditions. Chen et al.(2007)<sup>[3]</sup> and Park et al.(2002)<sup>[4]</sup> studied the static and dynamics characteristics and the effect of operating parameters through the linearized gas force load, respectively. In the paper, steady-state



nonlinear response characteristics and the effect of operating parameters of a linear compressor were investigated considering the linearized and nonlinear gas force load.

### 2. Dynamic Modeling of Linear Compressor System

The mechanical system can be modeled as a mass-spring-damper<sup>[5-6]</sup>, as is shown in Fig. 1. The moving mass consists of the mass of the piston, the carrier, the link, the mover, and part of the leaf springs. The governing equation of motion is

$$m \frac{d^2x}{dt^2} + c \frac{dx}{dt} + K_s(x - x_0) = F_g - F_e \tag{1}$$

where  $x$  is the mover position,  $m$  is the total moving mass,  $c$  is the viscous damping coefficient that represents the frictional effect between the cylinder and piston,  $K_s$  is the stiffness of the leaf springs,  $x_0$  is the initial position where the spring force is zero,  $F_e$  is the motor force,  $F_D$  is the damping force, and  $F_g$  is the gas force.

The gas force is given by the following equation:

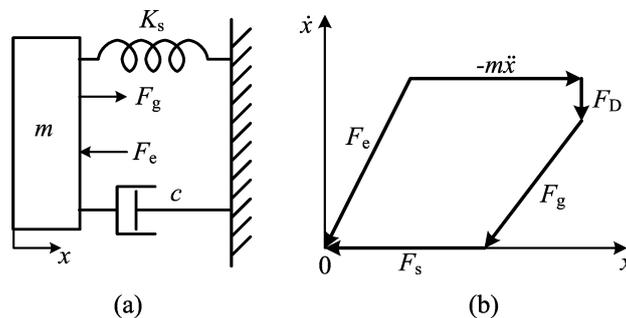
$$F_g(t) = [p(t) - P_s]A_p \tag{2}$$

$$s_0 = x_0 + \frac{(P_d - P_s)A_p}{2K_s} \tag{3}$$

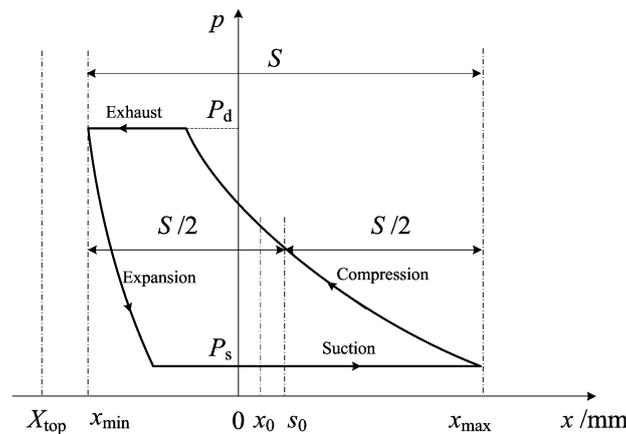
$$p(t) = \begin{cases} P_s & p < P_s & \text{Suction} \\ \left[ \frac{x_{\max} - X_{\text{top}}}{x(t) - X_{\text{top}}} \right]^q P_s & P_s \leq p \leq P_d & \text{Compression} \\ P_d & p > P_d & \text{Exhaust} \\ \left[ \frac{x_{\min} - X_{\text{top}}}{x(t) - X_{\text{top}}} \right]^q P_d & P_s \leq p \leq P_d & \text{Expansion} \end{cases} \tag{4}$$

where  $A_p$ ,  $P(t)$ , and  $P_s$  denote the cross sectional area of the piston, compression chamber pressure and suction pressure respectively,  $S_{\text{piston}}$  is the cross-sectional area of the piston,  $q$  is the polytropic coefficient,  $X_{\text{Top}}$  is the position of the cylinder top head. It is assumed that the gas behaves as an ideal gas in the compression cycle, that is, compression and expansion processes are described by polytropic processes as follows and shown in Fig. 2 and given as Eq. (4).

At the moment the mover's velocity is zero, the mover is at the maximum (or minimum) position, denoted  $x_{\max}$  (or  $x_{\min}$ ). The actual stroke is given by  $S = x_{\max} - x_{\min}$ . The equilibrium position  $s_0$  is given by  $(x_{\max} + x_{\min}) / 2$  and is given as Eq. (3), which is required to be controlled to zero.



**Figure 1.** Dynamic model and force vector diagram. (a) Dynamic model. (b) Force vector diagram.



**Figure 2.** The polytropic processes

### 3. Dynamics Analysis Based on the Linearized Model

As is known above,  $K_g$  is a nonlinear function of the piston stroke and is subjected to the operating conditions. Kornhauser studied the gas spring<sup>[7]</sup>, and regarded the mean spring stiffness as the equivalent stiffness, which is given by the following equation:

$$K_g = \frac{\beta q (P_d - P_s) A_p}{S} \tag{5}$$

where  $\beta$  is the stiffness coefficient of gas spring.

As long as the motion is harmonic, the energy loss of the mechanical system can be simulated by equivalent damping. The work on the gas, the heat exchange of the main cylinder and the loss of friction are considered in the form of energy dissipation. The first two items reflect the equivalent damping of the gas spring and can be estimated by the adiabatic work instead of the variable work. The indicated power of the compressor is given by the following equations:

$$W_g + W_h = \frac{q}{q-1} P_s [V_1 - (\frac{P_d}{P_s})^{\frac{1}{q}} V_s] [(\frac{P_d}{P_s})^{\frac{q-1}{q}} - 1] \tag{6}$$

$$c_g = \frac{W_g + W_h}{\pi \omega A^2} \tag{7}$$

where  $W_g$  and  $W_h$  is the work on the gas and the heat exchange of the main cylinder respectively,  $V_1$  is the maximum intake volume of cylinder,  $V_s$  is clearance volume at the top of a cylinder,  $A$  is the displacement amplitude of reciprocating piston,  $\omega$  is the angular frequency of reciprocating motion of piston,  $c_g$  is the equivalent damping coefficient.

Let  $c_{eq} = c + c_g$ ,  $K_{eq} = K_s + K_g$ , Eq. (1) is linearized as follows:

$$m \frac{d^2x}{dt^2} + c_{eq} \frac{dx}{dt} + K_{eq} (x(t) - x_0) = -F_e(t) \tag{8}$$

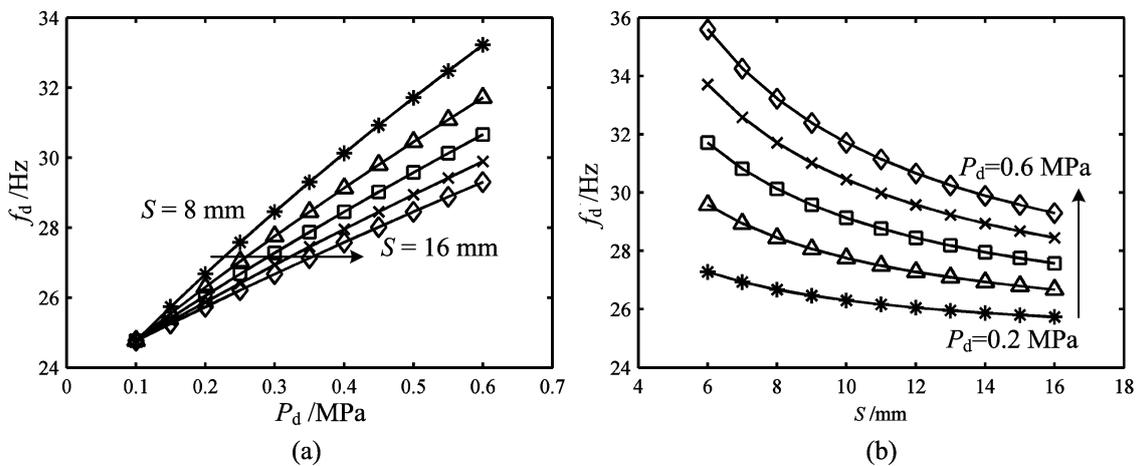
Let  $x_0 = 0$ ,  $\dot{x} = 0$ , the harmonic excitation force  $F_e(t) = F_{em} \sin(t)$ , the steady-state solution of Eq.(8) is given by the following equations:

$$x(t) = \frac{F_{em}}{K_{eq}} \frac{1}{\sqrt{(1 - \frac{\omega^2}{\omega_n^2})^2 + (\frac{2n_D}{\omega_n} \frac{\omega}{\omega_n})^2}} \sin(\omega t - \varphi) \tag{9}$$

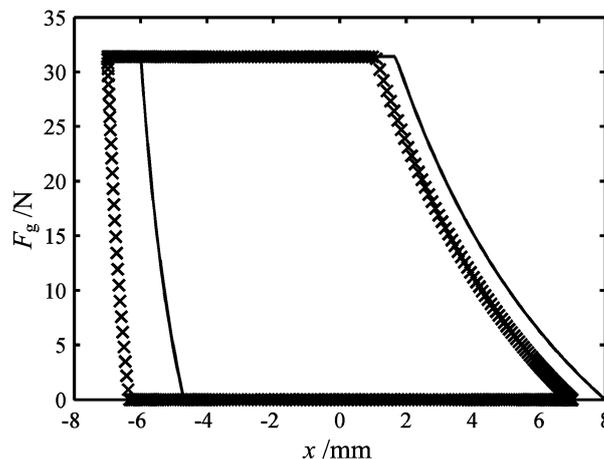
$$f_d = \frac{1}{2\pi} \sqrt{\frac{K_s}{m} + \frac{K_g}{m} - (\frac{c + c_g}{2m})^2} \tag{10}$$

where  $\varphi$  is the phase difference between displacement and exciting force, and  $f_d$  is the resonance frequency.

Resonant frequency is one of the important operating parameters<sup>[8]</sup>, for the function of piston diameter, compressor stroke, mass of the moving part, spring stiffness and exhaust pressure, and many other factors. Fig.3 shows the result that the resonance frequency varies with the exhaust pressure and stroke. As can be seen from the diagram, the resonance frequency increases with the increase of the exhaust pressure due to the damping of the gas. When the exhaust pressure is constant, the resonance frequency decreases with the increase of the stroke. Fig.4 shows the gas force with the same stroke and different equilibrium positions. The gas pressure at the initial equilibrium position, both sides of the piston and the mechanical spring stiffness changes will alter the equilibrium position. The greater the exhaust pressure and the smaller spring stiffness, then the equilibrium position deviation is bigger, and the clearance is greater. All that is resulting in decrease in emissions even without exhaust. The equilibrium position in the operation can be adjusted by adjusting the initial balance position of the installation.



**Figure 3.** The resonance frequency varies with the exhaust pressure and stroke. (a) The resonance frequency varies with the exhaust pressure. (b) The resonance frequency varies with stroke.



**Figure 4.** The gas force with the same stroke and different equilibrium positions

#### 4. Dynamics Analysis Based on the Nonlinear Model

For the resonance frequency, it is important to reasonably select the ratio of  $m$  and  $K_s$ . When  $K_s$  keeps unchanged,  $m$  changes the resonance frequency of the mechanical system to affect the overall performance of the linear compressor. The compressor with light weight moving mass has higher dynamic response and system bandwidth, thus can work at high frequency, output much power, and the motor efficiency will be higher. So  $m$  should be as small as possible. And the smaller the moving mass, the smaller the stiffness of the spring, and the less material to make spring. But spring is in high frequency, the structure and material of the spring, the dynamic structure and material support are put

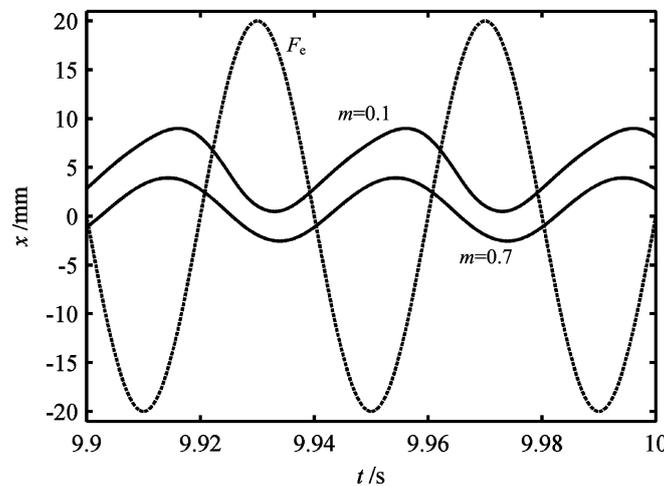
forward very high requirements. And the moving mass  $m$  is not as small as possible. While the inertia of motion is small, anti disturbance ability is weak, on the other hand, the matched mechanical spring stiffness is small, energy storage capacity weak equilibrium position also affected.

Based on the nolinear model, under the same excitation force, the mechanical position of a small mechanical system with a small mass movement is offset greatly, and the displacement waveform is non sinusoidal and asymmetrical, as is shown in Fig.5 and Fig.6.

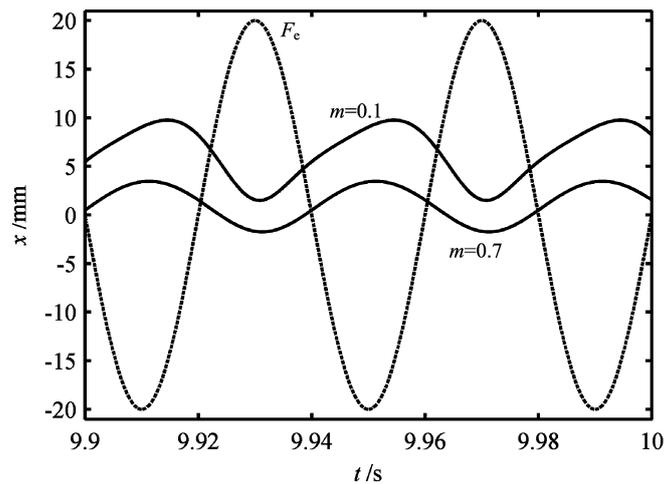
The power and the stroke vary with the frequency, and near the resonance frequency, the stroke and the input power of the exciting force is the biggest, but the balance position of the motion is shifted, as is shown in Fig.7.

When the frequency is unchanged, the stroke increases with the increase of the exciting force amplitude. When the exciting force amplitude is unchanged, the stroke at the resonant frequency is the largest. The resonance frequency decreases with the increase of the stroke, as is shown in Fig.8.

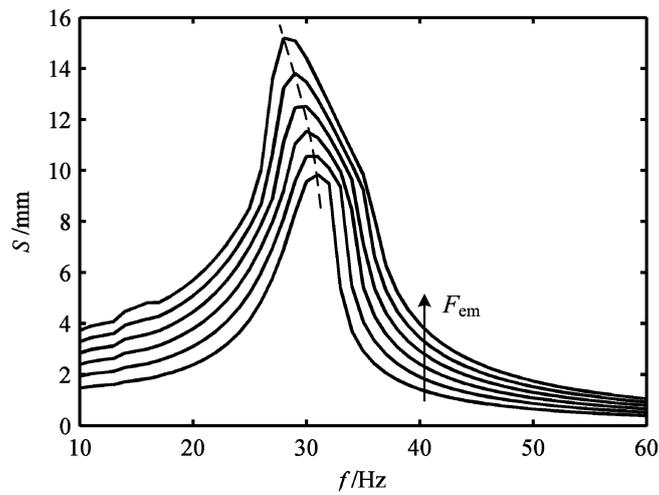
The model of an actual linear compressor is set up based on the nonlinear dynamics system, and the simulation result of displacement compared with the actual experimental result is shown in Fig. 9. The results are consistent, and the modeling method is proved to be correct.



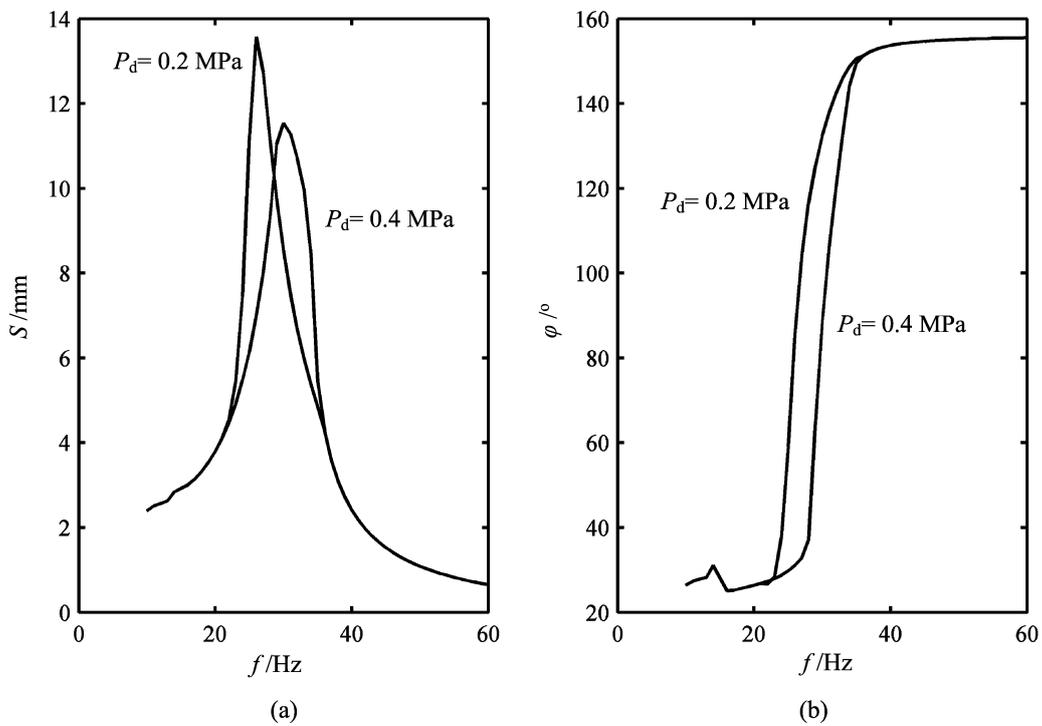
**Figure 5.** Displacement response at  $P_d=0.2$  MPa,  $m=0.1$ kg and  $m=0.7$  kg



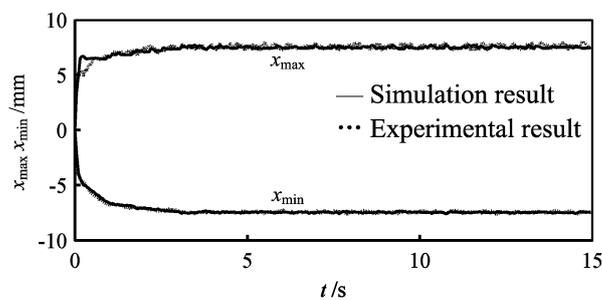
**Figure 6.** Displacement response at  $P_d=0.4$  MPa,  $m=0.1$  kg and  $m=0.7$  kg



**Figure 7.** The relationship of stroke and phase with various frequency ( $F_{em}=25N$ )



**Figure 8.** The relationship between stroke and frequency under different excitation amplitudes ( $P_d=0.4\text{ MPa}$ )



**Figure 9.** The simulation result and the experimental result

## 5. Summary

Steady-state nonlinear response characteristics of a linear compressor are investigated based on the linear and nonlinear model. The gas force can be expressed by the equivalent stiffness and equivalent damping, thus the nonlinear system is simplified as a linear model, which can be very convenient for the parameter analysis. Dynamics analysis based on the nonlinear model indicates that the moving mass is not the smaller the better; the mass-spring system can only absorb the excitation energy and have the maximum stroke when it works at the resonance frequency; the stroke increases with the increase of the excitation amplitude; the resonance frequency decreases with the increase of the stroke however increases with the increase of exhaust pressure.

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