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Numerical Flexural Strength Analysis of Thermally Stressed Delaminated Composite Structure under Sinusoidal Loading

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Abstract. In this article, we investigate the thermomechanical deflection characteristics of the debonded composite plate structure using an isoparametric type of higher-order finite element model. The current formulation is derived using higher-order kinematic theory and the displacement variables described as constant along the thickness direction whereas varying nonlinearly for the in-plane directions. The present mid-plane kinematic model mainly obsoletes the use of shear correction factor as in the other lower-order theories. The separation between the adjacent layers is modeled via the sub-laminate technique and the intermittent continuity conditions imposed to avoid the mathematical ill conditions. The governing equation of equilibrium of the damaged plate structure under the combined state of loading are obtained using the variational principle and solved numerically to compute the deflection values. Further, the convergence test has been performed by refining the numbers of elements and validated through comparing the present results with available published values. The usefulness of the proposed formulation has been discussed by solving the different kind of numerical examples including the size, location and position of delamination.

1. Introduction

With increasing application of the composite material in many weight-sensitive industries, the prediction of the structural behavior close to the actual one is very challenging and become a critical issue nowadays. Because when they exposed to the real-life situation, experience different direct (externally applied mechanical) and indirect (environmental loading, i.e., unlike temperature) loading. Additionally, the bonding between the laminas is not perfect as considered in the theoretical analysis, i.e., debonding always present due to different reasons (presence of foreign element during fabrication, matrix cracking, geometrical discontinuity, etc.). In view of the same (environmental exposer and delamination) the stiffness of the structure made using composite material reduces considerably. Therefore, the consideration of such factors or real-life situation that highly affects the structural integrity and the final performance, while prediction of structural responses is highly important.

Many works, presented in the past have considered such condition in their solution approach, few very important of them are discussed in following lines. The analytical solution approach used to analyze the bending behavior and strain energy release rate of the orthotropic debonded layered composite structure are presented by Davidson et al. [1] and Hamed et al. [2]. Further, Parhi et al. [3] used the first-order shear deformation theory (FSDT) to model the debonded composite plate structure under low-velocity impact and reported the bending as well as first-ply failure behavior. Bruno et al. [4] applied the fracture and contact mechanics approach with that of the FSDT and investigated the mixed-mode delamination. Similarly, the layerwise theory and the FSDT is used by Nanda [5] to obtain the transverse deformation

behavior of the centrally located debonded shell panel. Szekre´nyes [6] developed an exact kinematic condition so-called system of exact kinematic conditions (SEKC) for the delamination modeling in layered composite structure and applied that in FSDT model of the plate structure.
Different analytical [7-8], as well as numerical method [9-10] based on different mid-plane kinematic (lower and higher-order shear deformation theory (HSDT)), are also presented to investigate the central deflection responses of the composite structure subjected to thermo-mechanical loading. Further, the finite element (FE) model derived using four variable kinematics is presented by the Tounsi and his co-author [11-12] to investigate the effect of thermo-mechanical loading on the central deflection behavior of the functionally graded plate. A simulation model of the layered composite plate under elevated thermal environment is developed by Das et al. [13] in FE based software ANSYS and analyze the stress behavior of the same. Han et al. [14] also investigated the thermomechanical behavior of the composite plate using a mathematical model developed based on higher-order zig-zag theory.
It is clear from the review of the literature that many solution methodologies to obtain the structural responses of the layered structure with and without considering the effect of debonding and environmental condition have already been reported in open literature. It is also clear that methodology based on analytical approach and FSDT are major in number and hardly a work is reported using HSDT. In order to fulfill such gap in the literature, current work aim's to develop a FE model of the delaminated composite structure subjected to mechanical (sinusoidally distributed) and thermal (uniform unlike temperature) load using HSDT mid-plane kinematics to analyze the structural response (Bending) of the same. The square-shaped delamination is assumed at the center of the laminate using the sub-laminate method. Effect of the environmental condition is incorporate via constitutive relation. The model's accuracy is checked via comparison study. Further, the parametric study has been performed to show the influence of the input parameters (geometrical and the material) on bending behavior of the debonded structure.

2. Mathematical Formulations

Figure 1 shows a layered composite plate structure consisting a delamination at centre mid-plane whose geometrical parameter is defined as length, *a*; breadth, *b* and height, *h* in ξ_1, ξ_2 and ζ direction, respectively. The laminate is made of *n* number of equally thick laminae oriented at an angle θ .

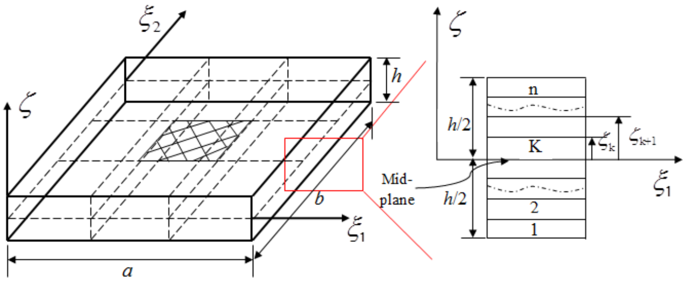

Figure 1. Geometry of the layered composite plate structure

The higher-order displacement kinematics (nine degrees of freedom) as presented in Reddy and Liu [15] is used for the current modeling purpose and shown as:

$$\left.\begin{aligned}\bar{u}(\xi_1,\xi_2,\zeta) &= u_0 + \zeta\phi_1 + \zeta^2\psi_1 + \zeta^3\theta_1 \\ \bar{v}(\xi_1,\xi_2,\zeta) &= v_0 + \zeta\phi_2 + \zeta^2\psi_2 + \zeta^3\theta_2 \\ \bar{w}(\xi_1,\xi_2,\zeta) &= w_0\end{aligned}\right\} \quad (1)$$

where $\bar{u}, \bar{v}$ and $\bar{w}$ represents the displacement of any point along ξ_1, ξ_2 and ζ directions, respectively. The u_0, v_0 and w_0 denotes the mid-plane displacement whereas ϕ_1 and ϕ_2 are the rotation at $(\zeta = 0)$ of the normal to mid-plane about ξ_2 and ξ_1, respectively. Rest of the terms $\phi_3, \psi_1, \psi_2, \theta_1$ and θ_2 are the higher order term of Taylor series expansion.
The laminate constitutive relations in terms of force and moments can be expressed as [16, 17]:

$$\{F_{ij}\} = [D]\{\{\varepsilon_{ij}\} - \{\alpha_{ij}\}\Delta T\} \quad (2)$$

where, $\{F_{ij}\},[D]$and$\{\varepsilon_{ij}\}$ are denoted the forces and moment vector, elastic constant matrix, mid-plane strain and curvature matrices, respectively. Further, 'ΔT' is the temperature gradient and $\{\alpha_{ij}\}=\{\alpha_{\xi_1\xi_1}\ \alpha_{\xi_2\xi_2}\ 0\ 0\ \alpha_{\xi_1\xi_2}\}$'' coefficient of thermal expansion along the principal material directions.

The $\{F_{ij}\}$ and $[D]$ matrix can be explored as:

$$\{F_{ij}\}=\left[N_{\xi_1\xi_1}\ N_{\xi_2\xi_2}\ N_{\xi_2\zeta}\ N_{\xi_1\zeta}\ N_{\xi_1\xi_2}\ M_{\xi_1\xi_1}\ M_{\xi_2\xi_2}\ M_{\xi_2\zeta}\ M_{\xi_1\zeta}\ M_{\xi_1\xi_2}\right]^T \tag{3}$$

$$[D]=\sum_{k=1}^{n}\int_{\zeta_{k-1}}^{\zeta_k}\left(\bar{Q}_{ij}\right)_k\left(1,\zeta,\zeta^2...\zeta^6\right)d\zeta \tag{4}$$

$\left(\bar{Q}_{ij}\right)_k$ in eqation (4) is the transformed reduced stiffness matrix whose detail can be seen in [18]. Similarly, the necessary strain of Equation (2) is elaborated as following and detail can be seen in Mahapatra and Panda [19]:

$$\{\varepsilon_{ij}\}=\left[\varepsilon_{0\xi_1\xi_1}\ \varepsilon_{0\xi_2\xi_2}\ \gamma_{0\xi_2\zeta}\ \gamma_{0\xi_1\zeta}\ \gamma_{0\xi_1\xi_2} K_{\xi_1\xi_1}\ K_{\xi_2\xi_2}\ K_{\xi_2\zeta}\ K_{\xi_1\zeta}\ K_{\xi_1\xi_2}\right]^T \tag{5}$$

The displacement vector using FEM concept (Cook et al. [20]) can be represented as:

$$\{\delta\}=\sum_{i=1}^{9}[N_i]\{\delta_i\} \tag{6}$$

where, $[N_i]$and $\{\delta_i\}$ are shape functions and the nodal displacement vector, respectively. The nodal displacement elaborated as:

Now, the mid-plane strain vector is expressed in terms of nodal displacement vector:

$$\{\bar{\varepsilon}\}=[B]\{\delta_i\} \tag{7}$$

where,$[B]$matrix which establish a relation between strain and displacement.

The strain energy and the total work done can be presented as follows:

$$U=\frac{1}{2}\iint\left\{\sum_{k=1}^{n}\int_{\zeta_{k-1}}^{\zeta_k}\{\sigma_{ij}\}\{\varepsilon_{ij}\}d\zeta\right\}d\xi_1 d\xi_2 \tag{8}$$

$$W=\int_A\{\delta\}^T\{q^e\}dA \tag{9}$$

where, $\{q^e\}=\{q^e_m\}+\{q^e_{th}\}$ the combined form of thermal and mechanical load vector.

The integral form of the elemental stiffness matrix can be represented as:

$$[k]=\int_A\left(\sum_{k=1}^{n}\int_{\zeta_{k-1}}^{\zeta_k}[B]^T[D][B]d\zeta\right)dA \tag{10}$$

The mathematical modelling steps for the delamination i.e. displacement kinematics, stiffness matrix and intermittent continuity condition can be seen in Hirwani et al. [21].

The governing equation of the deflection analysis obtained by minimizing the total energy functional via variational principle and presented as:

$$\partial\prod=\partial U-\partial W=0 \tag{11}$$

where, ∂ represents a variational symbol and $\prod$ is used to express the total potential energy functional.

Now, equation (8) and (9) is substituted into equation (11) to get the final equation and depicted as:

$$[K]\{\delta\}=\{q\} \tag{12}$$

The equation (12) is solved by applying different set of boundary condition as in Hirwani et al. [21]:

3. Result and discussion

Now, the above-discussed mathematical formulation is further converted into a MATLAB code using MATLAB-12. Four types of square delamination located at the center mid-plane of the laminate namely Dtype-1, Dtype-2, Dtype-3 and Dtype-4 are considered in this work. Dtype-1 is the healthy laminate, Dtype-2, Dtype-3 and Dtype-4 are the laminate having delamination with a side length of *a*/4, *a*/2 and 3*a*/4, respectively. The material property is used is given below. The external mechanical load is sinusoidally distributed on $\left(\xi_{1,}\ \xi_2\right)$plane as in equation below and the thermal load is considered uniform through the thickness as well as on $\left(\xi_{1,}\ \xi_2\right)$ plane. The nondimensional form of the external load and final central deflection responses are presented in equations below. Now, the convergence and accuracy of the developed code have been checked and discussed in the next section.

$$q_m = q_0 \sin\left(\frac{\pi\xi_1}{a}\right)\sin\left(\frac{\pi\xi_2}{b}\right); \qquad Q = q_0 * E_{\xi_2} * (a/h)^4; \quad W = w/h$$

$$E_{\xi_1} = 181\ GPa; E_{\xi_2} = 10.3 GPa; G_{\xi_1\xi_2} = G_{\xi_1\xi_3} = 7.17 GPa; G_{\xi_2\xi_3} = 6.21 GPa; \upsilon_{\xi_1\xi_2} = \upsilon_{\xi_1\xi_3} = \upsilon_{\xi_2\xi_3} = 0.25$$

3.1 Convergence and comparison study

In order to check the consistency of the current model, the nondimensional central deflection responses of the simply supported, symmetric laminated composite plate (0º/90º/90º/0º) subjected to thermomechanical load is obtained. The responses are obtained for five different amplitude of mechanical load (Q=30, 60, 90, 120 and 150) and eight different element sizes. The geometrical parameter (a/h) and temperature gradient (ΔT) is taken as 60 and 50, respectively. The responses are plotted and presented in Figure 2. The figure indicates that the responses are showing very good convergence for different mesh sizes. Based on this analysis a mesh size of (8×8) is utilized for further investigation.

For the validation study, a numerical example of cylindrical shell panel has been chosen because of unavailability of the result of delaminated plate bending (best of author knowledge) as the present model is so general that it can easily be converted into a plate to shell. The eight-layer simply supported cross-ply cylindrical shell panel (laminated and Dtype-2 delaminated) subjected to uniformly distributed mechanical load has been analyzed. The nondimensional deflection values are evaluated via current model and compare with the values of Nanda [5] and tabulated in Table 1. All the input parameter are taken as same as given in the corresponding reference. The table indicates that the reference (layerwise FE model) underpredict the responses. It is due to the lower DOF, i.e., 8 (eight) of the reference model compare to the present model, i.e., 9 DOF.

Table 1. Validation study of nondimensional central deflection responses of the simply supported laminated/delaminated cylindrical shell panel subjected to uniformly distributed load

(ξ_1 / a)	Laminate		Delaminate (a/4)	
	Present	Nanda [5]	Present	Nanda [5]
0	0	0	0	0
0.125	0.2322	0.21934	0.26	0.22607
0.25	0.4201	0.40915	0.4668	0.41656
0.375	0.5395	0.52964	0.5952	0.54027
0.5	0.5803	0.57273	0.6413	0.58611
0.625	0.5398	0.53105	0.5946	0.54193
0.75	0.4205	0.40928	0.4578	0.41804
0.875	0.2324	0.22268	0.2508	0.22762
1	0	0	0	0

3.2 Numerical illustrations

3.2.1 Effect of centrally located delamination size

The influence of the size of debonding on nondimensional central deflection response of laminated composite plate is analyzed in this illustration. For the illustration, a simply supported antisymmetric cross-ply (0°/90°/0°/90°) laminate with different type of delamination (Dtype-1, Dtype-2, Dtype-3 and Dtype-4) under thermo-mechanical (Q=50, 100, 150, 200, 250 and ΔT=80ºC) load is considered. The nondimensional response is evaluated using currently developed model and presented in Figure 3. The deflection responses are lower for Dtype-1 and following the increasing trend with Dtype-2, Dtype-3 and Dtype-4. This indicates that with the increase in debonding sizes the overall stiffness of the laminate reduces considerably resulting the deflection response increases.

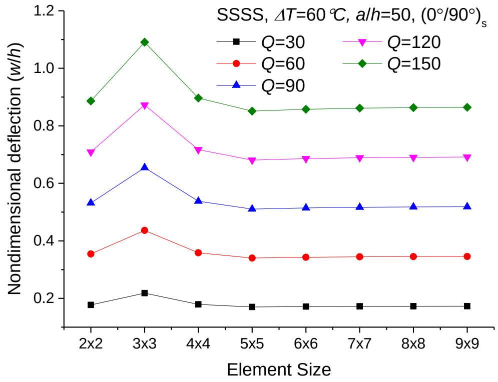

Figure 2. Convergence behavior of layered composite plate

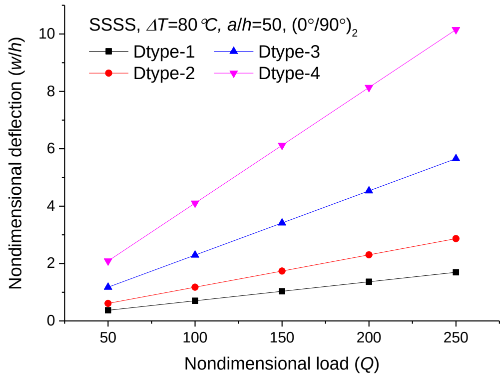

Figure 3. Nondimensional deflection responses of the antisymmetric delaminated composite plate

3.2.2 Influence of Young's modulus ratio

A clamped symmetric (0°/90°/90°/0°) laminate with delamination (Dtype-2) subjected to the combined effect of thermo-mechanical loading is analyzed for central deflection responses. The responses are obtained for five Young's modulus ratio (E_{ξ_1} / E_{ξ_1}) and five mechanical loading amplitude (Q=100, 200, 300, 400 and 500) and presented in Figure 4. The nondimensional response decreases with increase in Young's modulus ratio. In the current example, the transverse modulus is kept constant. Therefore longitudinal modulus increases with increase in Young's modulus ratio. The increase in longitudinal Young's modulus increases the overall stiffness and reduces the deflection responses.

3.2.3 Influence of aspect ratio

This example investigates the influence of aspect ratio on bending characteristic of the delaminated composite plate structure. For the investigation, cross-ply (0º/90º/90º/0º) delaminated (Dtype-2) composite plate (b/h=50) with simply supported edges under external mechanical and elevated temperature loading is considered. The nondimensional responses is examined for six aspect ratio (a/b=1, 1.4, 1.8, 2.2, 2.6 and 3) and five different load (Q=40, 80, 120, 160 and 200). The obtained responses are plotted and shown in Figure 5. It is evident from the figure that the nondimensional responses are increasing with the increase in (a/b) ratio. It is due to the reduction of the overall stiffness of the laminate with increasing the (a/b) ratio as b is kept constant and a is changing.

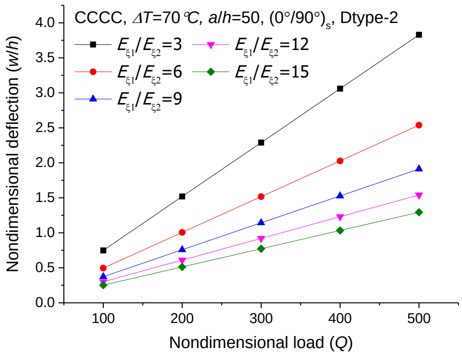

Figure 4. Effect of Young's modulus ratio on deflection responses of delaminated composite plate

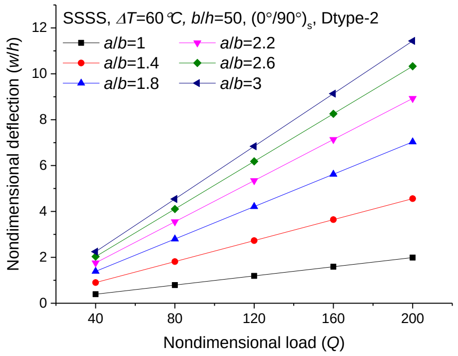

Figure 5. Effect of modulus ratio on deflection responses of delaminated composite plate

3.2.4 Influence of end conditions

To show the effect of boundary condition, eight-layer antisymmetric cross-ply $(0^o/90^o/0^o/90^o)_2$ delaminated (Dtype-3) composite plate structure is analyzed and central deflection responses are obtained for different boundary condition (CCCC, SCSC, CFCF, SSSS, CFFF). CCCC, SCSC, CFCF, SSSS and CFFF stand for all side clamped, two sides clamped two sides simply supported, two sides clamped two sides free and one side clamped and other three free, respectively. The plate geometry is considered as (a/h=70) under mechanical and thermal load (ΔT=70ºC). The computed responses are shown in Table 2. The table shows the nondimensional values is lowest for the CCCC and increasing with SCSC, CFCF, SSSS and CFFF. The trend in response is due to the increase in constrants degrees of freedom with CCCC, SCSC, CFCF, SSSS and CFFF.

Table 2. Nondimensional central deflection responses of the eight-layer antisymmetric cross-ply delaminated plate.

Load (Q)	Boundary condition				
	CCCC	SCSC	CFCF	SSSS	CFFF
40	0.1074	0.1527	0.1578	0.7052	2.5515
80	0.233	0.3364	0.3453	1.4123	3.697
120	0.3586	0.52	0.5327	2.1193	4.8426
160	0.4842	0.7037	0.7202	2.8264	5.9881
200	0.6098	0.8873	0.9076	3.5334	7.1337

4. Conclusion

The deformation behavior of the laminated and delaminated composite plate structure subjected to sinusoidal mechanical and uniform thermal loading is investigated in the present investigation. The mathematical model of the plate structure is developed using higher-order mid-plane kinematic. For the debonding modeling, a sub-laminate method is employed and continuity between the nodes of the laminated and delaminated boundary is established using intermittent continuity condition. A nine noded element with nine 81 DOF per element is used to discretize the entire plate structure. The final bending equation is obtained from variational principle and solved via FE approach. The necessary model’s consistency and accuracy have been checked via solving numerical examples. Finally, few parametric studies have been performed and based on that some conclusion has been drawn. The presence of delamination reduces the overall stiffness and as its size increases, the overall stiffness decreases. The deflection response increases with the increase in aspect ratio and reduces with the increase in Young's modulus and increasing constraint DOF.

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