

## Split delivery vehicle routing problem with time windows: a case study

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**Abstract.** This paper aims to implement an extension of VRP so called split delivery vehicle routing problem (SDVRP) with time windows in a case study involving pickups and deliveries of workers from several points of origin and several destinations. Each origin represents a bus stop and the destination represents either site or office location. An integer linear programming of the SDVRP problem is presented. The solution was generated using three stages of defining the starting points, assigning busses, and solving the SDVRP with time windows using an exact method. Although the overall computational time was relatively lengthy, the results indicated that the produced solution was better than the existing routing and scheduling that the firm used. The produced solution was also capable of reducing fuel cost by 9% that was obtained from shorter total distance travelled by the shuttle buses.

### 1. Introduction

A vehicle routing problem is a combinatorial integer programming problem that search for an optimal set of routes for a set of vehicles that move from a depot to one or more destinations to make a delivery to a set of given customers. Typical objective functions may include minimizing costs, fulfilling demands, maximizing profits, and maximizing utilities. The classical model that was first introduced in 1959 [1] has been extended to many different types such as: vehicle routing problem with time windows (VRPTW), vehicle routing problem with deliveries and pickups (VRPDP), split delivery vehicle routing problem (SDVRP), and capacitated vehicle routing problem (CVRP). More VRP extensions and solving methods were studied for both theoretical and practical purposes. As more manufacturers and distributors realized that transportation expense is one significant factor contributes to the cost of goods sold, the application of computer-generated solutions increased significantly. The transportation cost is especially more significant for a developed country like Indonesia where logistics sector like road and freight transport and distribution services underperformed [2][3]. Although VRP were mostly applied in product distribution, some were also applicable in other cases such as school bus [4][5], worker shuttle bus, other public transport [6], and marine border security patrol.

A study has revealed that computer-generated solution may reduce transportation costs by 5% to 20% [7]. With the advantages a VRP possibly offers, this paper aims to implement an extension of VRP for a case study in a coal-mining firm. In the case study, the firm provided shuttle buses for the workers that picked them up from several locations and drop them off at several destinations. However, the current approach the firm used has made the cost of transporting workers soaring. The transportation cost included fuel cost, maintenance cost, and driver cost. Thus, a better routing and scheduling solution



were required. The application of VRP, specifically SDVRP, in this case study was expected to produce a better routing and scheduling decisions for worker shuttle buses. The SDVRP is one of VRP extensions that drawn numerous attentions. It is applicable for many problems where customers or points can be visited more than once. It was introduced by Dror and Trudeau [8][9] and became widely studied later [10][11][12][13].

This paper presents the implementation of SDVRP and an exact method to generate a solution for the selected case study. The focus of the paper is on the construction of the problem formula and its implementation in the case study. No further analysis has been made in the performance of method used or comparison with any other methods. The remainder of the paper is organized as follows. The next section provides a description of the case study to be solved. It is followed by a mathematical formulation of the SDVRP, the computational procedure, results and discussion, and then concluding remarks.

## 2. Problem Description

The case study takes place in a coal-mining firm. The firm business process includes exploration, production, marketing, and distributing the products worldwide. The first two processes involve the use of heavy vehicles. Thus, to minimize the risk of accident in the work place and to provide a safer working environment, the firm decided to establish an access restriction for several offices and sites. This includes restricting any private vehicles use by the workers, except for the very important persons. As a consequence, the firm should provide official vehicles for the management and shuttle buses for all workers. When this study was performed, the firm was employing 49 shuttle buses to transport 1726 workers from 7 pickup points to 12 destinations – sites or offices – with different working hours. The shuttle bus schedule planning was done manually by considering only the capacity of the bus and the start of working hours. The shuttle buses were available in different capacity sizes; the small ones can take up to 29 persons, while the larger buses varied in sizes, ranged from up to 42 persons to 47 persons. More detailed information about the number of available buses and the capacity are described in Table 1. Working schedule at destination points, workers pickups and delivery points, as well as the number of workers to be picked up at each point are shown in Table 2.

**Table 1.** Capacity and number of busses available.

Max capacity	29	42	43	44	45	46	47
Number of buses	10	2	3	3	21	5	5

**Table 2.** Schedule (time windows) and numbers of workers at each points.

Destination (sites/offices)	Work starts at	Number of workers waiting at pick up points						
		1	2	3	4	5	6	7
1	7.00 am	3	0	1	8	2	38	2
2	7.00 am	18	7	0	6	1	36	0
3	7.00 am	42	0	23	0	0	0	25
4	6.00 am	7	14	3	5	5	9	3
5	6.00 am	7	9	1	9	2	21	3
6	6.00 am	34	35	16	92	16	73	8
7	7.00 am	56	49	56	11	5	44	16
8	7.00 am	42	56	25	31	11	21	24
9	6.00 am	31	27	10	26	17	30	8
10	7.00 am	39	16	25	0	4	12	5
11	6.30 am	54	24	63	0	11	0	34
12	6.00 am	26	34	15	36	12	43	11

## 3. Solving Procedure

While trying to solve the problem using exact method, we completely understand the consequence that it may take very long time to find the solution. Hence, we set up initial conditions that enable the exact method to work faster. The solving procedure was divided into three steps. The first step involved a set covering method to find the best possible points from where buses shall depart. This allowed the buses to start from the selected points instead of any points in the set. These selected points were then called as starting points. The second step is to assign each bus to a specific starting point that was selected in the first step. This made the SDVRP to be simpler to solve. And finally the last step aimed to find the routes that minimize the total distance travelled as formulated in Section 3. Formulation of set covering and bus assignment are described as follows.

### 3.1. Set Covering

The set covering aims to find a minimum number of starting points that covers all points on the given set. In this case, let  $L$  be the set of points with a total number of 7. The formulation can be described as follows.

#### Indexed Sets:

$i = \{1, 2, \dots, 7\}$ ; point index

$j = \{1, 2, \dots, 7\}$ ; point index

#### Parameters:

$n$ : the number of points

$C_{ij}$ : the value is 1 if point  $i$  is can covers point  $j$

#### Decision Variables:

$x_i$ : the value is one if point  $i$  is chosen as starting point

#### Objective:

$$\text{Minimize } Z = \sum_{i \in L} x_i \quad (1)$$

#### Constraints:

$$\sum_{j \in L} C_{ij} x_j \geq 1 \quad \forall i \in L \quad (2)$$

$$\sum_{i \in L} x_i \leq 3 \quad (3)$$

$$x_i \in \{1, 0\} \quad \forall i \in N \quad (4)$$

The objective is represented by (1), which is to minimize the total number of starting point. Constraint (2) ensures the selected points can cover all other points in the set. Constraint (3) was added because the firm would like to limit the number of selected starting points up to 3. Constraint (4) sets the decision variables to have binary value. The input value for constraint (4) is given in Table 3. A value of 1 is given for a point (in row) that may cover another point (in column). This value is given based on two considerations; the first one is that they are close in distance, and the second is that they are in the same direction given the last stops are the drop-off points.

**Table 3.** Set covering input matrix.

	1	2	3	4	5	6	7
1	1	0	0	0	0	0	0
2	1	1	0	0	0	0	0
3	1	1	1	0	0	0	0
4	1	1	1	1	0	0	0
5	1	1	1	1	1	1	1
6	0	0	0	0	0	1	1
7	0	0	0	0	0	0	1

### 3.2. Bus Assignment

For the bus assignment, let  $P$  be the set of starting points and  $M$  is a set of vehicles. The formulation can be explained as follows.

#### Indexed Sets:

$i = \{1, 2, \dots, p\}$ ; starting point index

$k = \{1, 2, \dots, m\}$ ; vehicle index

#### Parameters:

$p$ : the number of stopping points

$m$ : the number of vehicles

$Q^k$ : the capacity of vehicle  $k$

$d_i$ : the number of workers at stopping point  $i$

$A^k$ : the available number of vehicle  $k$

#### Decision Variables:

$x_i^k$ : the number of vehicles allocated in starting point  $i$

#### Objective:

$$\text{Minimize } Z = \sum_{k \in M} \sum_{i \in P} x_i^k \quad (5)$$

#### Constraints:

$$\sum_{k \in M} x_i^k Q^k \geq d_i \quad \forall i \in P \quad (6)$$

$$\sum_{i \in P} x_i^k \leq A^k \quad \forall k \in M \quad (7)$$

$$x_i^k \in \text{Integer} \quad \forall k \in M; i \in P \quad (8)$$

The objective of the bus assignment is to minimize the total vehicles assigned as represented by (5). Constraint (6) guarantees that the total capacity at each point is greater than the number of workers to be transported. Constraint (7) ensures that the number of vehicle allocated does not exceed the availability and constraint (8) sets the value of decision variables to be integer.

### 3.3. The SDVRP Model with Time Windows

The SDVRP formulation can be defined as follows. Let  $S$  be the set of destination points, while  $N = S \cup \{p\}$  is a set of all possible stopping points for all vehicles including the starting point ( $p$ ), and  $M$  is a set of vehicles. The variables in the SDVRP formulation include the following.

#### Indexed Sets:

$i = \{1, 2, \dots, n\}$ ; stopping point index

$j = \{1, 2, \dots, n\}$ ; stopping point index

$k = \{1, 2, \dots, m\}$ ; vehicle index

Parameters:

$n$ : the number of stopping points

$m$ : the number of vehicles

$Q^k$ : the capacity of vehicle  $k$

$J_{ij}$ : the distance from stopping point  $i$  to stopping point  $j$

$W_{ij}$ : the travel time from stopping point  $i$  to stopping point  $j$

$d_i$ : the number of workers at stopping point  $i$

$e_i$ : opening time of location  $i$

$l_i$ : the schedule at which the work starts of location  $i$

$r$ : distance travelled for each unit of fuel consumption (km/litre)

Decision Variables:

$x_{i,j}^k$ : 1 if vehicle  $k$  travels from stopping point  $i$  to stopping point  $j$

$y_i^k$ : the proportion of workers that are delivered to stopping point  $i$  by vehicle  $k$

$v_i^k$ : the number of workers that are delivered to stopping point  $i$  by vehicle  $k$

$t_i^k$ : departure time of vehicle  $k$  at stopping point  $i$

$u_i^k$ : the remaining number of workers in vehicle  $k$  after delivery at stopping point  $i$

Objective:

$$\text{Minimize } Z = \sum_{i \in N} \sum_{j \in N} \sum_{k \in M} \frac{1}{r} J_{ij} x_{ij}^k \quad (9)$$

Constraints:

$$\sum_{i \in N} x_{ip}^k = \sum_{j \in N} x_{pj}^k \quad \forall p \in N; k \in M \quad (10)$$

$$\sum_{k \in M} y_{ik}^k = 1 \quad \forall i \in S \quad (11)$$

$$\sum_{k \in M} v_i^k = d_i \quad \forall i \in S \quad (12)$$

$$v_i^k \geq x_{ij}^k \quad \forall i \in S; k \in M \quad (13)$$

$$\sum_{i \in S} d_i y_i^k \leq Q^k \quad \forall k \in M \quad (14)$$

$$\sum_{j \in S} x_{ij}^k \geq y_i^k \quad \forall i \in S; k \in M \quad (15)$$

$$d_i y_i^k \geq v_i^k \quad \forall i \in S; k \in M \quad (16)$$

$$x_{ij}^k + x_{ji}^k \leq 1 \quad \forall i \in N; j \in N; i \neq j; k \in M \quad (17)$$

$$u_i^k - u_j^k - Q^k x_{ij}^k \geq v_j^k - Q^k \quad \forall i \in S; j \in S; i \neq j; k \in M \quad (18)$$

$$d_i \leq u_i^k \leq Q^k \quad \forall i \in S; k \in M \quad (19)$$

$$x_{ijk} \leq \sum_{\substack{p \in N \\ p \neq i}} x_{jpk} \quad \forall i \in S; j \in S; i \neq j; k \in M \quad (20)$$

$$t_i^k + W_{ij} - t_j^k + 2M x_{ij}^k \leq 2M \quad \forall i \in S; j \in S; i \neq j; k \in M \quad (21)$$

$$e_i \leq t_i^k \leq l_i \quad \forall i \in N, k \in M \quad (22)$$

$$x_{ijv} \in \{0,1\} \quad \forall i \in N; j \in N; k \in M \quad (23)$$

$$v_{ik} \in \text{Integer} \quad \forall i \in N; k \in M \quad (24)$$

The objective function is represented by (9), which is to minimize the total distance travelled. Constraint (10) ensures that each route continues and finally ends up at the last destination. Constraints (11) and (12) ensure all workers with destination  $i$  have been served. Constraint (13) guarantees that if location  $i$  is visited, then at least one worker is served. Constraint (14) makes sure that the number of workers in a vehicle does not exceed the capacity of the vehicle. Constraint (15) guarantees that if location  $i$  is visited by vehicle  $k$ , then the same vehicle leaves the location. Constraint (16) arranges the workers to be delivered to node  $i$  by vehicle  $k$ . Constraints (17) to (19) ensure that sub tours are eliminated, while constraint (20) ensures the fractional cycle is eliminated. Time windows are represented by constraints (21) and (22). Finally, constraints (23) and (24) make sure that variable are binary or integer based as intended.

Input data for the SDVRP are given as follows. Table 4 shows the distance between two points within the stopping point set. Other data used in this case study include the distance between each starting point found in the set covering and the destinations as shown in Table 5 and the travel time required between a starting point and destinations as shown in Table 6.

**Table 4.** Distance matrix between stopping points in kilometer (km).

	1	2	3	4	5	6	7
1	-	1.6	3.9	4	7.1	8.1	12.3
2	1.6	-	2.2	2.4	5.4	6.4	10.7
3	3.9	2.2	-	0.12	3.2	4.2	8.4
4	4	2.4	0.12	-	3	4	8.3
5	7.1	5.4	3.2	3	-	2.2	6.5
6	8.1	6.4	4.2	4	2.2	-	4.3
7	12.3	10.7	8.4	8.3	6.5	4.3	-

**Table 5.** Distance matrix between starting points and destinations in kilometer (Km).

		Starting points		Destinations (offices/sites)											
		1	2	1	2	3	4	5	6	7	8	9	10	11	12
Starting points	1	-	-	10.4	21.5	10.5	10	11.2	9.4	15.8	20.9	10.4	9.8	15.8	9.4
	2	-	-	6.5	21	10	9.5	4.7	4.4	6.4	17	5.5	4.9	11.9	3.2
Destinations (offices/sites)	1	10.4	6.5	-	15.1	4.1	3.7	1.8	2.1	8.5	10.5	0.65	2.4	5.4	6
	2	21.5	21	15.1	-	12.4	12.1	15.9	16.2	22.5	25.6	15.3	16.5	20.5	20.1
	3	10.5	10	4.1	12.4	-	0.65	4.8	9.2	11.6	14.6	4.1	5.5	9.1	9.2
	4	10	9.5	3.7	12.1	0.65	-	4.5	4.8	11.2	14.2	3.7	5.1	9.1	8.7
	5	11.2	4.7	1.8	15.9	4.8	4.5	-	0.23	6.7	12.3	0.85	0.6	7.2	4.2
	6	9.4	4.4	2.1	16.2	9.2	4.8	0.23	-	6.5	12.1	1.2	0.4	7	4.1
	7	15.8	6.4	8.5	22.5	11.6	11.2	6.7	6.5	-	19	7.4	6.1	14	3.2
	8	20.9	17	10.5	25.6	14.6	14.2	12.3	12.1	19	-	11.2	13	5.1	16.5
	9	10.4	5.5	0.65	15.3	4.1	3.7	0.85	1.2	7.4	11.2	-	1.4	6.1	5.1
	10	9.8	4.9	2.4	16.5	5.5	5.1	0.6	0.4	6.1	13	1.4	-	7.8	3.7
	11	15.8	11.9	5.4	20.5	9.1	9.1	7.2	7	14	5.1	6.1	7.8	-	11.4
	12	9.4	3.2	6	20.1	9.2	8.7	4.2	4.1	3.2	16.5	5.1	3.7	11.4	-

**Table 6.** Travel time between starting points and destinations in minutes.

		Starting points		Destinations (offices/sites)											
		1	2	1	2	3	4	5	6	7	8	9	10	11	12
Starting points	1	-	-	24	38	24	23	24	24	34	37	23	25	32	30
	2	-	-	12	31	18	16	9	9	11	25	10	10	25	6
Destinations (offices/sites)	1	24	12	-	21	9	8	3	4	17	15	1	4	10	10
	2	38	31	21	-	18	17	22	22	33	38	20	23	30	29
	3	24	18	9	18	-	2	9	10	19	20	8	11	12	16
	4	23	16	8	17	2	-	7	7	18	22	7	8	17	14
	5	24	9	3	22	9	7	-	1	11	20	1	1	13	7
	6	24	9	4	22	10	7	1	-	11	19	2	1	12	7
	7	34	11	17	33	19	18	11	11	-	45	12	10	35	6
	8	37	25	15	38	20	22	20	19	45	-	17	21	8	27
	9	23	10	1	20	8	7	1	2	12	17	-	3	13	9
	10	25	10	4	23	11	8	1	1	10	21	3	-	13	6
	11	32	25	10	30	12	17	13	12	35	8	13	13	-	21
	12	30	6	10	29	16	14	7	7	6	27	9	6	21	-

#### 4. Results and Discussion

Performing the three stages procedure as aforementioned above, we started by defining two starting points where the busses should depart based on the result from set covering. At the second stage, we found that the total number of buses required in this case study was 25 at the first departure point and 12 for the second point. At the last stage, the SDVRP produces 23 routes in total, 12 routes for the first departure point and 11 routes for the second departure point. All buses were successfully scheduled within the given time windows.

The computational time required to solve the problem using the procedure was relatively long. Although the set covering and bus assignment were solved in relatively no time, the SDVRP in this case was solved in about 2.5 to 11 hours depending on the number of iterations that ranged between 10 million to 50 million. The SDVRP was solved for 2 different starting points. The first one did not improve after 20 million iterations, while for the second starting point we found the best result in 40 million iterations.

Based on the solution, several cost improvements were found. With the smaller number of buses to operate, the firm can save operational cost that included driver cost, bus maintenance cost, and fuel cost. With only fuel cost data available, we also found that this solution can save about 9% of the fuel cost in comparison with the existing routing and scheduling. In the case where the buses are not yet available, the use of computational solution would also benefit the company with less investment cost for buses. When more complete data is available, a study will be able provide important information on total cost reduction instead of fuel cost only.

#### 5. Conclusion

The solution to the problem from the case study was found through 3 stages of: 1) defining the starting point and clustering using set covering, 2) assigning bus from each starting point, and 3) determining the route by solving the SDVRP with time windows using an exact method. With 7 pickup points and 12 destinations, this combinatorial problem required a significant duration to find the global optimum solution when an exact method is used. This is not a problem when the VRP does not need to be solved quickly and it does not need to be run many times (i.e. when the problem is multiple), otherwise the use of metaheuristics is deemed more practical. Based on our solution, we found that all the pickups and

deliveries can be performed using 37 buses out of the available 49 buses that previously used. Different schedules for each bus were applied. Implementation of these schedules and routes allowed all shuttle buses to operate without violating the time windows and speed limit policy set by the firm. Additionally, this solution is capable of decreasing the fuel cost by 9% of the given initial cost. Other potential savings include reduction in investment cost (of buses) and operational cost that includes drivers and maintenance cost.

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