

# Policy for equipment's leasing period extension with minimum cost of maintenance

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**Abstract.** The cost structure for equipment investment including purchase cost and maintenance cost is getting more expensive. The company considers to lease the equipment instead of purchase it under a contractual agreement. Offering to extend the lease period, following to the base lease period, will provide more benefits for both the lessor (owner) and the lessee (user). Whenever the lease period extension offered at the beginning of the contract, there are some risks in finance e.g. uncertainty of the equipment performance and lessor responsibility. Therefore, this research attempts to model the optimal maintenance policy for lease period extension offered at the end of the contract. Minimal repair is performed to rectify a failed equipment, while imperfect preventive maintenance is conducted to improve the operational state of the equipment when reaches a certain control limit to avoid failures. The mathematical model is constructed to determine the optimal control limit, the number and degree of preventive maintenance, and the multiplication number of the lease period extension. Finally, numerical examples are given to illustrate the influences of the optimal length of the extended lease and the maintenance policy to minimize the maintenance cost.

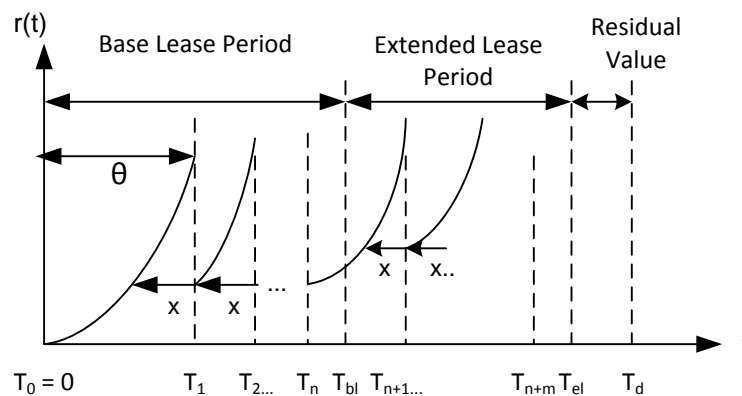
## 1. Introduction

Most companies need various types of equipment for running their business processes, either to produce goods or provide services to consumers. The purchase price and maintenance cost of the equipment is getting more expensive, it may not be economical for the company to purchase the expensive equipment. The company considers to lease the equipment instead of purchase it [1][2].

For repairable leased equipment, there are two types of maintenance actions, they are Corrective Maintenance (CM) and Preventive Maintenance (PM). CM is used to rectify failed equipment back to its operational state and PM is performed to improve the operational state of the equipment to avoid failures. In the CM action, minimal repair is the commonly used to restore failed equipment. After minimal repair, the equipment is under normal operation but the failure rate remains unchanged [3][4]. PM is classified into two major categories, perfect PM and imperfect PM [5]. There are two methods for describing PM degree, Age Reduction Method (ARM) and Failure Rate Reduction Method (FRRM). ARM is used to restore the age of the equipment younger than the current age, FRRM is used to reduce the failure rate of the equipment [6]. Pongpech & Murthy [7] utilized FRRM to describe PM degree and derived the optimal PM policy for leased equipment. Some other issues



associated with FRRM can be found in the literature [8][9][10][11]. Yeh et al. [12] utilized ARM to describe PM degree and derived the optimal PM policy for leased equipment with considering lease period extension at the beginning of the contract. Some other issues associated with ARM can be found in the literatures [13][14]. Most studies mentioned focus on determining the optimal PM policy for leased equipment with a specified lease period by minimizing cost or an extension of lease period that was offered at the begin of the contract by maximizing profit. On the other hand, Chang & Lin [15] used ARM to derive the optimal PM policy and length of extended warranty after the base warranty period expires. Thus, extension of the lease period after base lease period ends may be a critical issue to get more benefit for the lessor and the lessee.



**Figure 1.** ARM of PM within life cycle of the equipment.

Hence, this paper will adopt ARM to describe PM degree and determine the optimal length of extended lease and the corresponding optimal maintenance policy for the equipment with minimizing the expected total maintenance cost of the lessor. Organization of this paper is as follows. The mathematical formulation is shown in Section 2. Section 3 provides the properties of the optimal maintenance policy and length of extended lease and an effective algorithm. The performance of the maintenance policy and length of extended lease are evaluated and illustrated through numerical examples in Section 4. Finally, some conclusions are illustrated in Section 5.

## 2. Mathematical formulation

The following notations are used to construct the mathematical model in this paper:

$L$	Unit length of lease period
$k$	Number of extended lease periods of a fixed time length unit $L$
$r(t)$	Failure rate function of the leased equipment
$R(t)$	Cumulative failure rate function of the leased equipment
$C_m$	Minimal repair cost
$C_f$	Penalty cost for each failure
$\tau$	Pre-specified time limit for minimal repair action
$\theta$	Controlled-limit of age for performing a PM action
$x$	Maintenance degree of a PM action
$C_p(x)$	PM cost function with maintenance degree $x$
$n$	Total number of PM action within base lease period $(0, t_{bl}]$
$m$	Total number of PM action within extended lease period $(t_{bl}, t_{el}]$

$T_i$	Time epoch to perform the $i$ -th PM action
$t_{bl}$	Base lease period of leasing equipment
$t_{el}$	Extended lease period of leasing equipment, where $t_{el} = t_{bl} + kL$
$t_d$	Life cycle of the equipment
$E[TC]$	Expected total maintenance cost within the lease period

In Figure 1, there are three phases within life cycle of the equipment. They are base lease period  $(0, t_{bl}]$ , extended lease period  $(t_{bl}, t_{el}]$ , and residual value at time  $t_d$ . When the leased equipment fails, there is a penalty cost  $C_f > 0$  to the lessor. Each equipment failure is rectified by a minimal repair. The minimal repair time required is less than or equal to pre-specified time limit  $\tau$ . An imperfect PM action is performed with the same degree  $x$  when the age of the equipment reaches a controlled-limit  $\theta$  over the lease period  $(0, t_{bl} + kL)$ .  $T_i$ ,  $i = 1, 2, \dots, n, n+1, \dots, n+m$  is the time epochs to perform PM action. Thus,  $T_i = \theta$  can be assumed. The number of PM action within the base lease period and extended lease period are  $n$  and  $m$ .

In addition, the boundary conditions  $x \leq \theta \leq \min\{t_{bl}, kL\}$  and  $\max\{(t_{bl} - \theta)/n, (kL - \theta)/m\} \leq x \leq \min\{t_{bl}, kL\}$  create an equation  $T_i = T_{i-1} + x, i = 1, 2, \dots, n+m$ . This result shows that  $T_i = T_1 + (i-1)x, i = 1, 2, \dots, n+m$ . After substituting  $T_i = \theta$  into  $T_i = T_1 + (i-1)x$ , the equation becomes  $T_i = \theta + (i-1)x, i = 1, 2, \dots, n+m$ .

### 2.1. Repair cost to the lessor

With the failure rate of the product  $r(t)$  when  $r(0) = 0$ , the expected total number of failures within the interval  $[0, t]$  is  $R(t) = \int_0^t r(u) du$ . When using the two-parameter Weibull distribution, the equation becomes  $R(t) = (\lambda t)^\beta$  where  $\alpha > 0$  and  $\beta > 1$ . The failure process of the equipment in the interval  $[T_i, T_{i+1}]$  is a non-homogeneous Poisson process with intensity  $r(t - ix)$  when product failures are rectified using minimal repairs. A minimal repair is performed with a fixed cost  $C_m > 0$  within the lease period. From Figure 1, the expected total numbers of failures are  $\sum_{i=0}^{n-1} \int_{T_i}^{T_{i+1}} r(t - ix) dt + \int_{T_n}^{t_{bl}} r(t - nx) dt$ , where  $T_0 = 0$  and  $T_i = \theta$  within the base lease period  $(0, t_{bl}]$  and  $\int_{t_{bl}}^{T_{n+1}} r(t - nx) dt + \sum_{i=1}^m \int_{T_{n+i}}^{T_{n+i+1}} r(t - (n+i)x) dt$ , where  $T_{n+m+1} = t_{el}$  and  $t_{el} = t_{bl} + kL, k = 0, 1, 2, \dots$  within the extended lease period  $(t_{bl}, t_{el}]$ , the repair cost of the equipment over the lease period  $(0, t_{el})$  is

$$[C_m + C_f] \left\{ \sum_{i=0}^{n-1} \int_{T_i}^{T_{i+1}} r(t - ix) dt + \int_{T_n}^{t_{bl}} r(t - nx) dt + \int_{t_{bl}}^{T_{n+1}} r(t - nx) dt + \sum_{i=1}^m \int_{T_{n+i}}^{T_{n+i+1}} r(t - (n+i)x) dt \right\} \\ = [C_m + C_f] \left\{ (n+m) [R(\theta) - R(\theta - x)] + R(t_{bl} + kL - (n+m)x) \right\} \quad (1)$$

### 2.2. PM cost to the lessor

After an imperfect PM action is performed, the age of the equipment becomes  $x$  unit of time younger than before and each PM cost is  $C_p(x)$ . It is a non-negative and non-decreasing function of

maintenance degree  $x$ , i.e.,  $C_p(x) \geq 0$  and  $C_p'(x) \geq 0$  for all  $x \geq 0$ . Assuming that the time required to perform an imperfect PM is negligible. PM costs are  $\sum_{i=0}^n C_p(x) = nC_p(x)$  and  $\sum_{i=0}^m C_p(x) = mC_p(x)$  within the base and extended lease periode. Therefore, the total PM cost over the lease period  $(0, t_{bl} + kL)$  is

$$\sum_{i=0}^n C_p(x) + \sum_{i=0}^m C_p(x) = (n+m)C_p(x) \quad (2)$$

The repair and PM costs as in equation (1) and (2) are combined yield the expected total maintenance cost over the lease period  $(0, t_{bl} + kL)$ , which is

$$E[TC] = [C_m + C_f] \{ (n+m)[R(\theta) - R(\theta - x)] + R(t_{bl} + kL - (n+m)x) \} + (n+m)C_p(x) \quad (3)$$

### 3. Optimal policy

Based on the objective function equation (3), we will find an optimal maintenance policy  $(n^*, m^*, x^*, \theta^*)$  and number of extended lease period  $k^*$  that minimize the expected total maintenance cost of the lessor. The properties of them are investigated as followed. Taking the first and second partial derivatives of equation (3) with respect to  $k$ , we have

$$\frac{\partial E[TC]}{\partial k} = -L[C_m + C_f]r(t_{bl} + kL - (n+m)x) \quad (4)$$

and

$$\frac{\partial^2 E[TC]}{\partial k^2} = -[L^2[C_m + C_f]r'(t_{bl} + kL - (n+m)x)] \quad (5)$$

Equation (4) and (5) are observed by the following theorems.

**Theorem 1.** Given any  $n, m, x, \theta > 0$  when  $r'(t) > 0, \forall t > 0$ , the results can be written as followed.

- (i) If  $k^* = 0$ , then  $L[C_m + C_f]r(t_{bl} - (n+m)x) > 0$
- (ii) If  $k^* = \frac{t_d - t_{bl}}{L}$ , then  $L[C_m + C_f]r(t_d - (n+m)x) < 0$
- (iii) If  $-L[C_m + C_f]r(t_d - (n+m)x) > -L[C_m + C_f]r(t_{bl} - (n+m)x)$   
then there is a unique solution  $k^* \in \left(0, \frac{t_d - t_{bl}}{L}\right)$  such that equation (4) equals zero.

Substituting  $k^*$  into equation (3) yields the expected total total maintenance cost, which is

$$E[TC] = [C_m + C_f] \{ (n+m)[R(\theta) - R(\theta - x)] + R(t_{bl} + k^*L - (n+m)x) \} - (n+m)C_p(x) \quad (6)$$

**Theorem 2.** Given any  $n, m, x, > 0$ , when  $r'(t) > 0, \forall t > 0$ , the optimal controlled-limit  $\theta^* = x$ . The boundary condition becomes  $\max\{t_{bl}/(n+1), k^*L/(m+1)\} \leq x \leq \min\{t_{bl}, k^*L\}$  after substituting  $\theta^* = x$  into the boundary condition  $\max\{(t_{bl} - \theta)/n, (kL - \theta)/m\} \leq x \leq \min\{t_{bl}, kL\}$  and equation (6). Thus, the expected total maintenance cost can be rewritten as

$$E[TC] = [C_m + C_f] \{ (n+m)R(x) + R(t_{bl} + k^*L - (n+m)x) \} - (n+m)C_p(x) \quad (7)$$

**Theorem 3.** Given any  $n, m > 0$ , when  $r'(t) > 0, \forall t > 0$ , dan  $C_p'(x) \geq 0, \forall x > 0$ , the optimal PM degree is  $x^* = \max \left\{ \frac{t_{bl}}{n+1}, \frac{k^* L}{m+1} \right\}$ . Substituting  $x^*$  into equation (7) yields the expected total maintenance cost, which is

$$E[TC] = [C_m + C_f] \left\{ (n+m)R(x^*) + R(t_{bl} + k^* L - (n+m)x^*) \right\} - (n+m)C_p(x^*) \quad (8)$$

Finally,  $n$  and  $m$  are the decision variables in equation (8). Assuming that the time required to perform a minimal repair is less than or equal to pre-specified time limit  $\tau$ . Therefore, we have the boundary condition  $0 \leq n\tau \leq t_{bl}$  and  $0 \leq m\tau \leq k^* L$ . The trivial upper bounds are  $t_{bl} / \tau$  and  $k^* L / \tau$  for  $n$  and  $m$ . These upper bounds are used to find the optimal  $n^*$  in the interval  $[0, t_{bl} / \tau]$  and  $m^*$  in the interval  $[0, k^* L / \tau]$  because  $n$  and  $m$  are both integers. The following algorithm can be used to find the optimal maintenance policy  $(n^*, m^*, x^*, \theta^*)$  and number of extended lease period  $k^*$ .

#### Algorithm

1. Set  $k=1$ ,  $\min E[TC] = 999999999$ , compute  $\bar{k} = (t_d - t_{bl}) / L$ ,  $\bar{n} = t_{bl} / \tau$ .
2. Set  $n=1$  and compute  $m = kL / \tau$ .
3. Set  $m=1$ .
4. Compute  $\theta = x = \max \left\{ t_{bl} / (n+1), kL / (m+1) \right\}$  and  

$$E[TC] = [C_m + C_f] \left\{ (n+m)R(x^*) + R(t_{bl} + k^* L - (n+m)x^*) \right\} - (n+m)C_p(x^*)$$
5. If  $\min E[TC] > E[TC]$ , then set  $(k^*, n^*, m^*, x^*, \theta^*) = (k, n, m, x, \theta)$ , thus  $\min E[TC] = E[TC]$ .  
 Otherwise, set  $m = m+1$ . If  $m > m$ , then go to 6. Otherwise, go to 4.
6. Set  $n = n+1$ . If  $n > n$ , then go to 7. Otherwise, go to 3.
7. Set  $k = k+1$ . Jika  $k > \bar{k}$ , then stop. Otherwise, go to 2.

#### 4. Numerical example

Based on using a two-parameter Weibull distribution as the lifetime distribution of the equipment,  $r(t) = \lambda\beta(\lambda t)^{\beta-1}$ , where  $\alpha > 0$  and  $\beta > 1$ . The PM cost is  $C_p(x) = 30 + bx$  (\$). The following values are considered for the model parameter.

$$\begin{array}{llllll} L = 6 \text{ (months)} & t_d = 100 \text{ (months)} & C_m = 220 \text{ ($) } & C_f = 180 \text{ ($) } & b = 0, 10, 30, 50, 70, 90 \text{ ($) } \\ t_{bl} = 48 \text{ (months)} & \alpha = 5 & \beta = 1.5, 2, 2.3 & \tau = 0.1 \text{ (months)} & \end{array}$$

The optimal maintenance policy and number of extended lease period for the equipment are determined corresponding the minimal expected total maintenance cost of the lessor. One combination of values parameter has one result for the expected total maintenance cost of the lessor. The summary of all the results is shown at Table 1.

**Table 1.** Optimal policy and number of extended lease period.

$b$	$\beta=1.5$					$\beta=2$					$\beta=2.5$				
	$k^*$	$n^*$	$m^*$	$\theta^*=x^*$	$\min E[TC]$	$k^*$	$n^*$	$m^*$	$\theta^*=x^*$	$\min E[TC]$	$k^*$	$n^*$	$m^*$	$\theta^*=x^*$	$\min E[TC]$
<b>10</b>	1	33	4	1.4118	3937	1	34	4	1.3714	2862	1	32	3	1.5000	2285
<b>30</b>	1	31	3	1.5000	4971	1	35	4	1.3333	3903	1	30	3	1.5484	3330
<b>50</b>	1	31	3	1.5000	5991	1	31	3	1.5000	4938	1	31	3	1.4118	4352
<b>70</b>	1	31	3	1.5000	7011	1	31	3	1.5000	5958	1	31	3	1.5000	5372
<b>90</b>	1	31	3	1.5000	8031	1	31	3	1.5000	6978	1	31	3	1.5000	6392

Table 1 summarizes the optimal number of extended lease period  $k^*$  and maintenance policy  $(n^*, m^*, x^*, \theta^*)$  with  $\lambda=0.2$ . For example, when  $(\beta, b)=(1.5, 50)$  yields  $k^*=1$ ,  $(n^*, m^*, x^*, \theta^*)=(31, 3, 1.5)$ , and  $E[TC]=5991(\$)$ . This means that the optimal number of PM are  $n^*=31$  and  $m^*=3$  for the base and extended lease period. In this maintenance policy, PM degree are 1.5 months, thus the time epochs to perform PM are  $T_1 \approx 1.5, T_2 \approx 3, T_3 \approx 4.5, T_4 \approx 6, T_5 \approx 7.5, \dots, T_{30} \approx 45, T_{31} \approx 46.5$  within the base lease period and  $T_1 \approx 49.5, T_2 \approx 51, T_3 \approx 52.5$  within the extended lease period. Furthermore, we have some results from Table 1.

1. When the shape parameter ( $\beta > 2$ ) increases, the maintenance degree ( $x$ ) increase, the number of PM actions ( $n^* + m^*$ ) and the maintenance cost decrease.
2. When the shape parameter ( $\beta < 2$ ) increases, the maintenance cost and the maintenance degree ( $x$ ) decrease, but the number of PM actions ( $n^* + m^*$ ) increase.
3. When the marginal PM cost ( $b$ ) increases, the maintenance cost and the maintenance degree ( $x$ ) increase, but the number of PM actions decrease.

## 5. Conclusions

In this paper, PM degree is described by ARM to determine the optimal length of extended lease and the corresponding optimal maintenance policy for the equipment with minimizing the expected total maintenance cost of the lessor. Based on Table 1, the optimal length of extended lease period are one period for every variations of shape parameter and marginal PM cost.

## 6. References

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