

A Perishable Inventory Model with Return

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Abstract. In this paper, we develop a mathematical model for a perishable inventory with return by assuming deterministic demand and inventory dependent demand. By inventory dependent demand, it means that demand at certain time depends on the available inventory at that time with certain rate. In dealing with perishable items, we should consider deteriorating rate factor that corresponds to the decreasing quality of goods. There are also costs involved in this model such as purchasing, ordering, holding, shortage (backordering) and returning costs. These costs compose the total costs in the model that we want to minimize. In the model we seek for the optimal return time and order quantity. We assume that after some period of time, called return time, perishable items can be returned to the supplier at some returning costs. The supplier will then replace them in the next delivery. Some numerical experiments are given to illustrate our model and sensitivity analysis is performed as well. We found that as the deteriorating rate increases, returning time becomes shorter, the optimal order quantity and total cost increases. When considering the inventory-dependent demand factor, we found that as this factor increases, assuming a certain deteriorating rate, returning time becomes shorter, optimal order quantity becomes larger and the total cost increases.

1. Introduction

In managing goods, retailer should consider several costs such as purchasing, ordering, holding and possibly shortage costs. For some special goods, especially when dealing with perishable goods such as vegetables, milk, fruit or drugs, deteriorating rate or the product shelf life should be taken into account as well by considering return costs where goods can be returned to supplier at some costs. Considering shortage cost, there are two conditions faced by the retailer when goods are not available when demand comes. Customers can wait until goods are available in the next period (backorder) or they can try to find another retailer to fulfil their demand (lost sales).

In this paper, we develop a mathematical model for perishable goods with return cost. We assume that demand is deterministic and the amount of inventory depends on the demand (inventory-dependent demand). Customers tend to buy more products when there are more goods displayed in the shelf, therefore inventory-dependent demand is realistic to be included in the model. The developed model is based on the model developed by Nafisah, et.al (2016) and Nagare and Dutta (2012). Numerical examples are provided to illustrate our model and some sensitivity analysis are also performed by studying the effect of changes in the deterioration rate and inventory-dependent demand factor on the optimal return time, order quantity and total cost.



2. Literature Review

Perishable inventory models have become interesting topic of research for many years. Many papers deal with perishable inventory models with many variants that related to problems faced by industries. This paper contributes in the developing mathematical model for perishable inventory with deterministic demand by considering return cost, where all shortages are handled with backorder. In the area of perishable inventory models, a model for perishable goods where the purchasing cost is modelled as a function of the credit policy of the retailer has been developed by Dye (2012). Kumar et.al developed a model by considering demand as quadratic function and uncertain holding cost, while a model with Weibull distribution for the deteriorating rate and holding cost as a linear function of time has been developed by Mishra (2013). Nagare and Dutta (2012) have developed a continuous review model by considering deteriorating rate and inventory-dependent demand. Nafisah et.al (2016) proposed a multi-item model for goods that close to their expiration date by applying discount and return. They assumed that demand rate can be divided in several intervals while their values are constant, but different in each interval. Rikardo et al (2017) proposed a continuous review model for perishable goods with all-unit discount. In this paper we consider the demand function used by Nagare and Dutta (2012) from the beginning period until the return time and then constant after the return time.

The organization of this paper is as follows. In Chapter 3, we introduce and formulate our model and give the condition for the optimal return time and order quantity. Numerical examples and sensitivity analysis are provided in Chapter 4 while conclusions and further research are relegated in the last chapter.

3. The Model

3.1 Notations

The notations we use in this paper are as follows:

TC	: total inventory cost	$I(t)$: inventory level at time t .
PC	: purchasing cost	m	: number of inventory during interval $[0, T_r]$
OC	: ordering cost	S	: number of shortage during interval $[T_r, T]$
HC	: holding cost	P	: purchasing cost per unit
SC	: shortage cost	θ	: deteriorating rate
T	: time between replenishment (weeks)	h	: holding cost per unit per year
$D(t) = \alpha + \beta I(t)$: demand at time t	A	: ordering cost per order
β	: inventory-dependent demand factor	A_r	: return cost per return
Q	: order quantity	R	: return cost per unit
T_r	: return time (weeks)	π	: backorder cost per unit

3.2. Model Development

In developing our model, basic assumption we use is inventory-dependent demand as given by the following equation.

$$D(t) = \begin{cases} \alpha + \beta I(t) & , 0 \leq t \leq T_r \\ B_1 & , T_r \leq t \leq T \end{cases}$$

The demand representation is given by Figure 1. When $t = 0$, the replenishment of amount Q arrives so the inventory reaches its maximum level. As time goes by, inventory is depleted because of the demand as depicted in Figure 1. At $t = T_r$, all unsold goods are returned to the supplier. When return takes place, they are handled by *full backorder* because all returned goods will be available in the next

replenishment. Demand and deteriorating rate in the interval $[0, T]$ will decrease the inventory level according to the following first order differential equation:

$$\frac{dI(t)}{dt} = -\theta I(t) - D(t) , 0 \leq t \leq T$$

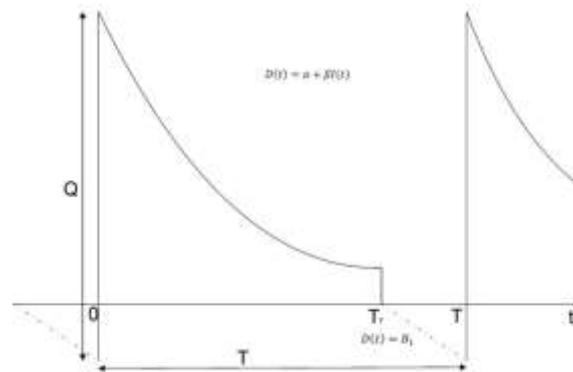


Figure1. Demand representation for one cycle T

Using the inventory-dependent demand, $D(t) = \alpha + \beta I(t)$, we have

$$\begin{aligned} \frac{dI(t)}{dt} &= -\theta I(t) - (\alpha + \beta I(t)) , 0 \leq t \leq T_r \\ \frac{dI(t)}{dt} &= -\theta I(t) - B_1 , T_r \leq t \leq T \end{aligned}$$

Using the boundary condition $I(T_r) = 0$, we have

$$I(t) = B_1(T_r - t)$$

In the interval $0 \leq t \leq T_r$ we have

$$\frac{dI(t)}{dt} = -\alpha - (\theta + \beta)I(t)$$

Let $\lambda = \theta + \beta$, and using the boundary condition $I(T_r) = 0$, we finally have

$$I(t) = \frac{\alpha}{\lambda} (e^{\lambda(T_r-t)} - 1)$$

Then

$$I(t) = \begin{cases} \frac{\alpha}{\lambda} (e^{\lambda(T_r-t)} - 1) , & 0 \leq t \leq T_r \\ B_1(T_r - t) , & T_r \leq t \leq T \end{cases}$$

When $t = 0$, $I(t) = Q - B_1(T_r - t)$ then we have

$$Q = \frac{\alpha}{\lambda} (e^{\lambda(T_r-t)} - 1) + B_1(T_r - t)$$

The number of inventory during $[0, T]$ is given by

$$m = \int_0^{T_r} I(t) dt = \int_0^{T_r} \frac{\alpha}{\lambda} (e^{\lambda(T_r-t)} - 1) dt = \frac{\alpha}{\lambda^2} (e^{\lambda T_r} - 1) - \frac{\alpha}{\lambda} T_r$$

The number of shortages during $[T_r, T]$ is

$$S = -I(t)(t - T_r) = B_1(T_r - t)^2$$

3.3. Model Formulation

The objective of the developed model is to find the optimal order quantity that minimizes the total annual cost. The total cost (TC) consists of the purchasing cost (PC), the ordering cost (OC), the annual holding cost (HC), the backorder/shortage cost (SC) and the return cost (RC). All those costs are given as follows.

$$\begin{aligned}
 PC &= P.D; \quad OC = \frac{A}{T}; \quad HC = \frac{hm}{T} = \frac{h}{2T} \left[\frac{\alpha}{\lambda^2} (e^{\lambda T_r} - 1) - \frac{\alpha}{l} T_r \right] \\
 SC &= \frac{\pi S}{T} = \frac{\pi}{2T} [B_1(T_r - t)^2]; \quad RC = \frac{1}{T} \left[A_r + \frac{RS}{2} \right] = \frac{1}{T} \left[A_r + \frac{RB_1}{2} (T_r - t)^2 \right] \\
 TC &= P.D + \frac{A}{T} + \frac{h}{2T} \left[\frac{\alpha}{\lambda^2} (e^{\lambda T_r} - 1) - \frac{\alpha}{l} T_r \right] + \frac{\pi}{2T} [B_1(T_r - t)^2] + \frac{\pi}{2T} [B_1(T_r - t)^2] \\
 &\quad + \frac{1}{T} \left[A_r + \frac{RB_1}{2} (T_r - t)^2 \right]
 \end{aligned}$$

Taking the first derivative of TC with respect to T_r and equates to zero we have

$$\frac{dTC}{dT_r} = \frac{h\alpha}{\lambda} (e^{\lambda T_r} - 1) + 2\pi B_1(T_r - t) + 2RB_1(T_r - t) = 0$$

The second derivative of TC with respect to T_r is

$$\frac{d^2TC}{dT_r^2} = h\alpha e^{\lambda T_r} + 2\pi B_1 + 2RB_1$$

Since the second derivative is always positive, then the value of T_r that satisfies $\frac{dTC}{dT_r} = 0$, will yield the optimal Q that minimizes TC

4. Numerical Experiments and Sensitivity Analysis

In this chapter, we give numerical experiments according to the following input parameters ($T = 24$ weeks):

$\alpha = 1000$ unit, $\beta = 0.1$, $\theta = 0.1$, $A = \$10$, $h = \$1$, $\pi = \$1.2$, $P = \$14.5$, $A_r = \$10$, $R = \$1$, $B = 250$ unit.

Using Maple we have the optimal return time (weeks) and order quantity (units) that minimizes the annual total cost as follows:

$$T_r = 7.59; Q = 19,692; TC = \$4,903.80$$

Results on the sensitivity analysis are given in Table 1. In Table 1, effects of β and θ on T_r , Q and TC are analyzed. Smaller value of θ represent lower deteriorating rate, meaning that it takes longer for the products to deteriorate. Smaller value of β indicates that the effect of inventory on demand is smaller or the inventory-dependent demand factor is smaller. If we look at Table 1 for certain value of β , we can see that as θ gets larger, the optimal return time gets shorter but the order quantity becomes larger and the total cost becomes larger. This is due to the fact that goods are deteriorating faster, so the return time becomes shorter and the order quantity becomes larger (to fulfill demand), so the total cost gets larger. For certain value of θ as β gets larger, the same situation occurs, due to the increase of demand. When β and θ get larger, the optimal return time becomes shorter, the optimal order quantity becomes larger and eventually the total cost becomes larger.

Table 1. Effect of β and θ on T_r , Q and TC

$\beta \backslash \theta$	0.1	0.2	0.3	0.4	0.5
Tr	7.59	6.39	5.53	4.89	4.39
Q	19,692	20,934	21,698	22,246	22,668
TC	4,903.80	5,422.29	5,768.55	6,019.14	6,211.29
Tr	6.39	5.53	4.89	4.39	4.00
Q	20,902	21,694	22,246	22,668	23,004
TC	4,941.41	5,406.28	5,729.16	5,969.63	6,157.49
Tr	5.53	4.89	4.39	4.00	3.67

Q		21,690	22,245	22,668	23,004	23,278
TC		5,044.00	5,439.18	5,727.96	5,950.35	6,128.08
Tr		4.89	4.39	4.00	3.67	3.40
Q	0.4	22,245	22,668	23,004	23,278	23,508
TC		5,149.20	5,486.30	5,743.21	5,946.83	6,112.97
Tr		4.39	4.00	3.68	3.40	3.17
Q	0.5	22,667	23,004	23,278	23,508	23,702
TC		5,244.63	5,536.06	5,765.58	5,951.86	6,106.59

5. Conclusions and Further Research

We have developed a perishable inventory model for inventory-dependent demand with return. All shortages are handled using backorder. Analysis of sensitivity of the model with respect to changes on the deterioration rate and inventory-dependent demand factor is given. We found that as the deterioration rate and inventory-dependent demand factor are both increasing, the optimal return time is shortened; the optimal order size and total cost are both increasing. In developing this model, we assume that demand is deterministic which is a bit unrealistic in most cases. Considering a probabilistic demand model is an avenue for further research. Also, incorporating discount factor from the supplier whether it is all-unit discount or incremental discount is a challenging development to pursue,

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