

# A Learning Trajectory for Teaching Social Arithmetic using RME Approach

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**Abstract.** This paper discusses the role of a learning trajectory (LT) in promoting students' reasoning when they learn social arithmetic using Realistic Mathematics Education (RME) approach. In our LT, we built the intertwining of the concepts such as profit, loss, percentage, discount, and interest rate, so that the students understand the relations among them. The LT was developed through a design research that consisted of a cyclic process of preparing for the experiment, conducting the experiment, and retrospective analysis. The research's subject was 32 students at grade 7 MTsN Sintoga, Pariaman, Indonesia. Data were collected through observations, interviews, checklist, videotaping, and analyzing the students' works. The results showed that the LT could help the students to reinvent the concepts in social arithmetic. The students had more confidence to use their own strategies in solving contextual problems. The most important thing, we discovered the growth in the students' mathematical reasoning.

## 1. Introduction

The needs to develop a learning trajectory (LT) for teaching a certain mathematics topic are increased nowadays. That is because researchers and mathematics educators realize that LTs play very important roles in building students' understanding of mathematical concepts. Gravemeijer [1] mentioned that if we want students to reinvent mathematics by doing mathematics, teachers have to adapt to how their students reason and help them build on their own thinking. To do so, teachers need to design a framework of reference (LT). LTs are very helpful for bridging the work of researchers and practitioners [2]. LTs also can help teachers evaluating and rethink teaching, which enable them to have a general vision of the class before they start teaching [3,4].

A LT is the sequences of activities and tasks that might support the development of students' understanding of a specific instructional goal [5]. Gravemeijer [6] said that neither teachers, nor researchers can rely on fixed teaching sequences, since a teacher continuously has to adapt to the actual thinking and learning of her students. Therefore, the preliminary version of a LT is in the form of a hypothetical one, and it is called a *hypothetical learning trajectory* (HLT)[5,6,7].

A HLT consists of three components: the learning goal that defines the direction, the learning activities, and the hypothetical learning processes—a prediction and anticipation of how the students' thinking and understanding will evolve in the context of the learning activities [5]. After a cyclic process of designing, testing, and re-designing, a HLT becomes a theory (LT) that can be used as a lesson learned by other mathematics educators to teach a certain mathematics topic. Gravemeijer [6] and Liljekvist [8] called the theory as a local instructional theory (LIT), while Cobb at al[9] called it as



domain specific theories. A local instruction theory consists of theories about both the process of learning a specific topic and the means to support that learning [1].

Many LTs in mathematics were developed by researchers. They were not only for teaching mathematics in primary and secondary education (see e.g. [10,11,12,13]) but also for teaching certain courses in higher education (see e.g. [14,15,16]). In general, the results of the researches revealed that the LTs were very helpful in building student's conceptual understanding.

In this research, we developed LT for teaching social arithmetic using RME approach. The LT was designed in the contrary to the way the concepts of social arithmetic were presented in mathematics textbooks which tend to be mechanistic, as can be seen on an example below.

### 1. *Persentase Keuntungan*

Persentase keuntungan digunakan untuk mengetahui persentase keuntungan dari suatu penjualan terhadap modal yang dikeluarkan.

Misal :       PU = Persentase keuntungan

                  HB = Harga beli (modal)

                  HJ = Harga jual (total pemasukan)

Persentase keuntungan dapat ditentukan dengan rumus

$$PU = \frac{HJ - HB}{HB} \times 100\%$$

**Figure 1.** A formula is presented in a mechanistic way

We can see in Figure 1 that the formula to calculate loss was given directly followed by the examples to show how the formula worked. This condition also influenced the way teachers teach mathematics which tend to be mechanistic [10,13,17].

We employed RME approach in this research because the idea of developing a LT is in line with the idea of how mathematics has to be taught in RME. The process of learning mathematics in RME can be described as a phenomenon of an iceberg below [16].

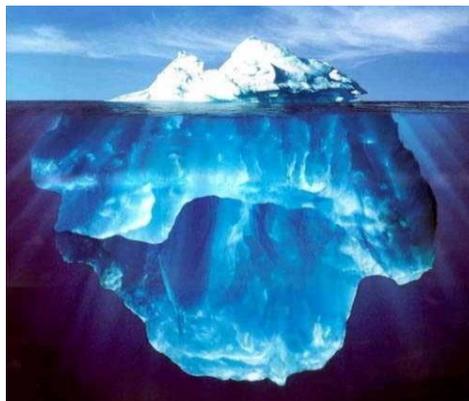


Photo: *Frans Moerlands*

**Figure 2.** RME as a phenomenon of an iceberg

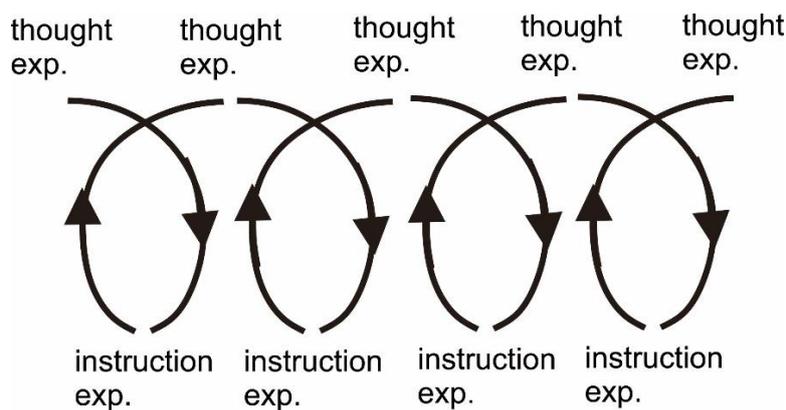
A very strong foundation is needed to support the top of the iceberg to appear on sea surface. In relation to this phenomenon, formal and abstract mathematical concepts are situated on the top of the iceberg. Mathematics educators or researchers need to provide a strong foundation and 'a best trajectory' for students to reach the top of the iceberg. To do so, at the beginning of the lesson, students are provided with contextual problems that can be solved using their informal knowledge. The contextual problems will also facilitate students to use their own symbols or their own strategy.

This process is called *horizontal mathematization*. After experiencing similar processes and empowering by *simplification* and *formalization* (see [18]), students will use more formal language or strategies in solving contextual problems. The journey, that will bring students to re-invent a formal mathematical, is called *vertical mathematization* [6, 19, 20].

The LT designed in this research was based on three key principles of RME for instructional designed, namely *guided reinvention trough progressive mathematization*, *didactical phenomenology*, and *emerging models* [6, 19, 21]. Meanwhile, the implementation of the LT in the classrooms was guided by RME's characteristics [17, 18, 22, 23]. To focus the research, we formulated two research questions. Firstly, what are the characteristics of a valid and practical learning trajectory for teaching social arithmetic using RME approach? Secondly, to what extend the learning trajectory for teaching social arithmetic using RME approach could stimulate student's reasoning?

## 2. Method

This research used design research approach proposed by Gravemeijer and Cobb [7]. We used the approach because design research aims at understanding more of the interrelatedness between teaching and learning in order to improve teaching [8]. Design research in this study consisted of a cyclic process of preparing for the experiment, conducting the experiment, and retrospective analysis. Gravemeijer and Cobb illustrated the cyclic process as can be seen in Figure 3.



**Figure 3.** A cyclic process of thought and instruction experiment

In preparing for the experiment we determined the end points of the instructions. The goals of our social arithmetic lessons where the students reinvent the concepts of profit, loss, discount, interest rate, tax, and the way to determine their percentages. As the students were already familiar with the activities such as counting money, buying or selling some things, then we used such activities as the starting point of the lessons. After we set the end and the starting points, we designed the HLT that consisted of five main activities and thirteen sub-activities of solving contextual problems that would facilitate students to do horizontal and vertical mathematization as well as stimulate students' thinking and reasoning. In addition, we also formulated the predictions of students' thinking and solutions, and the anticipations.

In the experimental phase, we tried out the HLT in two cycles. The first try out was conducted in small groups that involved six students at grade seven MTsN Sintoga Padang Pariaman, Indonesia. After the retrospective analysis and re-design processes, the HLT was tried out to 32 students at grade seven in the same school. The retrospective analysis involved the research team, a teacher, and an observer. Besides focusing our attention to develop the HLT, we also observed and analyzed the impact of the HLT on the development of students' confidence in using their own strategies when solving the contextual problems and the development of students' reasoning during the tryout. Data of the research were collected through observations, interviews, checklist, videotaping, and analyzing the students' works.

### 3. Results and Discussion

The HLT for teaching social arithmetic was validated by three mathematic education experts in Indonesia during preparing for the experiment phase. The result showed that the HLT met the criteria of validity [24], with the characteristics; the activities of solving contextual problems in the HLT were potential to facilitate the students to reinvent the concepts in social arithmetic; the activities were well sequenced, the HLT suit the key principles and characteristics of RME; and the components in the HLT were well designed and consistent between one and another.

The HLT also satisfied the criteria of practicality [24], in which it worked as intended during the tryout. The students understood the contextual problems and they conducted ‘doing math’ activities without major obstacle. The probing questions that were prepared as the anticipations of students’ thinking and solutions also helped the students to achieve the goals of the activities. In addition, the time provided for doing the activities of solving contextual problems was well planned.

The next example (Figure 4) is a contextual problem provided in the HLT that aimed at facilitating the students to find the concepts of profit and loss.

Mr. Andi was a fruit seller. At the end of the week he calculated his earning after making the following list.

Fruit	Quantity (kg)	Buying Price (Rp/kg)	Selling Price (Rp/kg)	Earning
Oranges	50	10,000	15,000	600,000
Apples	25	15,000	20,000	300,000
Salak	40	12,500	15,000	750,000
Mangoes	60	15,000	20,000	800,000

Mr. Andi also noted that he still had 10 kg Oranges, Apples and Salak respectively, and they could not be sold anymore, while Mangoes were sold out.

- What do you think about the earning of Mr. Andi regarding to each fruit that he sold? Explain your answer!
- If you want to present your findings in the form of percentages, how are you going to do that?

**Figure 4.** An example is a contextual problem provided in the HLT

Although the students were not formally introduced yet to the concepts of loss and profit, and the way to calculate loss, profit, and their percentages, but most of the students could reinvent the concepts using their own strategies and then mentioned the concepts using their own words. We observed that the contextual problem could stimulate the students to think about profit and loss by analyzing the context and relations among the data on the table. An example of student’s answer can be seen in Figure 5.

- Jeruk bersisa 10 kg. dgn modal 10.000/kg. dgn kuantitas 50 kg dan warga jual 15.000/kg. maka 20 kg x 15.000/kg = 600.000 rupiah. Sedangkan modalnya 50 x 10.000/kg = 500.000. maka dari penjualan jeruk mengalami untung 100.000 rupiah.  
 - apel bersisa 10 kg. modal 15 x 20.000/kg = 300.000 rupiah modal 25 x 15.000/kg = 375.000 rupiah. maka dari penjualan apel mengalami kerugian 75.000 rupiah.  
 - salak bersisa 10 kg. maka 50 x 15.000/kg = 750.000 rupiah, modal 60 x 12.500/kg = 750.000 rupiah, maka tidak mengalami untung maupun rugi.

**Figure 5.** An example of student’s answer in finding the concept of profit, loss, and balance

In Figure 5, the student mentioned that Mr. Andi got profit =100,000 *rupiah* from selling the Oranges because he earned  $40 \times 15,000 = 600,000$  *rupiah*, while the capital was  $50 \times 10,000 = 500,000$  *rupiah*. For the Apples, the earning was  $15 \times 20,000 = 300,000$  *rupiah* and the capital was  $25 \times 15,000 = 375,000$  *rupiah*, so he got loss = 75,000 *rupiah*. The earnings for Salak was  $50 \times 15,000 = 750,000$ , the capital =  $60 \times 12,500 = 750,000$ , so he did not get profit or loss.

Most students also found the way to calculate the percentages of profit and loss after they discuss the contextual problem in the group. They found by themselves that the profit or loss was compared to the capital, in calculating the percentage, as can be seen in Figure 6.

**Jawab:**

$$\text{manga} : \frac{200}{6000} \times 100 = 33,3\%$$


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$$\text{jeruk} : \frac{100}{5000} \times 100 = 20\%$$

$$\text{salak} : \frac{0}{750} \times 100 = 0\%$$


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$$\text{apel} : \frac{75.000}{375.000} \times 100 = 20\%$$

**Figure 6.** An example of student's answer in finding the percentage of profit and loss

The student's answers presented above indicate that the didactics phenomenology, as a key principle of RME [6], worked as intended in our experiment. The students used the context in the problem to reinvent the concepts. Besides, the contexts in the problems were facilitated the students to use their own strategies, which met one of the characteristics of RMW namely 'student's free production' [17, 18]. This situation helped the students building their confidence in learning mathematics. Finally, we observed the development in students' reasoning ability. They started giving an argument, a reason, or an explanation when solving the contextual problems. The reasoning test that we gave at the end of the experiment showed that 78,1% of the students achieved the score greater than 75. This finding confirmed that LT and RME approach are potential to improve students' reasoning [25,26].

#### 4. Conclusion

The LT for teaching social arithmetic using the RME approach developed in this research met the criteria of validity, practicality, and effectiveness. A design research approach that we used to develop the LT was very helpful in reaching our goal. The LT for teaching social arithmetic reflected the state of the art of RME and it worked as intended in the classroom. Moreover, the LT could help the students to reinvent the concepts in social arithmetic. The students had more confidence to use their own strategies in solving contextual problems. The most important thing, we discovered the growth in the student's mathematical reasoning.

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