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Rabies epidemic model with uncertainty in parameters: crisp and fuzzy approaches

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Abstract. A deterministic mathematical model is formulated to investigate the transmission dynamics of rabies. In particular, we investigate the effects of vaccination, carrying capacity and the transmission rate on the rabies epidemics and allow for uncertainty in the parameters. We perform crisp and fuzzy approaches. We find that, in the case of crisp parameters, rabies epidemics may be interrupted when the carrying capacity and the transmission rate are not high. Our findings suggest that limiting the growth of dog population and reducing the potential contact between susceptible and infectious dogs may aid in interrupting rabies epidemics. We extend the work by considering a fuzzy carrying capacity and allow for low, medium, and high level of carrying capacity. The result confirms the results obtained by using crisp carrying capacity, that is, when the carrying capacity is not too high, the vaccination could confine the disease effectively.

Keywords: Rabies, mathematical model, crisp and fuzzy approaches, uncertainty

1. Introduction

Rabies is a zoonotic disease that is transmitted to humans via an animal bite in particular dogs. It is estimated that approximately 55,000 cases happens annually, with higher number of cases in Asia and Africa [1]. Rabies is also a problem in several places in Indonesia such as Bali and Flores islands [2, 3]. The fatality rate can reach 100 % once symptoms of disease develop.

A number of control measures such as mass vaccination and culling have been implemented to stop rabies epidemics, but they are not perfectly effective. For example, in Flores Island, Indonesia, although vaccination has been implemented [3], rabies cases are still found. This is likely due to low vaccination uptake level [4]. Therefore, this paper aims to investigate the dynamics of rabies transmission with vaccination strategy.

A mathematical model can be used to investigate the dynamics and the effectiveness of the intervention on reducing disease transmission [5-13]. A number of mathematical models have been developed to understand the rabies epidemics [14-18]. Huo et al. used a mathematical model to investigate rabies epidemics and found that vaccination rate is one of the key components determining rabies transmission [17]. Zinsstag et al. developed a mathematical model to investigate the

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transmission dynamics. They found that a 70% vaccination coverage level is sufficient to interrupt rabies epidemics [18]. In this paper, we develop a mathematical model to investigate rabies epidemics by including carrying capacity and allowing intrinsic stochasticity in the transmission rate. We explore the effects of carrying capacity and the transmission rate on rabies epidemics. We extend the work by considering fuzzy carrying capacity to investigate the effect of uncertainty on the dynamics of rabies transmission.

This paper is organized as follows. Section 2 presents a formulation of mathematical model. Section 3 presents the parameter exploration, and finally results and discussion are presented.

2. Model Formulation

A deterministic ordinary differential equation model is developed for the transmission of rabies for dogs. The dog's population is divided into four compartments: Susceptible (*S*), Exposed but not yet infectious (*E*), Infectious (*I*) and Vaccinated (*V*). In this model, dogs remain infectious except if they are vaccinated. Therefore, no recovered class is included.

Dogs are born susceptible and their growth is limited by carrying capacity, *C*. Rabies is transmitted to susceptible dogs at rate $\boldsymbol{\beta}$ when there is a contact between infectious and susceptible dogs. The parameter $\boldsymbol{\beta}$ is the transmission rate, which is the contact rate between susceptible and infectious dogs, and the probability that contact successfully transmits disease. The susceptible dogs get vaccinated at a rate *g* and die due to natural death at rate *d*. The exposed dogs progress to infectious class at rate $\boldsymbol{\gamma}$, and die because of natural death rate, $\boldsymbol{d}$, and disease related death, $\boldsymbol{d_r}$. The vaccinated dogs loss immunity at rate *l*, which is then re-susceptible to the virus.

The model is then governed by the following system of differential equations:

$$\frac{dS}{dt} = BN(1 - N/C) - \lambda S - dS - g\epsilon S + lV \tag{1}$$

$$\frac{dE}{dt} = \lambda S - \gamma E - dE - g\epsilon E \tag{2}$$

$$\frac{dI}{dt} = \gamma E - (d + d_r)I \tag{3}$$

$$\frac{dV}{dt} = g\epsilon(S + E) - lV - dV \tag{4}$$

where $\boldsymbol{\lambda = \beta I}$, and the total dog population is $\boldsymbol{N = S + E + I + V}$.

3. Crisp Approach

Since most parameter values of the model are largely unknown, we explore different parameter values to understand their effects on the dynamics of rabies epidemics. First, we explore different values of carrying capacity and vaccination coverage level. Then, we investigate the effects of the transmission rate on the rabies epidemics by allowing intrinsic stochasticity in the transmission rate. In our exploration, we use carrying capacity 5×10^5, which is around two times the number of dogs in Flores island, Indonesia [3], 7.5×10^5 and 10^6, and vaccination coverage level of 50 % and 75 %. The parameter values are given in Table 1.

In our investigation of stochasticity in the transmission rate, we assigned 1000 different sets of normally distributed transmission rates as

$$\beta = \beta_0 + \sigma_\beta N(0,1)$$

where β_0 is the average transmission rate, σ_β is the variance, and $N(0,1)$ is a Gaussian random variable with a mean of zero and variance equal to one. The values of the parameters β_0 and σ_β are 10^{-5} and 10^{-6}, respectively.

Table 1. *Parameter descriptions,* value and sources. The transmission rate, β, carrying capacity, C, and vaccination coverage level, ϵ, are explored in this paper

Parameters	Description	Value	Unit	Sources
β	Transmission rate	10^{-5}	year^{-1}	Assumed
B	Birth rate	0.013×52	year^{-1}	[18]
C	Carrying capacity	5×10^5	N/A	see text for explanation
d	Natural death rate	0.006638×52	year^{-1}	[18]
d_r	Disease-induced death rate	1	year^{-1}	[19]
g	Vaccine efficacy	0.94	N/A	[18]
ϵ	Vaccination coverage level	0.5	year^{-1}	Explored
l	Vaccination loss rate	0.0081×52	year^{-1}	[18]
γ	Progress from exposed to infectious class	$12/3$	year^{-1}	[1]

In exploring the effects of carrying capacity and vaccination coverage level on rabies epidemics, we found that when carrying capacity is higher, an outbreak occurs although the vaccination coverage level is high (see Figure 1). For example, if carrying capacity is 7.5×10^5 and vaccination coverage level is 75%, an outbreak can still occur, with around 1.98×10^4 cases. The same behaviour is found when carrying capacity is 10^6. A 50% vaccination coverage level is sufficient to stop rabies epidemics if the carrying capacity is low (5×10^5). If the carrying capacity is higher, a higher vaccination coverage level is not sufficient to stop rabies epidemics because of higher number of dogs the population. Carrying capacity affects the growth of dog population. If carrying capacity is high, it is likely that there will be many dogs in the population. Therefore our findings imply that limiting the growth of dogs is necessary in order to interrupt rabies epidemics.

When investigating the effect of the transmission rate with intrinsic stochasticity, we are interested in situations where an outbreak occurs. Therefore, in our exploration, we use carrying capacity of 7.5×10^5 and 10^6. We found that the average outbreak size is $1.9 \times 10^4 \pm 1.96 \times 140.63$ if $K = 7.5 \times 10^5$ and the vaccination coverage level is 50%. If $K = 10^6$ and the vaccination coverage level is 50%, the outbreak size is $3.6341 \times 10^4 \pm 1.96 \times 78.18$. The outbreak sizes are of similar range as when the transmission rate is fixed. This may suggest the role of carrying capacity determining rabies epidemics is a bit stronger than the transmission rate. Table 2 and 3 show the number of infected dogs at time $t = 50$ (Table 2) and $t = 100$ (Table 3) for various values of carrying capacity with the crisp classification (low, medium, and high carrying capacity).

Table 2. The number of infected individuals with different carrying capacity (C) and vaccine scenarios at t = 50. The values for low (C_{low}), medium (C_{medium}) and high (C_{high}) carrying capacity are 5×10^5, 7.5×10^5, and 10^6, respectively.

	Different Vaccine Scenarios		
	No Vaccination	50%	75%
C_{low}	1.7297×10^4	0.0908	1.8×10^{-5}
C_{medium}	3.0203×10^4	1.9826×10^4	1.788×10^3
C_{high}	3.9322×10^4	3.6835×10^4	2.9270×10^4

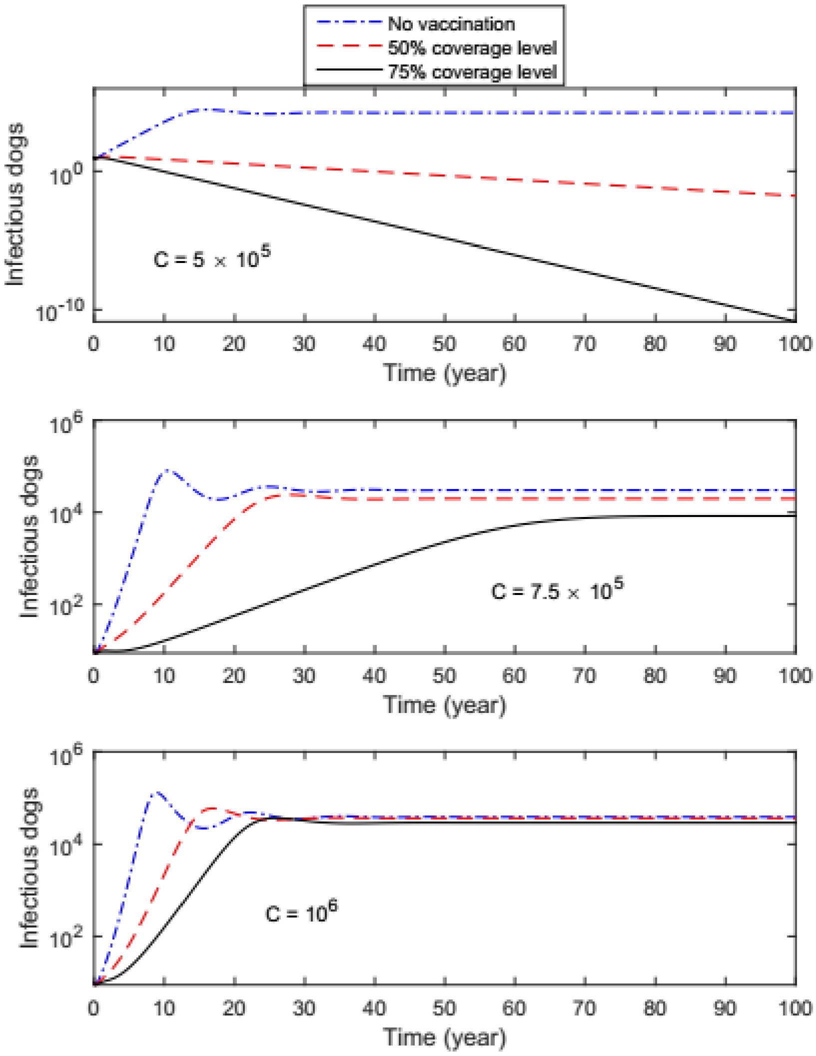

Figure 1. Plot of infectious dogs vs. time with different vaccination coverage levels. The top, middle and bottom plots are for $C = 5 \times 10^5$, $7{:}5 \times 10^5$, and 10^6, respectively.

Table 3. The number of infected individuals with different carrying capacity (C) and vaccine scenarios at $t = 100$. The values for low (C_{low}), medium (C_{medium}) and high $\left(C_{high}\right)$ carrying capacity are 5×10^5, 7.5×10^5, and 10^6, respectively.

	Different Vaccine Scenarios		
	No Vaccination	50%	75%
C_{low}	1.7269×10^4	0.0269	6.3×10^{-11}
C_{medium}	3.0347×10^4	1.979×10^4	8.350×10^3
C_{high}	3.9018×10^4	3.6836×10^4	2.9162×10^4

4. Fuzzy Approach

The number of infected individuals with crisp carrying capacity and initial conditions $I = 50$, $S = 1.5 \times 10^5$ and $V = 0 = 50$, are given in Tables 2 and 3. In reality the exact value of carrying capacity is strongly uncertain and hence it is difficult to be determined. In the following discussion we assume that there are three different levels of carrying capacity: low, medium, and high. However, there is no clear way to classify the numerical values of these carrying capacities into the low, medium, and high category. The numerical value of the boundary between the carrying capacity category is blur. To model the levelling of the carrying capacity, we use the fact that the carrying capacity level of 7.5×10^5 is regarded as the medium carrying capacity level. We then use triangular and trapezoidal fuzzy number to make classification. The membership function of the Triangular and Trapezoidal fuzzy number used in the simulation (in 10^5 individuals) are

$$\mu_{low}(C) = \begin{cases} 1, & if\ C \le 5 \\ \dfrac{C-7.5}{5-7.5}, & if\ 5 < C \le 7.5 \\ 0, & Otherwise \end{cases}$$

$$\mu_{medium}(C) = \begin{cases} 0, & if\ C \le 5 \\ \dfrac{C-5}{7.5-5}, & if\ 5 < C \le 7.5 \\ \dfrac{C-10}{7.5-10}, & if\ 7.5 < C \le 10 \\ 0, & Otherwise \end{cases}$$

$$\mu_{high}(C) = \begin{cases} 0, & if\ C \le 7.5 \\ \dfrac{C-7.5}{10-7.5}, & if\ 7.5 < C \le 10 \\ 1, & Otherwise \end{cases}$$

The graph of these fuzzy numbers is presented in Figure 2.

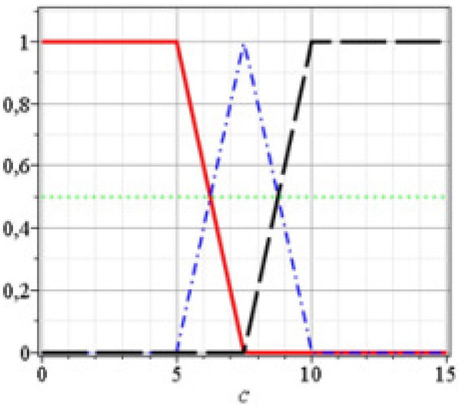

Figure 2. The graph of low-, medium-, and high- level of carrying capacities membership function (in red, blue, and black color, respectively). Horizontal line is in 10^5 individuals.

The number of the infectious dogs when there is no vaccination is presented in Figure 3, for low-, medium-, and high- level of carrying capacities. It is clear that the higher the carrying capacity, the higher the number of infectious dog population. Figures 4 shows the resulting infectious dog populations when there is 50% and 75% of vaccination respectively. The left part

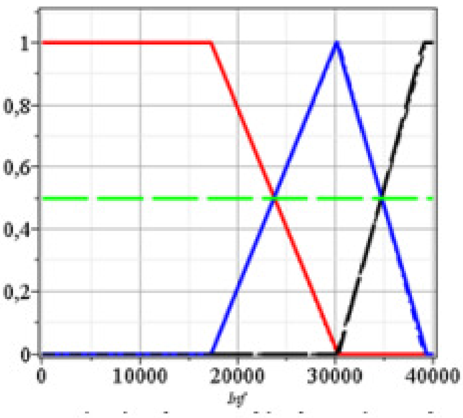

Figure 3. The graph of infectious dog population numbers at $t = 50$ (solid) and $t = 100$ (dashdot) for low-, medium-, and high- level of carrying capacities membership function (in red, blue, and black color, respectively) in the absence of vaccination. The initial values are crisp initial values $I = 50$, $S = 1.5 \times 10^5$ and $V = 0$.

of Figure 4 shows the graph of infectious dog population numbers at $t = 50$ (solid) and $t = 100$ (dash-dot) for low-, medium-, and high- level of carrying capacities membership function (in red, blue, and black color, respectively) for 50% vaccination rate. The graph for $t = 50$ and $t = 100$ coincide, showing that the population is reaching equilibrium. The initial values are the same as in the crisp approach, which are $I = 50$, $S = 1.5 \times 10^5$ and $V = 0$. The right part of Figure 4 shows the same thing as in the left part but for 75% vaccination rate at $t = 50$ (solid) and $t = 100$ (dash-dot). All centroid are lower than that for 50% vaccination indicating the more effective vaccination. The centroid for the low carrying capacity is close to zero.

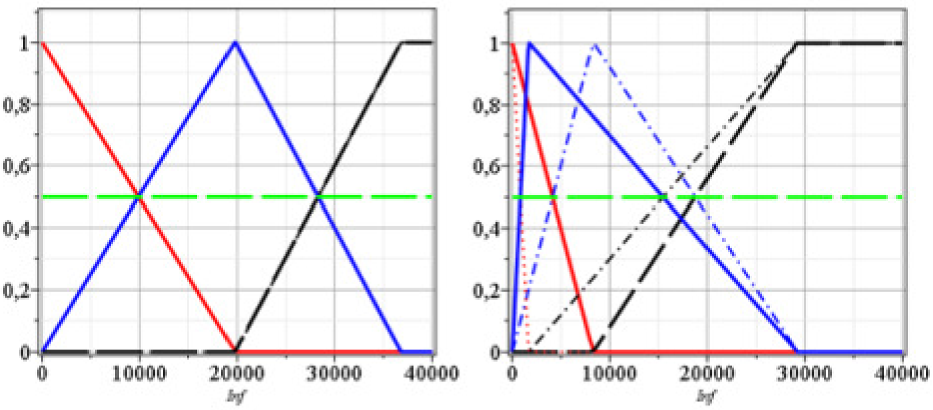

Figure 4. Infected dog population with 50% (left) and 75% (right) vaccination coverage.

5. Conclusions

In this paper, we develop a simple mathematical model and investigate the dynamics of rabies epidemics using crisp and fuzzy approaches to capture uncertainty in the parameters. We found that the results are similar for both approaches. That is, rabies epidemics may be interrupted when the carrying capacity and the transmission rate are not high. Given the complexity of rabies epidemics, a mathematical model for rabies epidemics incorporating human population and other control measures such as culling can be developed to understand the transmission dynamics of rabies. Furthermore, a spatial model can also be formulated to understand the spread of rabies. In general, our findings suggest that if the growth of dog population is limited and the potential contact between dogs is reduced, they may help to stop rabies epidemics.

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