

# Handling Uncertain Gross Margin and Water Demand in Agricultural Water Resources Management using Robust Optimization

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**Abstract.** In this paper, an application of Robust Optimization in agricultural water resource management problem under gross margin and water demand uncertainty is presented. Water resource management is a series of activities that includes planning, developing, distributing and managing the use of water resource optimally. Water resource management for agriculture can be one of the efforts to optimize the benefits of agricultural output. The objective function of agricultural water resource management problem is to maximizing total benefits by water allocation to agricultural areas covered by the irrigation network in planning horizon. Due to gross margin and water demand uncertainty, we assume that the uncertain data lies within ellipsoidal uncertainty set. We employ robust counterpart methodology to get the robust optimal solution.

## 1. Introduction

Refers to Sabouni and Mardani in [12], an agricultural irrigation network is comprised of a water supply source (river), a water distribution network (primary and branch irrigation canals), and demand centers (the areas that are covered by the irrigation network). Optimization of an irrigation system for the allocation of available water and cultivable area is one of the best methods of increasing a farmer's income and can be facilitated by an irrigation water distribution network. Irrigation water that is allocated to various crops is usually based on predictive factors such as the water demand for each crop and the availability of water for an irrigation network. Utilization of water resources can be done almost on every aspect of human life for both daily necessities and for businesses that use basic water or as a support, including business in agriculture. The fulfillment of raw water for agriculture is carried out with the development of an irrigation system, as mentioned in [13]. Referring to [10], farmers have the discretion to conduct farming enterprises that generate the highest financial returns.

The optimization of irrigation systems for the allocation of available water and land is one of the best methods to increase farmers' income and can make farmland well facilitated by irrigation water distribution networks, see [12]. Water allocated to various farms is usually based on predictable factors such as water demand for every farmland and water availability for irrigation networks. Uncertainty is an inherent aspect of predicting these factors. The main concern to reduce costs or increase profits from agricultural irrigation systems, without



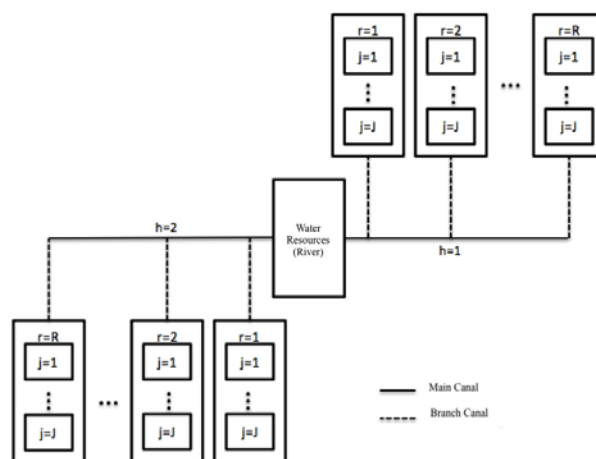
considering uncertainty, can cause many problems related to their future development. Possible consequences of not paying attention to uncertainty include the formation of less profits from expectations and possible system failures where non-fulfillment of demand or other constraints, see [14]. Other study about water distribution management and its robust version can be seen in [9] and [4].

Uncertainty in predicting data is an important issue in water resources management. There are several classical methods of overcoming the uncertainty parameters, among them Sensitivity Analysis and Stochastic Optimization. However, Sensitivity Analysis is just a tool for analyzing the goodness of the solution. It is not very helpful to generate a robust solution to data changes. In Stochastic Optimization, opportunity constraints can damage the convex nature and significantly increase the complexity of the original problem as stated in [15]. In this thesis, the authors use Robust Optimization approach of [1], which decision maker must choose strategy without knowing exact value taken by uncertain parameter.

## 2. Problem Description

### 2.1. Agricultural Water Resources Management

Water resources management is the activity of planning, developing, distributing and managing the optimal use of water resources. Management of water resources can be done almost on every aspect of human life for both daily necessities and for businesses that use basic water or as a support, including business in agriculture. Surface water and groundwater are the main water resources used to meet agricultural needs. But now most of the needs still rely on surface water sources, therefore the surface water source needs to be managed properly so as to provide benefits for the development of the agricultural sector. Surface water is all water found on the ground (rivers, lakes, springs, waterlogging). Flooded and flooded surface waters (lakes, reservoirs, swamps) and some subsurface water will collect and flow to form rivers and ends at sea. The process of water travel in the mainland occurs in the components of the hydrological cycle that form the system of Watershed (DAS). In this paper, the distribution of water from water resources to farms is illustrated as in Figure 1.



**Figure 1.** Illustration of Water Distribution from River to Agricultural Fields

In Figure 1, the river as water resource is illustrated to have two branches  $h$  which is the main channel for the left side of the river and the right side of the river. The main channels  $h$  distribute water to areas where  $r$  in each region  $r$  there are branch channels to drain water to some agricultural land  $j$  in  $r$  area. According to Rossiter and Wambeke [11], gross margin is

the average amount of income minus the average amount of all costs incurred on a certain land area within a certain period. Basically, gross margin for the farmer is the income from total agricultural production minus operational cost for agriculture.

The Linear Programming Model for water resource management in this paper refers to Sabouni and Mardani, 2013 [12] with objective function is to maximize the total profit of farmers in a horizon planning. The allocation of net flow of water on each farm land  $j$  in the  $r$  area of the main channel  $h$  denoted by  $q_{hrjt}$ . The total area allocation of agricultural land  $j$  in area  $r$  in horizon planning  $t$  for main channel  $h$  denoted by  $y_{hrjt}$ . The gross margin of the water allocation to land  $j$  in the  $r$  region of the channel  $h$  in the horizon planning  $t$  denoted by  $n_{hrjt}$ .

The objective function is obtained by calculating the difference between the gross margin of the water allocation to land  $j$  in the  $r$  region of the channel  $h$  in the horizon planning  $t$  and  $p_{hrjt}$ , i.e., the price of water allocated to land  $j$  in the  $r$  region of the channel  $h$  in horizon planning  $t$ , goals can be formulated as follows.

$$\max \sum_{h=1}^H \sum_{r=1}^R \sum_{j=1}^J \sum_{t=1}^T n_{hrjt} q_{hrjt} - \sum_{h=1}^H \sum_{r=1}^R \sum_{j=1}^J \sum_{t=1}^T p_{hrjt} q_{hrjt} \quad (1)$$

Furthermore, it is assumed that the water allocated to each land  $j$  is limited by the available irrigation water. Therefore, the first constraint is the amount of water allocated to each agricultural land  $j$  in the  $r$  area of the main channel  $h$  and the water lost from the main channel  $h$  for the agricultural land  $j$  in the  $r$  area with the total quantity parameter of water lost in the main channel  $h$  and the branch channel in region  $r$  is denoted by  $\delta_{hr}$  and should not exceed  $k_t$  i.e. the quantity of water available for each horizon plan  $t$ , so that the first constraint function can be formulated as follows:

$$\sum_{h=1}^H \sum_{r=1}^R \sum_{j=1}^J (1 + \delta_{hr}) q_{hrjt} \leq k_t, \quad \forall t \quad (2)$$

Then a second constraint function is created. the amount of water allocated to land  $j$  in the  $r$  region of the main channel  $h$  in the horizontal and wasted water planning in the main channel  $h$  and the branch channel in  $r$  area in horizon planning  $t$  should not exceed the value of  $c_{ht}$ , i.e., capacity the main channel  $h$  on horizon planning  $t$ , so obtained.

$$\sum_{r=1}^R \sum_{j=1}^J (1 + \delta_{hr}) q_{hrjt} \leq c_{ht}, \quad \forall h, t \quad (3)$$

Subsequently, a third constraint function is formed which is the sum of the total water allocated to land  $j$  in the  $r$  region of the main channel  $h$  in the horizontal and wasted water planning in the branch channel in  $r$  area with the water level wasted in the branch channel at  $r$  region of the canal the primary  $h$  for the land  $j$  is denoted by, should not exceed the value of  $b_{hrjt}$  is the capacity of the branch channel in  $r$  region for the total area of agricultural land  $j$  in horizon planning  $t$ , so that it is obtained:

$$\sum_{j=1}^J (1 + \xi_{hrj}) q_{hrjt} \leq b_{hrjt}, \quad \forall h, r, t \quad (4)$$

The fourth constraint function, i.e., water allocated to land  $j$  in the  $r$  area of the main channel  $h$  in horizon planning  $t$  shall be more than or equal to the minimum demand for irrigation water and less than or equal to the maximum demand for irrigation water, meaning that any land

must obtain the amount of water sufficient, not less or more than the limit of its request, so obtained:

$$(d_{\min})_{hrjt} \leq q_{hrjt} \leq (d_{\max})_{hrjt}, \quad \forall h, r, j, t \quad (5)$$

The total area of agricultural land  $j$  in area  $r$  in the horizon year plan  $t$  for the main channel  $h$  denoted by  $y_{hrjt}$  is influenced by the irrigation demand unit of the agricultural land  $j$  for region  $r$  on the main channel  $h$  denoted by  $w_{hrj}$ , and the flow water allocated to land  $j$  in region  $r$  of the main channel  $h$  in horizon planning  $t$ , is formulated into the fifth constraint function as follows

$$w_{hrj}y_{hrjt} = q_{hrjt}, \forall h, r, j, t \quad (6)$$

To obtain the value of the irrigation demand unit of the  $j$  plant in the  $r$  area for the main channel  $h$ , suppose  $ni_{hrj}$  is the irrigation network needed for the  $j$  plant in the  $r$  area on the main channel  $h$  and  $SR$  is an irrigation efficiency worth  $0 < SR \leq 1$ , it can be calculated as follows:

$$w_{hrj} = \frac{ni_{hrj}}{SR} \quad (7)$$

At the last constraint all decision variables must be non-negative, so they are obtained

$$q_{hrjt}, y_{hrjt} \geq 0, \quad \forall h, r, j, t \quad (8)$$

Thus the complete the linear programming model for water resource management in agriculture (Sabouni and Mardani [12]) is formulated in the following equations:

$$\max \sum_{h=1}^H \sum_{r=1}^R \sum_{j=1}^J \sum_{t=1}^T n_{hrjt} q_{hrjt} - \sum_{h=1}^H \sum_{r=1}^R \sum_{j=1}^J \sum_{t=1}^T p_{hrjt} q_{hrjt} \quad (9)$$

$$s.t \sum_{h=1}^H \sum_{r=1}^R \sum_{j=1}^J (1 + \delta_{hr}) q_{hrjt} \leq k_t, \quad \forall t \quad (10)$$

$$\sum_{r=1}^R \sum_{j=1}^J (1 + \delta_{hr}) q_{hrjt} \leq c_{ht}, \quad \forall h, t \quad (11)$$

$$\sum_{j=1}^J (1 + \xi_{hrj}) q_{hrjt} \leq b_{hrjt}, \quad \forall h, r, t \quad (12)$$

$$(d_{\min})_{hrjt} \leq q_{hrjt} \leq (d_{\max})_{hrjt}, \quad \forall h, r, j, t \quad (13)$$

$$w_{hrj}y_{hrjt} = q_{hrjt}, \forall h, r, j, t \quad (14)$$

$$q_{hrjt}, y_{hrjt} \geq 0, \forall h, r, j, t. \quad (15)$$

### 3. Methodology: Robust Optimization

In this section we present a brief discussion about the theory of Robust Optimization (RO) as proposed by [1] and discussed in [6], [7], also discussed in [3] and [2]. Let  $c \in R^n, b \in R^m$  and  $A \in R^m \times R^n$  be the parameters of the canonical linear programming problem

$$\min_x \{c^T x : Ax \leq b\} \quad (LP)$$

The main idea of Robust Optimization is we are interested in a version of (LP) problem in which the data  $(c, A, b)$  are uncertain, but are known to reside in an uncertain set  $U$ . In this case the

(LP) is not a single deterministic problem, but a family of problems, one for each realisation  $(c, A, b) \in U$ :

$$\min_{x \in R^n} \{c^T x : Ax \leq b\}_{(c,A,b) \in U} \quad (\text{ULP}).$$

We assume that the decision environment is such that

- (i) Decision is offline: the entire decision vector  $x$  is to be fixed prior to knowing which value the actual parameters take "here-and-now-decision".
- (ii) In the dynamic (ULP) case only part of the variables  $(x_1, \dots, x_n)$  need to be determined off-line. The rest may be determined after some of the uncertain parameters become known ("wait and see" decision).
- (iii) The prior information on the data  $(c, A, b)$  is crude and is captured by a compact uncertainty set  $U$ .
- (iv) The inequality constraints  $Ax \leq b$  are hard constraints, i.e., they must all be satisfied whenever the uncertain parameters reside in  $U$ .

Under the above conditions the robust optimization approach converts the uncertain family of the problems (ULP) into the following single deterministic problem, which we call *Robust Counterpart*:

$$\pi^* = \min_{x \in R^n} \{c^T x : Ax \leq b, (c, A, b) \in U\} \quad (\text{RC}).$$

A vector  $x^*$  is called a *robust optimal solution* if for all realizations  $(c, A, b) \in U$ ,  $x^*$  is feasible and the value of objective function is guaranteed to be at most  $\pi^*$ . Problem (RC) can be written equivalently as a problem with a linear certain objective function and only uncertain constraints as follows:

$$\min t \quad (16)$$

$$s.t. \ c^T x - t \leq 0, \quad (17)$$

$$a_i^T x - b_i \leq 0, i = 1, \dots, m, \forall (c, A, b) \in U \quad (18)$$

### 3.1. The Treatment of Robust Linear Optimization (RLO)

We cite from [6] that for the treatment of Robust Linear Optimization (RLO) we will assume without loss of generality that,

- (i) Firstly, the objective  $c^T x$  is certain. If there is uncertainty in the objective, we can reformulate the problem such that this uncertainty appears in a constraint see (16).
- (ii) Secondly, the right-hand-side  $b$  is certain. If  $b$  is uncertain, we can introduce an extra variable  $x_{n+1}$  and change the problem into

$$\min c^T x \quad (19)$$

$$s.t. \ a_i^T x - b_i x_{n+1} \leq 0, \quad (20)$$

$$x_{n+1} = 1, i = \dots, m, \forall (A, b) \in U. \quad (21)$$

This assumption is made to make the notation for the uncertainty region and the resulting robust counterpart formulations easier.

- (iii) Third, the robustness with respect to  $U$  can be formulated constraint-wise and the last, the uncertainty set  $U$  is closed and convex.

Notes that because this inequality on  $x$  is of the semiinfinite type (it should be satisfied for an infinite number of values of the parameters  $A$  and  $b$ ), it cannot directly be efficiently solved by standard optimization methods. The challenge is to find for which types of uncertainty sets problem (RC) can be reformulated into a tractable optimization problem.

### 3.2. Types of uncertainty sets

The challenge is to find  $U$  for which types of uncertainty sets problem can be reformulated into a tractable optimization problem. Since the robustness with respect to  $U$  can be formulated constraint-wise, thus we can reformulate each constraint which involves the uncertain data. Refers to [6], consider the canonical robust semi-infinite constraint

$$a^T x - b \leq 0, \forall (a, b) \in U. \quad (22)$$

Here  $a$  is a vector in  $R^n$  and  $b$  which are the general representatives of  $a_i$  and  $b_i$ .

Similarly,  $U$  stands for  $U_i$ . Its convenient to describe the uncertainty parameters  $a$  and  $b$  and the uncertainty set  $U$  in terms of a primitive factor  $\zeta \in R^L$ . Namely,

$$a = \bar{a} + Q\zeta, b = \bar{b} + q^T \zeta \quad (23)$$

where  $\bar{a} \in R^n, Q \in R^{n \times L}, \bar{b} \in R$  and  $q \in R^L$  and

$$U = \left\{ \begin{pmatrix} a = \bar{a} + Q\zeta \\ b = \bar{b} + q^T \zeta \end{pmatrix} : \zeta \in Z \right\} \quad (24)$$

where  $Z \subset R^L$  is the uncertainty set for the primitive factors. The fixed vector  $\bar{a}$  and the scalar  $\bar{b}$  will thereafter be called nominal. In the view of representation (24) the alternative formulation of (22). Replacing  $a$  and  $b$  by their expressions in  $\zeta$ , we have

$$(\bar{a}^T x - \bar{b}) + (Q^T x - q)^T \zeta \leq 0, \forall \zeta \in Z. \quad (25)$$

**3.2.1. Case 1: Using Interval or Box Uncertainty Set** In general, a tractable robust counterpart formulation for RLO of (22) with interval (or box) uncertainty regions can be stated as the following formulation.

$$(\bar{a} + P\zeta)^T x \leq b, \forall \zeta : \|\zeta\|_\infty \leq \mu. \quad (26)$$

This problem is a semi-infinite programming problem.

**3.2.2. Case 2: Using Ellipsoidal Uncertainty Set** Refers to [6], in the case of ellipsoidal uncertainty, the robust counterpart of (22) becomes

$$\begin{aligned} & (\bar{a} + P\zeta)^T x \leq b, \forall \zeta : \|\zeta\|_2 \leq \mu. \\ & \equiv \bar{a}^T x + \max_{\zeta : \|\zeta\|_2 \leq \mu} (P^T x)^T \zeta \leq b. \end{aligned} \quad (27)$$

It can easily be verified that

$$\max_{\zeta : \|\zeta\|_2 \leq \mu} (P^T x)^T \zeta = \mu \|P^T x\|_2. \quad (28)$$

Hence  $x$  satisfies (27) if and only if

$$\bar{a}^T x + \mu \|P^T x\|_2 \leq b. \quad (29)$$

Now the constraint does not have the semi-infinite structure as the original one in (27). This is a conic quadratic programming problem. Again as in the case of box uncertainty, in the final robust counterpart an extra safety term  $\mu \|P^T x\|_2$  is added to account for uncertainty and it depends on the value of  $x$ .

#### 4. Result and Discussion

##### 4.1. Data Uncertainty In Agricultural Water Resources Management

Assume that the model of water resource management problems in agriculture contains uncertainty on gross margin and water demand, i.e.,

$$n_{hrjt}, (d_{\min})_{hrjt}, (d_{\max})_{hrjt} \in U. \quad (30)$$

Under Robust Optimization assumption the right-hand side must be a certain parameter, thus define a new variable  $x_0 = 1$ . Based on these assumptions, the model of the water resource management problem can be reformulated into a robust counterpart model as follows:

$$\max \omega \quad (31)$$

$$s.t \left[ \sum_{h=1}^H \sum_{r=1}^R \sum_{j=1}^J \sum_{t=1}^T n_{hrjt} q_{hrjt} - p_{hrjt} q_{hrjt} \right] - \omega \leq 0, \quad (32)$$

$$\sum_{h=1}^H \sum_{r=1}^R \sum_{j=1}^J (1 + \delta_{hr}) q_{hrjt} \leq k_t, \forall t \quad (33)$$

$$\sum_{r=1}^R \sum_{j=1}^J (1 + \delta_{hr}) q_{hrjt} \leq c_{ht}, \forall h, t \quad (34)$$

$$\sum_{j=1}^J (1 + \xi_{hr}) q_{hrjt} \leq b_{hrjt}, \forall h, r, t \quad (35)$$

$$q_{hrjt} - (d_{\max})_{hrjt} x_0 \leq 0, \forall h, r, j, t \quad (36)$$

$$q_{hrjt} - (d_{\min})_{hrjt} x_0 \geq 0, \forall h, r, j, t \quad (37)$$

$$w_{hrjt} y_{hrjt} - q_{hrjt} = 0, \forall h, r, j, t \quad (38)$$

$$q_{hrjt}, y_{hrjt} \geq 0, \forall h, r, j, t \quad (39)$$

$$x_0 = 1 \quad (40)$$

$$n_{hrjt}, (d_{\min})_{hrjt}, (d_{\max})_{hrjt} \in U. \quad (41)$$

##### 4.2. Robust Version for Water Management Problem with Box Uncertainty Set

For water resource management problems in agriculture with uncertainty as described by simple intervals as follows:

$$U = \left\{ (n, d_{\max}, d_{\min}) : \begin{aligned} & (n_{hrjt})_{\min} \leq n_{hrjt} \leq (n_{hrjt})_{\max} \\ & \left( (d_{\max})_{hrjt} \right)_{\min} \leq (d_{\max})_{hrjt} \leq \left( (d_{\max})_{hrjt} \right)_{\max}, \forall h, r, j, t \\ & \left( (d_{\min})_{hrjt} \right)_{\min} \leq (d_{\min})_{hrjt} \leq \left( (d_{\min})_{hrjt} \right)_{\max} \end{aligned} \right\} \quad (42)$$

Based on (Hertog, 2015), we may select the set of  $U$  uncertainties that satisfy representatives of (19) with the following nominal data conditions.

$$\begin{aligned} \bar{n}_{hrjt} &= \frac{(n_{hrjt})_{\max} + (n_{hrjt})_{\min}}{2} \\ (Q_{hrjt})_n &= \text{diag} \left( \frac{(n_{hrjt})_{\max} - (n_{hrjt})_{\min}}{2} \right) \end{aligned} \quad (43)$$

$$\begin{aligned} (\bar{d}_{\max})_{hrjt} &= \frac{((d_{\max})_{hrjt})_{\max} + ((d_{\max})_{hrjt})_{\min}}{2} \\ (Q_{hrjt})_{d_{\max}} &= \text{diag} \left( \frac{((d_{\max})_{hrjt})_{\max} - ((d_{\max})_{hrjt})_{\min}}{2} \right) \end{aligned} \quad (44)$$

$$\begin{aligned} (\bar{d}_{\min})_{hrjt} &= \frac{((d_{\min})_{hrjt})_{\max} + ((d_{\min})_{hrjt})_{\min}}{2} \\ (Q_{hrjt})_{d_{\min}} &= \text{diag} \left( \frac{((d_{\min})_{hrjt})_{\max} - ((d_{\min})_{hrjt})_{\min}}{2} \right) \end{aligned} \quad (45)$$

Notice the constraint equation containing uncertain parameters  $n_{hrjt}$ , e.g.

$$n_{hrjt} = \bar{n}_{hrjt} + (Q_{hrjt})_n (\zeta_{hrjt})_n. \quad (46)$$

Now we derive first the part of (25) that contains the uncertainty parameter as follows.

$$\begin{aligned} \sum_{h=1}^H \sum_{r=1}^R \sum_{j=1}^J \sum_{t=1}^T (n_{hrjt} q_{hrjt} - p_{hrjt} q_{hrjt}) &= \sum_{h=1}^H \sum_{r=1}^R \sum_{j=1}^J \sum_{t=1}^T ((\bar{n}_{hrjt} + (Q_{hrjt})_n (\zeta_{hrjt})_n) q_{hrjt} - p_{hrjt} q_{hrjt}) \\ &= \left[ \sum_{h=1}^H \sum_{r=1}^R \sum_{j=1}^J \sum_{t=1}^T (\bar{n}_{hrjt} q_{hrjt} - p_{hrjt} q_{hrjt} + (Q_{hrjt})_n (\zeta_{hrjt})_n q_{hrjt}) \right] \\ &= \left( \sum_{h=1}^H \sum_{r=1}^R \sum_{j=1}^J \sum_{t=1}^T (\bar{n}_{hrjt} q_{hrjt} - p_{hrjt} q_{hrjt}) \right) + \\ &\quad \max_{\zeta_n: |(\zeta_{hrjt})_n| \leq \sigma} \sum_{h=1}^H \sum_{r=1}^R \sum_{j=1}^J \sum_{t=1}^T (Q_{hrjt})_n (\zeta_{hrjt})_n q_{hrjt} \end{aligned} \quad (47)$$

Thus the robust counterpart of (25) is obtained as follows:

$$\begin{aligned} &\left[ \left( \sum_{h=1}^H \sum_{r=1}^R \sum_{j=1}^J \sum_{t=1}^T (\bar{n}_{hrjt} q_{hrjt} - p_{hrjt} q_{hrjt}) \right) + \sigma \left\| \sum_{h=1}^H \sum_{r=1}^R \sum_{j=1}^J \sum_{t=1}^T (Q_{hrjt})_n q_{hrjt} \right\|_1 \right] - \omega = \\ &\left[ \left( \sum_{h=1}^H \sum_{r=1}^R \sum_{j=1}^J \sum_{t=1}^T (\bar{n}_{hrjt} q_{hrjt} - p_{hrjt} q_{hrjt}) \right) + \sigma \left\| \sum_{h=1}^H \sum_{r=1}^R \sum_{j=1}^J \sum_{t=1}^T (Q_{hrjt})_n q_{hrjt} \right\|_1 \right] - \omega \leq 0 \end{aligned} \quad (48)$$

Use the same way for constraints that contain uncertain parameters

$$(d_{\max})_{hrjt} = (\bar{d}_{\max})_{hrjt} + (Q_{hrjt})_{d_{\max}} (\zeta_{hrjt})_{d_{\max}} \quad (49)$$

$$[(d_{\min})_{hrjt} = (\bar{d}_{\min})_{hrjt} + (Q_{hrjt})_{d_{\min}} (\zeta_{hrjt})_{d_{\min}}] \quad (50)$$

then we obtain the robust counterpart as follows:

$$\begin{aligned} q_{hrjt} - (d_{\max})_{hrjt} x_0 &= q_{hrjt} - ((\bar{d}_{\max})_{hrjt} + (Q_{hrjt})_{d_{\max}} (\zeta_{hrjt})_{d_{\max}}) x_0 \\ &= q_{hrjt} - (\bar{d}_{\max})_{hrjt} x_0 - (Q_{hrjt})_{d_{\max}} (\zeta_{hrjt})_{d_{\max}} x_0 \\ &= q_{hrjt} - (\bar{d}_{\max})_{hrjt} x_0 - \max_{\zeta_n: |(\zeta_{hrjt})_n| \leq \varepsilon} |(Q_{hrjt})_{d_{\max}} (\zeta_{hrjt})_{d_{\max}} x_0| \\ &= q_{hrjt} - (\bar{d}_{\max})_{hrjt} x_0 - \varepsilon \left\| (Q_{hrjt})_{d_{\max}} x_0 \right\|_1 \\ &= q_{hrjt} - (\bar{d}_{\max})_{hrjt} x_0 - \varepsilon \left| (Q_{hrjt})_{d_{\max}} x_0 \right| \leq 0 \end{aligned} \quad (51)$$

and

$$\begin{aligned}
 q_{hrjt} - (d_{\min})_{hrjt} &= q_{hrjt} - \left( (\bar{d}_{\min})_{hrjt} + (Q_{hrjt})_{d_{\min}} (\zeta_{hrjt})_{d_{\min}} \right) x_0 \\
 &= q_{hrjt} - (\bar{d}_{\min})_{hrjt} x_0 - (Q_{hrjt})_{d_{\min}} (\zeta_{hrjt})_{d_{\min}} x_0 \\
 &= q_{hrjt} - (\bar{d}_{\min})_{hrjt} x_0 - \max_{\zeta_n: |(\zeta_{hrjt})_n| \leq \tau} \left| (Q_{hrjt})_{d_{\min}} (\zeta_{hrjt})_{d_{\min}} x_0 \right| \\
 &= q_{hrjt} - (\bar{d}_{\min})_{hrjt} x_0 - \tau \left\| (Q_{hrjt})_{d_{\min}} x_0 \right\|_1 \\
 &= q_{hrjt} - (\bar{d}_{\min})_{hrjt} x_0 - \tau \left| (Q_{hrjt})_{d_{\min}} x_0 \right| \geq 0
 \end{aligned} \tag{52}$$

So that obtained model robust counterpart for water resource management problem in agriculture with box uncertainty approach as follows.

$$\begin{aligned}
 &\max \omega \\
 &s.t \\
 &\left[ \left( \sum_{h=1}^H \sum_{r=1}^R \sum_{j=1}^J \sum_{t=1}^T (\bar{n}_{hrjt} q_{hrjt} - p_{hrjt} q_{hrjt}) \right) + \sigma \left\| \sum_{h=1}^H \sum_{r=1}^R \sum_{j=1}^J \sum_{t=1}^T (Q_{hrjt})_n q_{hrjt} \right\| \right] - \omega \leq 0 \\
 &\sum_{h=1}^H \sum_{r=1}^R \sum_{j=1}^J (1 + \delta_{hr}) q_{hrjt} \leq k_t, \forall t \\
 &\sum_{r=1}^R \sum_{j=1}^J (1 + \delta_{hr}) q_{hrjt} \leq c_{ht} \\
 &q_{hrjt} - (\bar{d}_{\max})_{hrjt} x_0 - \varepsilon \left| (Q_{hrjt})_{d_{\max}} x_0 \right| \leq 0, \forall h, r, j, t \\
 &q_{hrjt} - (\bar{d}_{\min})_{hrjt} x_0 - \tau \left| (Q_{hrjt})_{d_{\min}} x_0 \right| \geq 0, \forall h, r, j, t \\
 &w_{hrjt} y_{hrjt} - q_{hrjt} = 0, \forall h, r, j, t \\
 &q_{hrjt}, y_{hrjt} \geq 0, \forall h, r, j, t \\
 &x_0 = 1
 \end{aligned} \tag{53}$$

#### 4.3. Robust Version for Water Management Problem with Ellipsoidal Uncertainty Set

For the case of ellipsoidal uncertainty set, assume that the uncertain data lies in  $n_{hrjt}$ ,  $(d_{\min})_{hrjt}$  and  $(d_{\max})_{hrjt}$ . The following is how we define the robust counterpart for  $n_{hrjt}$ .

$$\begin{aligned}
 &\left[ \sum_{h=1}^H \sum_{r=1}^R \sum_{j=1}^J \sum_{t=1}^T (n_{hrjt} q_{hrjt} - p_{hrjt} q_{hrjt}) \right] - \omega \leq 0, \\
 &\equiv \left[ \left( \sum_{h=1}^H \sum_{r=1}^R \sum_{j=1}^J \sum_{t=1}^T (\bar{n}_{hrjt} q_{hrjt} - p_{hrjt} q_{hrjt}) \right) + \sigma \left\| \sum_{h=1}^H \sum_{r=1}^R \sum_{j=1}^J \sum_{t=1}^T (Q_{hrjt})_n q_{hrjt} \right\|_2 \right] - \omega \\
 &= \left[ \left( \sum_{h=1}^H \sum_{r=1}^R \sum_{j=1}^J \sum_{t=1}^T (\bar{n}_{hrjt} q_{hrjt} - p_{hrjt} q_{hrjt}) \right) + \sigma \sqrt{\sum_{h=1}^H \sum_{r=1}^R \sum_{j=1}^J \sum_{t=1}^T ((Q_{hrjt})_n q_{hrjt})^2} \right] - \omega \leq 0.
 \end{aligned} \tag{54}$$

The following is how we define the robust counterpart for  $(d_{\max})_{hrjt}$ :

$$\begin{aligned}
 q_{hrjt} - (d_{\max})_{hrjt} &\equiv q_{hrjt} - (\bar{d}_{\max})_{hrjt} x_0 - \varepsilon \left\| (Q_{hrjt})_{d_{\max}} x_0 \right\|_2 \\
 &= q_{hrjt} - (\bar{d}_{\max})_{hrjt} x_0 - \varepsilon \sqrt{(Q_{hrjt})_{d_{\max}} x_0^2} \geq 0
 \end{aligned} \tag{55}$$

The following is how we define the robust counterpart for  $(d_{\min})_{hrjt}$ .

$$\begin{aligned} q_{hrjt} - (d_{\min})_{hrjt} &\equiv q_{hrjt} - (\bar{d}_{\min})_{hrjt}x_0 - \tau \left\| (Q_{hrjt})_{d_{\min}} x_0 \right\|_2 \\ &= q_{hrjt} - (\bar{d}_{\min})_{hrjt}x_0 - \tau \sqrt{(Q_{hrjt})_{d_{\min}} x_0^2} \leq 0. \end{aligned} \quad (56)$$

Thus the complete formulation for Robust Version for Water Management Problem with Ellipsoidal Uncertainty Set can be written as follows.

$$\begin{aligned} \max \quad & \omega \\ \text{s.t.} \quad & \left[ \left( \sum_{h=1}^H \sum_{r=1}^R \sum_{j=1}^J \sum_{t=1}^T (\bar{n}_{hrjt} q_{hrjt} - p_{hrjt} q_{hrjt}) \right) + \sigma \sqrt{\sum_{h=1}^H \sum_{r=1}^R \sum_{j=1}^J \sum_{t=1}^T ((Q_{hrjt})_n q_{hrjt})^2} \right] - \omega \leq 0 \\ & \sum_{h=1}^H \sum_{r=1}^R \sum_{j=1}^J (1 + \delta_{hr}) q_{hrjt} \leq k_t, \forall t \\ & \sum_{r=1}^R \sum_{j=1}^J (1 + \delta_{hr}) q_{hrjt} \leq c_{ht}, \forall h, t \\ & \sum_{j=1}^J (1 + \xi_{hr}) q_{hrjt} \leq b_{hrjt}, \forall h, r, t \\ & q_{hrjt} - (\bar{d}_{\max})_{hrjt}x_0 - \varepsilon \sqrt{((Q_{hrjt})_{d_{\max}} x_0)^2} \leq 0, \forall h, r, j, t \\ & q_{hrjt} - (\bar{d}_{\min})_{hrjt}x_0 - \tau \sqrt{((Q_{hrjt})_{d_{\min}} x_0)^2} \geq 0, \forall h, r, j, t \\ & w_{hrjt} y_{hrjt} - q_{hrjt} = 0, \forall h, r, j, t \\ & q_{hrjt}, y_{hrjt} \geq 0, \forall h, r, j, t \\ & x_0 = 1 \end{aligned} \quad (57)$$

#### 4.4. Numerical Experiment

The data used is secondary data. For water availability data on water resources, the quantity of water in the main canal, the quantity of water in branch channels, and the water demand unit is referenced from Sabouni and Mardani [12], for the price of water referenced from [5], and for gross margin of each plant is referenced from [8].

The data quantity of water available for irrigation networks in two years of horizon planning are  $k_1 = 470 \times 10^6 m^3$ ,  $k_2 = 343 \times 10^6 m^3$ . The irrigation flow required for each crop in a different area is presented in the following Table 1.

From the irrigation flow data required for each plant in different areas, with an irrigation efficiency of 35 percentage, an irrigation demand unit of plant  $j$  in  $r$  area for main channel  $h$  ( $w_{hrj}$ ) as shown in Table 2. Main channel capacity for left side ( $c_{1t}$ ) is  $1.577 \times 10^6 m^3$  per year and right side ( $c_{2t}$ ) is  $473 \times 10^6 m^3$  per year. The main channel capacity is assumed to be constant for each year. The capacity of branch canals for each region ( $b_{hrjt}$ ) is presented in Table 3. Assume 1 percentage of water allocated per kilometer is lost on the main channel  $h$ . The distance between the starting point of each branch's branch and the dam that divides the main channel is presented in Table 4. The water lost in each crop is assumed to be 15 percentage. The price of water for agriculture ( $p_{hrjt}$ ) is 0.8 USD/1000m<sup>3</sup> or 25,8065 rial/m<sup>3</sup>. Revenues earned from each crop ( $n_{hrjt}$ ) and water demand from each crop are presented on Table 5.

**Table 1.** Water Distribution for different area and plants

Area	Plants ( $m^3/ha$ )			
	Wheat $j = 1$	Barley $j = 2$	Rice $j = 3$	Alfalfa $j = 4$
Mobarake $r = 1$	4.970	4.460	7.090	9.280
Nadjafabad $r = 2$	4.270	3.590	7.940	9.310
Lenjan $r = 3$	4.270	3.590	6.890	4.460
Flavarjan $r = 4$	4.910	6.400	6.890	4.460

**Table 2.** Unit of irrigation demand from plant  $j$  in region  $r$  for main channel  $h$

Main Canal	Area	The required irrigation flow / irrigation efficiency ( $m^3/ha$ )			
		Wheat $j = 1$	Barley $j = 2$	Rice $j = 3$	Alfalfa $j = 4$
Left side $h = 1$	Mobarake $r = 1$	14.200,00	12.742,86	20.257,14	26.514,29
	Nadjafabad $r = 2$	12.200,00	10.257,14	22.685,71	26.600,00
	Lenjan $r = 3$	12.200,00	10.257,14	19.685,71	12.742,86
	Falavarjan $r = 4$	14.028,57	18.285,71	19.685,71	12.742,86
Right side $h = 2$	Mobarake $r = 1$	14.200,00	12.742,86	20.257,14	26.514,29
	Nadjafabad $r = 2$	-	-	-	-
	Lenjan $r = 3$	-	-	-	-
	Falavarjan $r = 4$	14.028,57	18.285,71	19.685,71	12.742,86

**Table 3.** Water Distribution for different area and plants

Main Canal	Kapasitas ( $m^3 \times 10^6$ )			
	Mobarakeh	Nadjafabad	Lenjan	Flavarjan
Leftside	79	349	79	349
Rightside	213	-	-	213

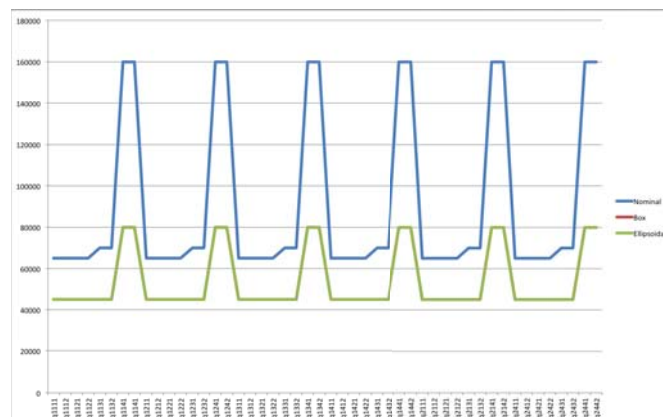
**Table 4.** The distance between the starting point and the dam.

Main Canal	Distances ( <i>kilometer</i> )			
	Mobarakeh	Nadjafabad	Lenjan	Flavarjan
Leftside	10,3	52,1	24,2	18,2
Rightside	30,6	-	-	35,1

**Table 5.** The distance between the starting point and the dam.

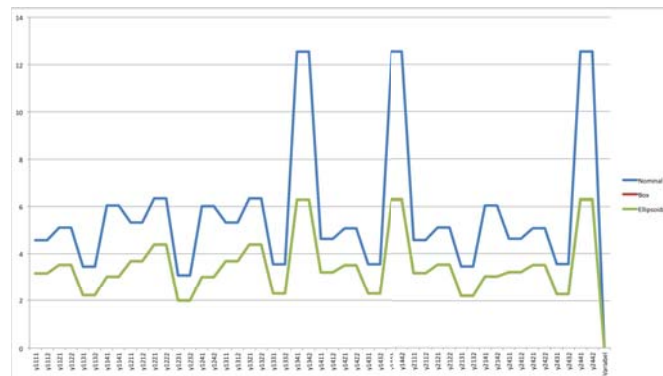
Crops	Revenues ( <i>rial/m<sup>3</sup></i> )			
	Mobarakeh	Nadjafabad	Lenjan	Flavarjan
Leftside	10,3	52,1	24,2	18,2
Rightside	30,6	-	-	35,1

The nominal model for water resource management problems in agriculture in numerical simulations involves 162 constraints and 96 variables, while for the Robust Optimization model involving 164 constraints for box uncertainty approach and 163 constraints for ellipsoidal uncertainty approach and both involving 97 variables with additional variables  $\omega$  as delimiter of purpose function. In the nominal model it takes 405 milliseconds and uses 11.54 MiB memory, the Robust Optimization Model with the uncertainty box takes 381 milliseconds using 12 MiB memory, and in Robust Optimization Model with ellipsoidal uncertainty it takes 144 milliseconds using 6.79 MiB. In this numerical experiment, the result for box and ellipsoidal uncertainty give the same solution, as seen in Figure 2 and 3. The objective function result can be seen in Figure 4.

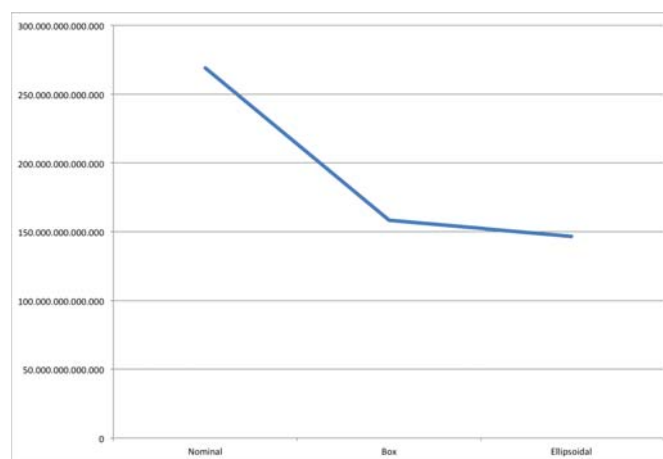
**Figure 2.** Water Allocation Calculation Result for Nominal, Box and Ellipsoidal Model

## 5. Conclusion and Future Research

Robust optimization is applied to water resource management issues in agriculture with uncertainty of gross margin of water demand. The robust counterpart form becomes a linear optimization problem in case of box uncertainty set and becomes conic quadratic optimization model in case of ellipsoidal uncertainty set. This shows us that the robust counterpart is guaranteed computationally tractable. Referring to Sabouni and Mardani [12] this study can be continued taking into consideration the uncertainties in the quantity of water available in water resource management issues in agriculture. In addition, referring to Hertog [6] on Robust Optimization, the problem of water resources management in agriculture can be continued with the formulation of Adjustable Robust Counterpart.



**Figure 3.** Area Allocation Calculation Result for Nominal, Box and Ellipsoidal Model



**Figure 4.** Maximum Profit for Nominal, Box and Ellipsoidal Model

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