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A hybrid Dantzig-Wolfe, Benders decomposition and column generation procedure for multiple diet production planning under uncertainties

A Udomsungworagul[1*], P Charnsethikul[2]

[1]Network Development Division, Strategic Retail Marketing Department, PTT Public Company Limited, Bangkok, Thailand
[2]Department of Industrial Engineering, Faculty of Engineering, Kasetsart University, Bangkok, Thailand

*Corresponding author: atit.u@pttplc.com

Abstract. This article introduces methodology to solve large scale two-phase linear programming with a case of multiple time period animal diet problems under both nutrients in raw materials and finished product demand uncertainties. Assumption of allowing to manufacture multiple product formulas in the same time period and assumption of allowing to hold raw materials and finished products inventory have been added. Dantzig-Wolfe decompositions, Benders decomposition and Column generations technique has been combined and applied to solve the problem. The proposed procedure was programmed using VBA and Solver tool in Microsoft Excel. A case study was used and tested in term of efficiency and effectiveness trade-offs.

1. Introduction

In the animal feed commerce or industry has a majority costs in feed mix. Feed producers need to consider finding the optimal feed mix with the minimum cost fulfilling nutrients requirements. The classical diet model was first studied by (Stigler. 1945). Since then, the feed mix production planning has also been an important decision for avoiding stock out or over stock manufactured that effect its related cost.

This paper propose methodology for solving stochastic linear programming in form of two-stage linear programming (Wagner. 1975) combining two type of problem, diet problem under uncertainty of nutrient in feed mix by (Thammanivit and Charnsethikul. 2013) and inventory problem under uncertainty of mixed products demand in finite multiple time period by (Pisit *et al*. 2008). The assumption of allowing to produce many product formulas in same time period is added. Dantzig-Wolfe decomposition, column generation and Benders decomposition algorithms have been used to solve this proposed model. Dantzig-Wolfe decomposition (Dantzig and Wolfe. 1960) has been used to solve large scale block angular problem of production planning part in the model. Column generation by Charnsethikul (2011) was proposed on case of relaxing the infinite number decision variables problem controlled from product tolerances allowed to manufacture possibility. Benders decomposition (Infanger. 1994) can also be applied to the case of large scale diet problem.

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2. Mathematical model

The objective of model in this article (Atit and Peerayuth 2017) is to seek for the production plan of manufacturing and to store feed mix products in each period against uncertain demands. The production planning part dealt with raw materials purchasing cost, raw materials holding cost, finished products holding cost and finished product back ordering cost with inventory balance constraint as equations (1)-(4) are considered. Set up cost between successive periods are not considered in this model. The corresponding diet model part is related with corrective action cost of rework for out of specification nutrients manufactured with the constraint of nutrition mixed from uncertain nutrients of raw materials and the constraint of raw materials proportion boundary. The hypothesis of allowing to produce many product formulas in the same time period is assumed and presented in equations (5)-(12).

$$\text{Minimize } z = \sum_{t=1}^{T}\left(\sum_{j=1}^{\infty} c_{jt}x_{jt} + g_{R_{i,t}} I_{R_{i,t}} + h_{R_{i,t}} Q_{i,t} + g_{D_t} I^{+}_{D_{k,t}} + h_{D_t} I^{-}_{D_{k,t}}\right) \tag{1}$$

Subject to

$$\sum_{j=1}^{\infty} a_{ijt}x_{jt} + I_{R_{i,t}} - Q_{i,t} - I_{R_{i,t-1}} = 0 \; ; t = 1,\ldots,T \; , i = 1,2,\ldots,m \; , \; I_{R_{i,0}} \text{ is a constant} \tag{2}$$

$$I^{+}_{D_{k,t-1}} - I^{-}_{D_{k,t-1}} + \sum_{j=1}^{\infty} x_{jt} - I^{+}_{D_{i,k,t}} + I^{-}_{D_{k,t}} = d_{k,t} \quad ; k = 1,\ldots,K \; , t = 2,\ldots,T \; , I^{+}_{D_{k,0}} \; I^{-}_{D_{k,o}} \text{ are constant} \tag{3}$$

$$x_{jt}, I_{R_{i,t}}, Q_{i,t}, I^{+}_{D_{k,t}}, I^{-}_{D_{k,t}} \geq 0 \quad \forall j,k,t \tag{4}$$

where

T : Number of time period considered,
K : Number of possible demand scenario considered,
x_{jt} : Decision variable of producing product formula j quantity at time period t,
a_{ijt} : Raw material i percentage in product formula j at time period t,
$I_{R_{i,t}}$: Decision variable quantity of holding raw material i at time period t,
$Q_{i,t}$: Decision variable quantity of ordering raw material i at time period t,
$g_{R_{i,t}}$: Raw material i holding cost per unit at time period t,
$h_{R_{i,t}}$: Raw material i ordering cost per unit at time period t,
$I^{+}_{D_{k,t}}$: Decision variable quantity of holding finished product at scenario k time period t,
$I^{-}_{D_{k,t}}$: Decision variable quantity of back ordering finished product at scenario k time period t,
g_{D_t} : Finished product holding cost per unit at time period t,
h_{D_t} : Finished product back ordering cost per unit at time period t,
$d_{k,t}$: Finished product demand at scenario k time period t and
c_{jt} : Corrective action cost of manufacturing product formula j at time period t coming from reworking in the case of under and over nutrients requirement as equations (6)-(7).

$$\text{Let } a_{ijt} = x_i^{*(jt)} \tag{5}$$

$$c_{jt} = c^{(jt)} \tag{6}$$

$$c^{(jt)} = \sum_{s=1}^{N}\sum_{l=1}^{m}\left(g_{ls1}u_{ls1}^{(jt)} + h_{ls1}v_{ls1}^{(jt)}\right) + \sum_{s=1}^{N}\sum_{l=1}^{m}\left(g_{ls2}u_{ls2}^{(jt)} + h_{ls2}v_{ls2}^{(jt)}\right) \tag{7}$$

$x_i^{*(jt)}$ can be found from following conditions :

$$\sum_{i=1}^{m} a_{ils}^{*} x_i^{*(jt)} + u_{ls1}^{(jt)} - v_{ls1}^{(jt)} = b_{l1}^{*} \; ; l = 1,2,...,p \quad ; s = 1,2,...,N \tag{8}$$

$$\sum_{i=1}^{m} a_{ils}^{*} x_i^{*(jt)} + u_{ls2}^{(jt)} - v_{ls2}^{(jt)} = b_{l2}^{*} \; ; l = 1,2,...,p \quad ; s = 1,2,...,N \tag{9}$$

$$l_i^{*} \le x_i^{*(jt)} \le u_i^{*} \tag{10}$$

$$\sum_{i=1}^{m} x_i^{*(jt)} = 1 \quad \forall j,t \tag{11}$$

$$x_i^{*(j)}, u_{kl1}^{(j)}, v_{kl1}^{(j)}, u_{kl2}^{(j)}, v_{kl2}^{(j)} \ge 0 \quad \forall_{i,j,k,l} \tag{12}$$

where
N : A number of possible scenarios considered,
$u_{ls1}^{(jt)}$: Slack of the amount of minimum nutrient in nutrient l scenario s product formula j at time period t,
$v_{ls1}^{(jt)}$: Surplus of the amount of minimum nutrient in nutrient l scenario s product formula j at time period t,
$u_{ls2}^{(jt)}$: Slack of the amount of maximum nutrient in nutrient l scenario s product formula j at time period t,
$v_{ls2}^{(jt)}$: Surplus of the amount of maximum nutrient in nutrient l scenario s product formula j at time period t,
g_{ls1} : Expected cost per unit of $u_{ls1}^{(jt)}$,
h_{ls1} : Expected cost per unit of $v_{ls1}^{(jt)}$,
g_{ls2} : Expected cost per unit of $u_{ls2}^{(jt)}$,
h_{ls2} : Expected cost per unit of $v_{ls2}^{(jt)}$,
$\tilde{a}_{ils}^{*}$: Amount per unit of nutrient l in raw material i scenario s,
$\tilde{b}_{l1}^{*}$: Minimum acceptable quantity of nutrient l,
$\tilde{b}_{l2}^{*}$: Maximum acceptable quantity of nutrient l,
l_i^{*} : Smallest allowable fraction of raw material i and
u_i^{*} : Largest allowable fraction of raw material i.

3. Methodology

Three major algorithms were used to solve the model described in previous section. Dantzig-Wolfe decomposition is applied for solving large scale two-stage linear programming for inventory problem (equations (1)-(4)). The column generation technique is used for the problem of infinite number of manufacturing quantity decision variables controlled from possibility in continuous fraction formula equations (6)-(12) while Benders decomposition can be utilized for solving large scale diet problem equations (7)-(12).

3.1. Dantzig-Wolfe Decomposition

Consider equations (1)-(4). This can be solved by using algorithm for two-stage linear programming such as Benders decomposition or Dantzig-Wolfe decomposition. Dantzig-Wolfe decomposition has been chosen in this step because it is easier to find directly both primal and dual solutions together. Equations (1)-(4) need to be transformed to the corresponding dual problem as equations (13)-(19).

$$\text{Maximize } w = \sum_{i=1}^{m} I_{R_{i,0}} y_{R_{i,1}} + \sum_{k=1}^{K}\sum_{t=1}^{T} d_{k,t} y_{D_{k,t}} \tag{13}$$

Subject to

$$\sum_{i=1}^{m} a_{ijt} y_{R_{i,t}} + \sum_{k=1}^{K} y_{D_{k,t}} \le c_{jt} \quad ; j=1,...,J \ ; t=1,...,T \tag{14}$$

$$y_{R_{i,t}} - y_{R_{i,t+1}} \le g_{R_{i,t}} \quad ; t=1,...,T-1 \ ; \forall i \tag{15}$$

$$y_{R_{i,T}} \le g_{R_{i,T}} \quad ; \forall i \tag{16}$$

$$y_{R_{i,t}} \ge -h_{i,t} \quad ; \forall i,t \tag{17}$$

$$-h_{D_t} \le y_{D_{k,t}} - y_{D_{k,t+1}} \le g_{D_t} \quad ; t=1,...,T-1 \ ; \forall k \tag{18}$$

$$-h_{D_T} \le y_{D_{k,T}} \le g_{D_T} \quad ; \forall k \tag{19}$$

Equations (13)-(19) are combined from two independent constraints set, Equations (14)-(17) and Equations (18)-(19), and linking with coupling constraint equations (14) and J is a finite number of feed mix formula initially used and sampled from infinite number of alternatives. This pattern can be transformed to convex combination model with initial solutions set $\theta = 1,...,\eta$ as equations (20)-(23).

$$\text{Maximize } w = \sum_{\theta=1}^{\eta} \Big(\sum_{i=1}^{m} I_{R_{i,0}} \vec{y}^{\alpha}_{R_{i,1}} + \sum_{k=1}^{K}\sum_{t=1}^{T} d_{k,t} \vec{y}^{\alpha}_{D_{k,t}} \Big) \hat{\lambda}_{\theta} \tag{20}$$

$$\sum_{\theta=1}^{\eta} \Big(\sum_{i=1}^{m} a_{ijt} \vec{y}^{\theta}_{R_{i,t}} + \sum_{k=1}^{K} \vec{y}^{\theta}_{D_{k,t}} \Big) \hat{\lambda}_{\theta} \le c_{jt} \quad ; j=1,...,n \ ; k=1,...,K \tag{21}$$

$$\sum_{\theta=1}^{\eta} \hat{\lambda}_{\theta} = 1 \tag{22}$$

$$\hat{\lambda}_{\theta} \ge 0 \quad \forall \theta \tag{23}$$

where
η: Number of solutions set considered,
$\vec{y}^{\theta}_{R_{i,t}}$: Optimal solutions of $\dot{y}_{D_{k,t}}$ equations (15)-(17) solutions set θ and
$\vec{y}^{\theta}_{D_{k,t}}$: Optimal solutions of $\dot{y}_{D_{k,t}}$ equations (18)-(19) solutions set θ.

According to equations (20)-(23), Initial solutions set are needed then searching for new potential solutions set $\eta+1$ iteratively. Searching algorithm is still using equations (13)-(19) concept by relaxing the coupling constraint equation (14) as Lagrange multiplier terms in the objective function as equation (23).

$$\text{Maximize } w = \sum_{i=1}^{m} I_{R_{i,0}} \vec{y}^{\theta}_{R_{i,1}} + \sum_{k=1}^{K}\sum_{t=1}^{T} \dot{d}_{k,t} \vec{y}^{\theta}_{D_{k,t}} - \sum_{t=1}^{T}\sum_{j=1}^{n} \lambda'_{jt} \Big(\sum_{i=1}^{m} \dot{a}_{ijt} \vec{y}^{\theta}_{R_{i,t}} + \sum_{k=1}^{K} \vec{y}^{\theta}_{D_{k,t}} - c_{jt} \Big) \tag{23}$$

Equation (23) can be reformed as equation (24).

$$\text{Maximize } w = \sum_{i=1}^{m} \Big(I_{R_{i,0}} - \sum_{t=1}^{T}\sum_{j=1}^{n} \lambda'_{jt} \dot{a}_{ijt} \Big) y_{R_{i,t}} + \sum_{k=1}^{K}\sum_{t=1}^{T} \Big(\dot{d}_{k,t} - \sum_{j=1}^{n} \lambda'_{jt} \Big) y_{D_{k,t}} + \sum_{t=1}^{T}\sum_{j=1}^{n} c_{jt} \lambda'_{jt} \tag{24}$$

Subject to

$$\vec{y}^{\theta}_{R_{i,t}} - \vec{y}^{\theta}_{R_{i,t+1}} \le g_{R_{i,t}} \quad ; t=1,...,T-1 \ ; \forall i \tag{25}$$

$$y_{R_{i,t}}^{\theta} \quad \le g_{R_{i,T}} \quad ;\forall i \tag{26}$$

$$y_{R_{i,t}}^{\theta} \quad \ge -h_{i,t} \quad ;\forall i,t \tag{27}$$

$$-h_{D_t} \le y_{D_{k,t}} - y_{D_{k,t+1}} \le g_{D_t} \quad ; t=1,...,T-1 \quad ;\forall k \tag{28}$$

$$-h_{D_T} \le y_{D_{k,T}} \quad \le g_{D_T} \quad ;\forall k \tag{29}$$

where λ'_{jt} is a dual decision variable of equation (14) formula j at time period t.

Solving equations (20)-(23) can only find the optimal solutions λ'_{jt} for the solutions set $\theta = 1,...,\eta$. Adding the optimal solutions of equations (24)-(29) into solutions set $\eta + 1$ equations (20)-(22) can improve the objective function value. Repeating these steps until the objective function value equations (20) converges will make the solutions close to optimal.

Equations (24)-(29) can be separated into two independent problems, constraints (25)-(27) and (28)-(29), equation with constraints (25)-(27) can be solved by using the direct simplex method but equation constraints (28)-(29) may not be proper because of the larger size involved. This pattern can written as equations (30)-(33) and solved by using Special purpose method (Apisak *et al.* 2016) introducing the new variable u'_{kt} as follows.

$$\text{Maximize } w = \sum_{k=1}^{K}\sum_{e=1}^{T-1}\left(\sum_{t=1}^{e}(d_{k,t} - \sum_{j=1}^{n}\lambda'_{jt})u'_{kt}\right) + \sum_{k=1}^{K}\left((\sum_{t=1}^{T}(d_{k,t} - \sum_{j=1}^{n}\lambda'_{jt}))\dot{y}_{D_{k,T}}\right) \tag{30}$$

Subject to

$$y_{D_{k,t}} = y_{D_{k,t+1}} + u'_{k,t} \quad ;t=1,...,T-1 \tag{31}$$

$$-h_{D_T} \le y_{D_{k,T}} \le g_{D_T} \tag{32}$$

$$-h_{D_t} \le u'_{k,t} \le g_{D_t} \quad ;t=1,...,T-1 \tag{33}$$

According to the new structure as described by equations (30)-(33), All of decision variables $\dot{y}_{D_{k,T}}, u'_{k,t}$ are independent. Thus, the decisions are depended upon on the objective function coefficients. If the coefficients are positive the variables are equal to $g_{D_t}, \forall t$. In contrast, if the coefficients are negative the variables are equal to $-h_{D_t}, \forall t$.

Finally, an optimal solution of equations (13)-(19) are equal to the product of the solutions set and their optimal weight as shown by equations (34)-(35).

$$y_{R_{i,t}} = \sum_{\theta=1}^{\eta}\hat{\lambda}_{\theta}\vec{y}_{R_{i,t}}^{\theta} \tag{34}$$

$$y_{D_{k,t}} = \sum_{\theta=1}^{\eta}\hat{\lambda}_{\theta}\vec{y}_{D_{i,t}}^{\theta} \tag{35}$$

In conclusion, the optimal solution for both primal problem equations (1)-(4) and dual solutions λ'_{jt} from equations (21) can be simultaneously found using the above procedure.

After the optimal solutions x_{jt} from equations (1)-(4) are obtained. $I_{R_{i,t}}, Q_{i,t}, I_{D_{k,t}}^{+}$ and $I_{D_{k,t}}^{-}$ can be calculated in their relative equations. This method can be used with only finite decision variables problem ($j = 1,...,n$). Therefore, the column generation approach is needed to expand the subproblem size to approximate closely to the master problem.

3.2. Column generation

In other issues, equations (1)-(4) still have the problem of infinite number of decision variables of manufactured quantity consequence from continuous coefficient equations (7)-(12). This can be solved by using the column generations technique (Charnsethikul. 2011); create the restricted master problem and solve for minimum reduced cost of non-considering decision variables then add an improved one to the restricted master problem iteratively until all reduced cost of non-considering decision variables are nonnegative, To solve the minimum reduced cost search equations for expanding the size of restricted master problem. Benders decomposition is also needed as a sub-procedure when the number of scenarios becomes too large (Artit and Peerayuth, 2016).

Reduced cost of any x_{jt} from equations (1)-(4) can be written as the right hand side of equation (36). By maximizing the objective function, if there exist no positive $\dot{R}$, the current best solution is already the optimal solution. Otherwise, the solution of equations (36) with most positive $\dot{R}$ will lead solution of equations (1)-(4) to a better optimal solution (Charnsethikul, 2011).

$$\text{Maximize } R = (\sum_{i=1}^{m} y_{R_{i,t}} x_i^{*(jt)} + \sum_{k=1}^{K} y_{D_{k,t}}) - \sum_{s=1}^{N}\sum_{l=1}^{m} (g_{ls1}u_{ls1}^{(jt)} + h_{ls1}v_{ls1}^{(jt)}) \tag{36}$$

Objective function equation (36) can be written as equation (37) to standardize two-stage linear programming form (Wagner. 1975) as follows.

$$\text{Minimize } R = -\sum_{i=1}^{m} y_{R_{i,t}} x_i^{*(jt)} + \sum_{s=1}^{N}\sum_{l=1}^{m} (g_{ls1}u_{ls1}^{(jt)} + h_{ls1}v_{ls1}^{(jt)}) - \sum_{k=1}^{K} y_{D_{k,t}} \tag{37}$$

Solving equation (37) is still constrained by raw material fraction usage condition equation (8)-(12). Adding this solution to restricted master problem of equations (1)-(4) and resolve equations (37) until no negative objective value detected will lead to the solution optimality.

3.3. Benders decomposition

In order to solve equation (37) constrained by equations (8)-(12), the direct Simplex method may not be the proper choice because the problem can be too large. This depends on number of nutrient scenarios considered. Benders decomposition can be applied to this case. Given the first stage decision variables be any feasible constant solution $x_i^{*(jt)} = x_i^{**(jt)}$. Reduced cost search equations are formed into equations (38)-(41) as follows.

$$\text{Minimize } R = \sum_{s=1}^{N}\sum_{l=1}^{m} (g_{ls1}u_{ls1}^{(jt)} + h_{ls1}v_{ls1}^{(jt)}) - \sum_{i=1}^{m} y_{R_{i,t}} x_i^{**(jt)} - \sum_{k=1}^{K} y_{D_{k,t}} \tag{38}$$

Subject to

$$u_{ls1}^{(jt)} - v_{ls1}^{(jt)} = b_{l1}^{*} - \sum_{i=1}^{m} a_{ils}^{*} x_i^{**(jt)} \;; l = 1,2,...,p \;\; ; s = 1,2,...,N \tag{39}$$

$$u_{ls2}^{(jk)} - v_{ls2}^{(jk)} = b_{l2}^{*(k)} - \sum_{i=1}^{m} a_{ils}^{*} x_i^{**(jk)} \;\; ; l = 1,2,...,p \;\; ; s = 1,2,...,N \tag{40}$$

$$u_{kl1}^{(j)}, v_{kl1}^{(j)}, u_{kl2}^{(j)}, v_{kl2}^{(j)} \geq 0 \quad \forall_{i,j,k,l} \tag{41}$$

The dual problem of equations (38)-(41) can be formulated as following equations (42)-(44).

$$\text{Maximize } \dot{w} = \sum_{l=1}^{p}\sum_{s=1}^{N}(b_{ls1} - \sum_{i=1}^{m} a_{ils}^{*} x_i^{**(jt)}) y_{lsq1}^{*(jt)} + \sum_{l=1}^{p}\sum_{s=1}^{N}(b_{ls2} - \sum_{i=1}^{m} a_{ils}^{*} x_i^{**(jt)}) y_{lsq2}^{*(jt)} - \sum_{i=1}^{m} y_{R_{i,t}} x_i^{**(jt)} - \sum_{k=1}^{K} y_{D_{k,t}} \tag{42}$$

Subject to

$$-h_{ls1} \leq y_{lsq1}^{*(jt)} \leq g_{ls1} \tag{43}$$

$$-h_{ls2} \leq y_{lsq2}^{*(jt)} \leq g_{ls2} \tag{44}$$

where
$\dot{w}$: Dual objective value of equations (38)-(41),
$y_{lsq1}^{*(jt)}$: Dual decision variable of nutrient lower bound constraint for nutrient l scenario s computing in round q of formula j at time period t from equation (39).
$y_{lsq2}^{*(jt)}$: Dual decision variable of nutrient upper bound constraint for nutrient l scenario s computing in round q formula j at time period t from equation (40).

The optimal solutions of equations (42)-(44) are trivial. Independent constraint causes the solutions depending upon the corresponding objective function coefficients. If the coefficient is positive the solution is equal to the related upper bound. Otherwise, the solution is equal to the related lower bound.

After solving equations (42)-(44), the next step is to solve the cuts equations for deciding the new first stage decision variables $x_i^{*(jt)} = x_i^{**(jt)}$ which can be shown in the following equations (45)-(46).

$$\text{Minimize } \dot{T} \tag{45}$$

Subject to

$$T + \sum_{l=1}^{p}\sum_{i=1}^{m}\sum_{s=1}^{N}((y_{lsq1}^{*(jt)} + y_{lsq2}^{*(jt)})a_{ils}^{*} + y_{R_{i,t}})x_i^{*(jt)}$$
$$\geq \sum_{l=1}^{p}\sum_{s=1}^{N}(b_{ls1}^{*}\dot{y}_{lsq1}^{*(jt)} + b_{ls2}^{*} y_{lsq2}^{*(jt)}) - \sum_{i=1}^{m}\dot{y}_{R_{i,t}} x_i^{**(jt)} - \sum_{k=1}^{K} y_{D_{k,t}} \quad \forall q \tag{46}$$

where
$\dot{T}$: The smallest upper bound of the reduced cost search problem.

Replacing new $x_i^{*(jt)}$ by $x_i^{**(jt)}$ in the equation (38)-(41) then resolve with the same algorithm until $T \rightarrow R$ will lead to the converged optimal solutions of equations (37) subjecting to constraint equations (8)-(12). In summary, figure 1 is illustrated the proposed method flowchart.

4. Computational test

To test the method capability, Visual basic for application in Microsoft Excel has been developed. Working parallel with automatic computing in spread sheets cells and Solver tool. In this study, the computer processor is Intel(R) Core(TM) i5-3450S CPU @ 2.80GHz. The experimental case consists of 8 types of raw materials. Nutrients percentage in raw materials, Nutrients requirement, raw materials fraction boundary, corrective action cost and corrective action cost per unit data are based on the pig diet problem by (Wanida *et al*. 2008) shown in appendix (Table: 1-Table: 4).

As (Wanida *et al*. 2008), assume the cost of raw materials equal to 42, 31, 34, 65, 22, 87, 120 and 15 baht per unit respectively. The assumption of no corrective action cost in the case of producing nutrient at its lower bound and under the upper bound has been used. Testing with 6 time periods, inventory holding cost are 5, 15, 20, 5, 5, 10, 15, 5 baht per unit per time period respectively, Inventory level at initial time period are 200, 300, 150, 150, 500, 150, 700, 650 units respectively. Finished products holding cost in each time period are 10, 15, 10, 12, 10, 20 baht per unit. Back ordering cost in each time period are 30, 24, 20, 16, 25, 22 baht per unit. Finished product demand in each time periods are 1,000±20, 1,200±20, 1,600±20, 1,800±10, 900±10, 1,300±40 units. Uncertainties of nutrients in raw material and demand are both simulated as discrete uniform distribution with 10,000 scenarios simulation size.

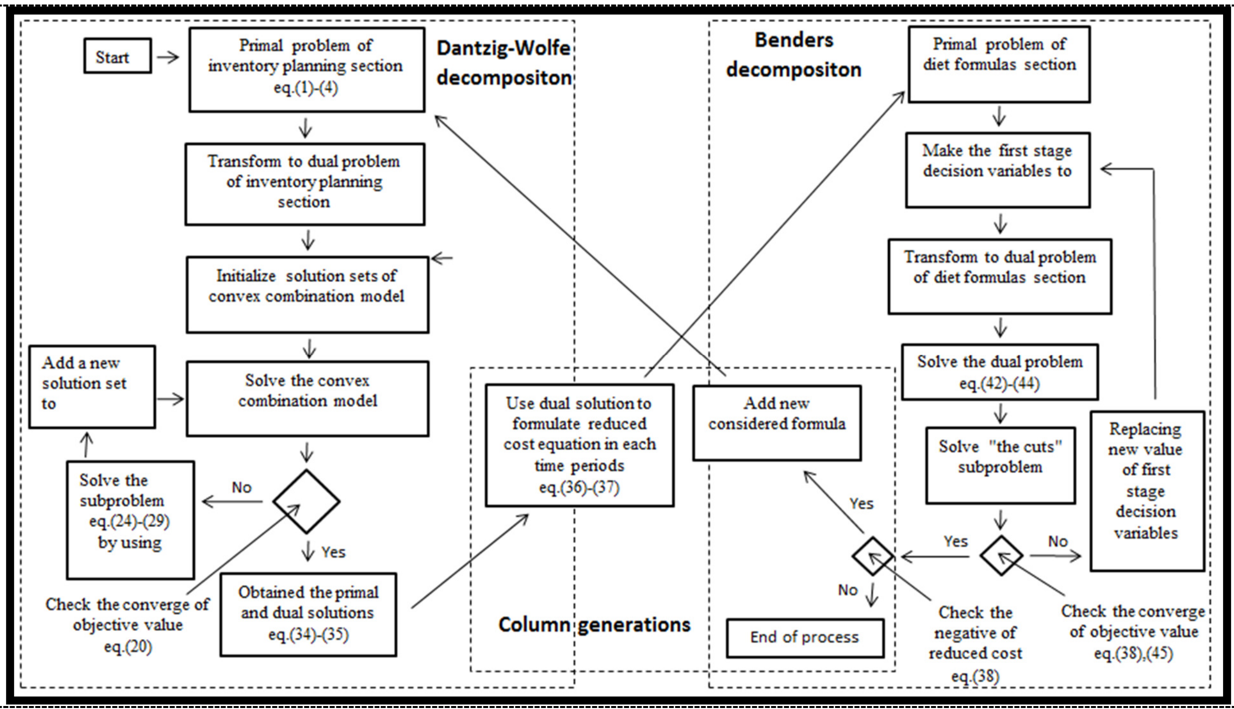

Figure 1. Proposed method flow chart.

5. Result and discussion

The solution obtained is to produce only the first 3 out of 6 periods shown that holding cost impact is less than back ordering cost. The optimal overall expected value is 524,373 baht with 2 iterations of column generations being used while 15, 14 and 11 iterations of Dantzig-Wolfe decomposition has been used in each iteration of the column generation method. Figure 2 shows the objective value updating result.

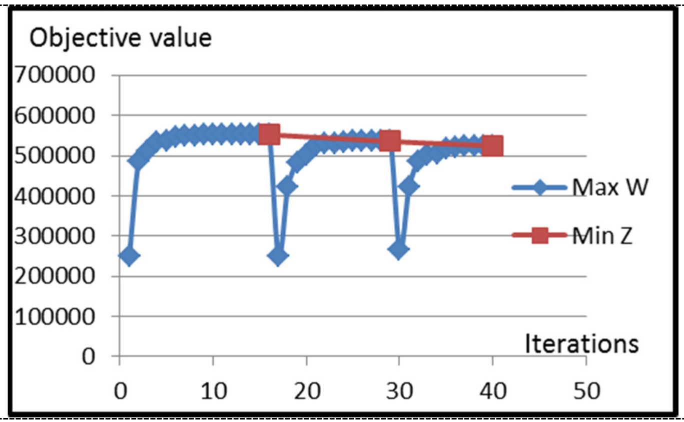

Figure 2. Objective value updating thru iterations.

The total runtime is 3525.39 second including from 3 iterations of column generation working with Benders decomposition and 3 iterations of Dantzig-Wolfe decomposition. Runtime per iteration and cumulative runtime are shown in figures 3 and 4 indicating that Dantzig-Wolfe decomposition consume more computing time because of the lack of any special propose technique to replace using simplex method in equations (24)-(27).

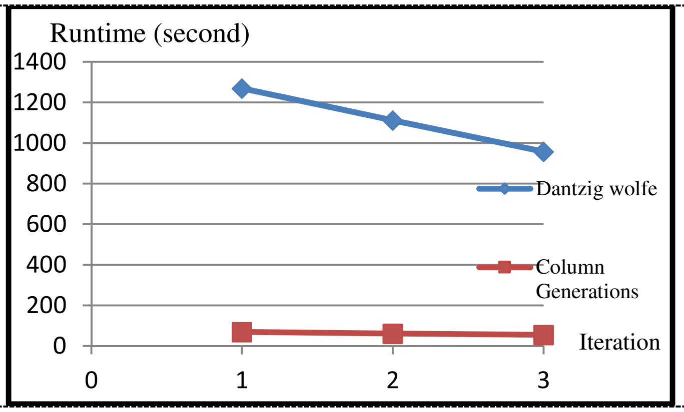

Figure 3. Runtime per iteration.

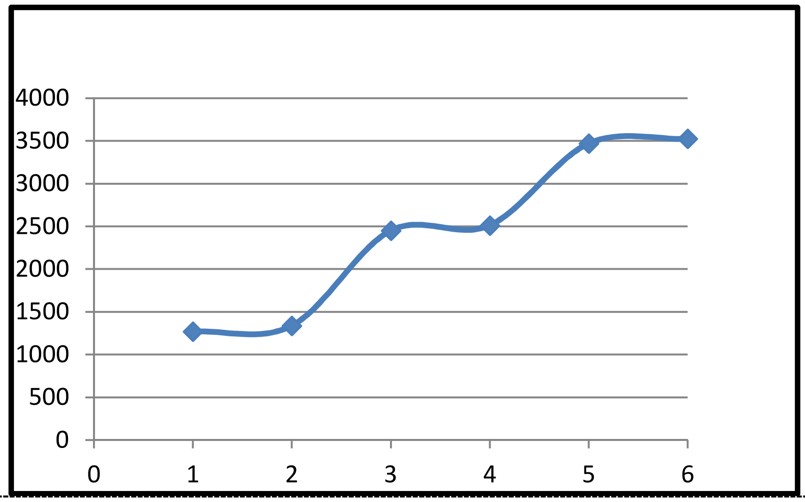

Figure 4. Cumulative runtime.

6. Conclusion

Two stage linear programming can be applied with the simultaneous inventory and diet planning problem. Dantzig-Wolfe decomposition and Benders decomposition are compatible in this case. The column generation technique is helpful for relaxing infinite number of related decision variables. These three algorithms can also be working as a hybrid approach. The preliminary on both computing time consumed and the solution quality when solving the case problem are reasonable and can be further improved.

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Appendix

Table 1. Nutrient percentage in raw materials.

Materials	Corn	Fat(%)	Calcium(%)	Phosphorus(%)	Energy (kcal)
Corn	40±1	22±1	11±1	7±2	1,800±50
Millet	5±0.25	4±1	8±1	5±0.5	2,200±100
Cassava	1±0.01	5±2	12±2	14±1	2,000±25
Fishmeal	60±2	10±1	15±2	10±1	1,750±50
Rice bran	-	13±2	6±1	5±1	1,800±50
Concentrated feed	-	-	17±2	15±3	500±20
Vitamin	-	-	25±1	24±2	200±10
Salt	7±1	5±1	30±2	21±1	1,200±100

Table 2. Nutrients requirement.

Nutrients	Requirement
Protein (%)	16-22
Fat (%)	≤ 14
Calcium (%)	≥ 7
Phosphorus (%)	≥ 5
Energy (kcal)	1700-2200

Table 3. Raw material fraction boundary.

Materials	Boundary (%)
Corn	15-25
Millet	20-30
Cassava	5-15
Fishmeal	10-15
Rice bran	15-20
Concentrated feed	2-3
Vitamin	1-2
Salt	4-8

Table 4. Corrective action cost.

Nutrients	Corrective action cost (Baht/unit)
Protein (%)	3
Fat (%)	5
Calcum (%)	4
Phosphorus (%)	6
Energy (kcal)	0.25