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A new two-scroll chaotic attractor with three quadratic nonlinearities, its adaptive control and circuit design

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Abstract. A 3-D new two-scroll chaotic attractor with three quadratic nonlinearities is investigated in this paper. First, the qualitative and dynamical properties of the new two-scroll chaotic system are described in terms of phase portraits, equilibrium points, Lyapunov exponents, Kaplan-Yorke dimension, dissipativity, etc. We show that the new two-scroll dissipative chaotic system has three unstable equilibrium points. As an engineering application, global chaos control of the new two-scroll chaotic system with unknown system parameters is designed via adaptive feedback control and Lyapunov stability theory. Furthermore, an electronic circuit realization of the new chaotic attractor is presented in detail to confirm the feasibility of the theoretical chaotic two-scroll attractor model.

1. Introduction

In the last few decades, chaotic and hyperchaotic systems have been applied in several areas of science and engineering [1-2]. Some important applications of chaotic systems can be listed out such as chemical reactors [3-5], oscillators [6-8], neural networks [9-10], memristors [11-12], ecology [13-14], robotics [15-16], Tokamak reactors [17-18], finance [19-20], etc.

In the chaos literature, there is good interest in shown in the modeling of chaotic systems with multi-scroll attractors such as two-scroll attractors [21-25], three-scroll attractors [26-28], four-scroll attractors

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[29-30], etc. There are also many chaotic systems with quadratic nonlinearities in the chaos literature [31-36].

In this work, we derive a new 3-D new dissipative chaotic system with three quadratic nonlinearities in this paper. The new chaotic system displays a two-scroll chaotic attractor.

This paper is organized as follows. Section 2 describes the new two-scroll chaotic system with three quadratic nonlinearities. This section also details dynamical properties such as phase portraits, Lyapunov exponents and Kaplan-Yorke dimension. Section 3 describes the global chaos control of the new chaotic system with unknown parameters. In Section 4, we use MultiSIM to build an electronic circuit realization of the new two-scroll chaotic system. The circuit experimental results of the new chaotic attractor show agreement with the numerical simulations. Section 5 contains the conclusions.

2. A new two-scroll chaotic system with three quadratic nonlinearities

In this paper, we design a new two-scroll chaotic system with three quadratic nonlinearities given by

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) + x_2 x_3 \\ \dot{x}_2 = b x_1 - x_2 + c x_1 x_3 \\ \dot{x}_3 = -x_3 - x_1 x_2 \end{cases} \tag{1}$$

where x_1, x_2, x_3 are state variables and a, b, c are positive constants.

In this paper, we show that the system (1) is *chaotic* for the parameter values

$$a = 10, \quad b = 20, \quad c = 30 \tag{2}$$

For numerical simulations, we take the initial values of the system (1) as

$$x_1(0) = 0.1, \quad x_2(0) = 0.1, \quad x_3(0) = 0.1 \tag{3}$$

Figure 1 shows the phase portraits two-scroll strange attractor of the new chaotic system (1) for the parameter values (2) and initial conditions (3). Figure 1 (a) shows the 3-D phase portrait of the new chaotic system (1). Figures 1 (b)-(c) show the projections of the new chaotic system (1) in (x_1, x_2), (x_2, x_3) and (x_1, x_3) coordinate planes, respectively.

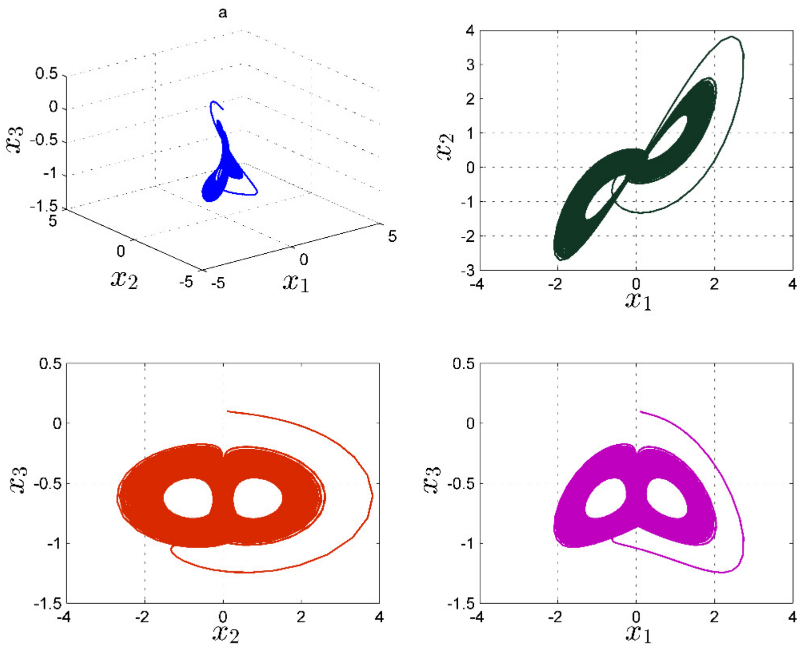

Figure 1. Phase portraits of the new chaotic system (1) for $a = 10,\ b = 20,\ c = 30$

For the rest of this section, we take the parameter values as in the chaotic case (2).

The equilibrium points of the new chaotic system (1) are obtained by solving the system of equations

$$\begin{cases} a(x_2 - x_1) + x_2 x_3 = 0 \\ b x_1 - x_2 + c x_1 x_3 = 0 \\ -x_3 - x_1 x_2 = 0 \end{cases} \tag{4}$$

Solving the equations in (4) we obtain the equilibrium points of the system (1) as

$$E_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad E_1 = \begin{bmatrix} 0.7689 \\ 0.8207 \\ -0.6311 \end{bmatrix}, \quad E_2 = \begin{bmatrix} -0.7689 \\ -0.8207 \\ -0.6311 \end{bmatrix} \tag{5}$$

It is easy to verify that E_0 is a saddle point, while E_1 and E_2 are saddle-focus points.

For the parameter values as in the chaotic case (2) and the initial state as in (3), the Lyapunov exponents of the new 3-D system (2) are determined using Wolf's algorithm as

$$L_1 = 0.4260, \quad L_2 = 0, \quad L_3 = -12.4260 \tag{6}$$

Since $L_1 > 0$, the new 3-D system (1) is chaotic. Thus, the system (1) exhibits a two-scroll chaotic attractor. Also, we note that the sum of the Lyapunov exponents in (6) is negative. This shows that the new two-scroll chaotic system (1) is dissipative.

The Kaplan-Yorke dimension of the new 3-D system (1) is determined as

$$D_{KY} = 2 + \frac{L_1 + L_2}{|L_3|} = 2.0343, \tag{7}$$

which indicates the complexity of the new two-scroll chaotic system (1).

Figure 2 shows the Lyapunov exponents of the new chaotic system (1) with a strange attractor.

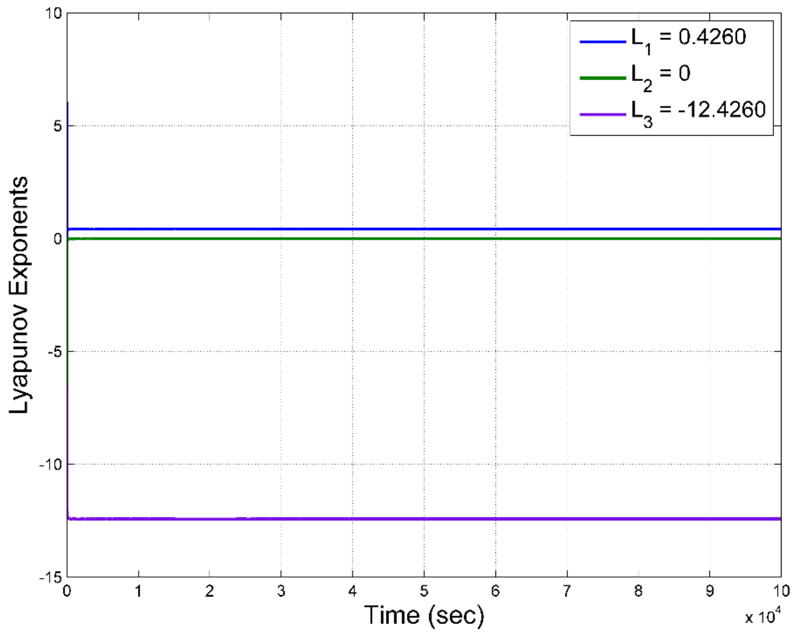

Figure 2. Lyapunov exponents of the new chaotic system (1) for $a = 10, \quad b = 20, \quad c = 30$

3. Global chaos control of the new two-scroll chaotic system via adaptive control method

In this section, we devise adaptive controller so as to globally stabilize all the trajectories of the new two-scroll chaotic system. The main result is proved via Lyapunov stability theory.

In this section, we consider the controlled chaotic system given by

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) + x_2 x_3 + u_1 \\ \dot{x}_2 = b x_1 - x_2 + c x_1 x_3 + u_2 \\ \dot{x}_3 = -x_3 - x_1 x_2 + u_3 \end{cases} \tag{8}$$

where x_1, x_2, x_3 are the states and a, b are unknown parameters.

We consider the adaptive control defined by

$$\begin{cases} u_1 = -\hat{a}(t)(x_2 - x_1) - x_2 x_3 - k_1 x_1 \\ u_2 = -\hat{b}(t) x_1 + x_2 - \hat{c}(t) x_1 x_3 - k_2 x_2 \\ u_3 = x_3 + x_1 x_2 - k_3 x_3 \end{cases} \tag{9}$$

where k_1, k_2, k_3 are positive gain constants.

Substituting (9) into (8), we obtain the closed-loop system

$$\begin{cases} \dot{x}_1 = [a - \hat{a}(t)](x_2 - x_1) - k_1 x_1 \\ \dot{x}_2 = [b - \hat{b}(t)] x_1 + [c - \hat{c}(t)] x_1 x_3 - k_2 x_2 \\ \dot{x}_3 = -k_3 x_3 \end{cases} \tag{10}$$

We define the parameter estimation errors as

$$\begin{cases} e_a(t) = a - \hat{a}(t) \\ e_b(t) = b - \hat{b}(t) \\ e_c(t) = c - \hat{c}(t) \end{cases} \tag{11}$$

Using (11), we can simplify (10) as

$$\begin{cases} \dot{x}_1 = e_a (x_2 - x_1) - k_1 x_1 \\ \dot{x}_2 = e_b x_1 + e_c x_1 x_3 - k_2 x_2 \\ \dot{x}_3 = -k_3 x_3 \end{cases} \tag{12}$$

Differentiating (11) with respect to t, we obtain

$$\begin{cases} \dot{e}_a(t) = -\dot{\hat{a}}(t) \\ \dot{e}_b(t) = -\dot{\hat{b}}(t) \\ \dot{e}_c(t) = -\dot{\hat{c}}(t) \end{cases} \tag{13}$$

Next, we consider the Lyapunov function defined by

$$V(x_1, x_2, x_3, e_a, e_b, e_c) = \frac{1}{2}\left(x_1^2 + x_2^2 + x_3^2 + e_a^2 + e_b^2 + e_c^2\right) \tag{14}$$

which is positive definite on $\boldsymbol{R}^6$.

Differentiating V along the trajectories of (12) and (13), we obtain

$$\dot{V} = -k_1 x_1^2 - k_2 x_2^2 - k_3 x_3^2 + e_a\left[x_1(x_2 - x_1) - \dot{\hat{a}}\right] + e_b\left[x_1 x_2 - \dot{\hat{b}}\right] + e_c\left[x_1 x_2 x_3 - \dot{\hat{c}}\right] \tag{15}$$

In view of the equation (15), we take the parameter update law as

$$\begin{cases} \dot{\hat{a}} = x_1(x_2 - x_1) \\ \dot{\hat{b}} = x_1 x_2 \\ \dot{\hat{c}} = x_1 x_2 x_3 \end{cases} \tag{16}$$

Theorem 1. The novel two-scroll chaotic system (8) is globally and exponentially stabilized by the adaptive control law (9) and the parameter update law (16), where k_1, k_2, k_3 are positive constants.

Proof. The Lyapunov function V defined by (14) is quadratic and positive definite on R^6.
By substituting the parameter update law (16) into (15), we obtain the time-derivative of V as

$$\dot{V} = -k_1 x_1^2 - k_2 x_2^2 - k_3 x_3^2 \tag{17}$$

which is negative semi-definite on R^6.

Thus, by Barbalat's lemma [37], it follows that the closed-loop system (15) is globally exponentially stable for all initial conditions $x(0) \in R^3$. This completes the proof. ■

For numerical simulations, we take the gain constants as $k_i = 10$ for $i = 1, 2, 3$.

We take the parameter values as in the chaotic case (2), i.e. $a = 10$, $b = 20$ and $c = 30$.

We take the initial conditions of the states of the novel chaotic system (8) as $x_1(0) = 12.3$, $x_2(0) = 7.4$ and $x_3(0) = 19.2$. We take the initial conditions of the parameter estimates as $\hat{a}(0) = 4.7$, $\hat{b}(0) = 10.4$ and $\hat{c}(0) = 5.8$.

Figure 3 shows the time-history of the controlled states x_1, x_2, x_3. Thus, Figure 3 illustrates the control law stated in Theorem 1 for the global chaos control of the novel chaotic system (8).

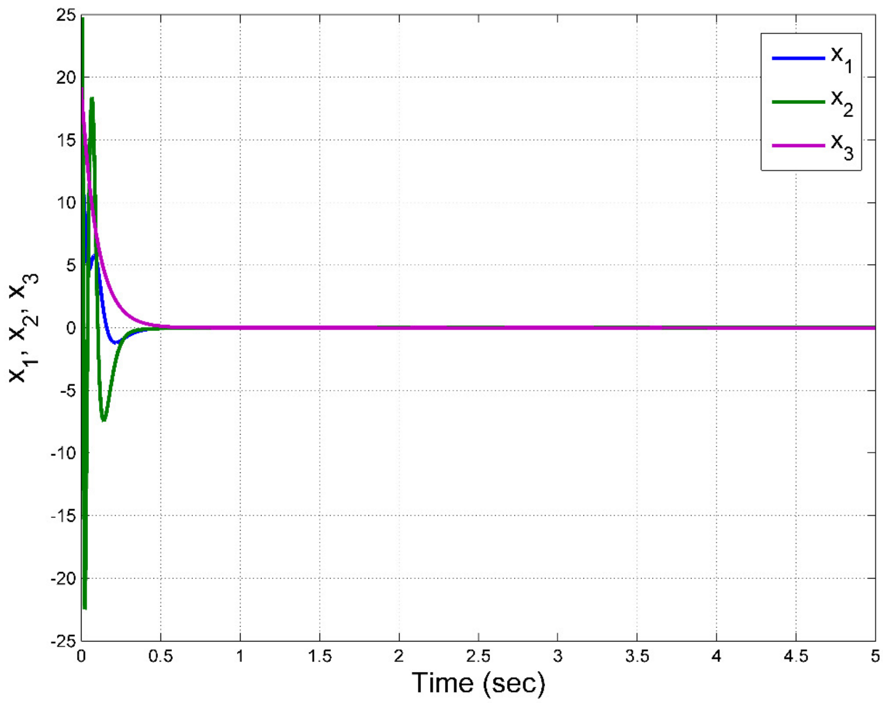

Figure 3. Time-history of the controlled chaotic system (8)

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4. Circuit implementation of the new two-scroll chaotic system

In this section, the new two-scroll chaotic system (1) is designed as an electronic circuit as seen on **Figure 4** and set in MultiSIM. As seen on **Figure 4**, 3 integrators, 3 multipliers and 2 inverters were used in the circuit in order to implement 3 differential equations that make up the chaotic system. By applying Kirchhoff's circuit laws, the corresponding circuital equations of the designed circuit can be written as::

$$\left.\begin{aligned} \dot{x}_1 &= \frac{1}{C_1R_1}x_2 - \frac{1}{C_1R_2}x_1 + \frac{1}{10C_1R_3}x_2x_3 \\ \dot{x}_2 &= \frac{1}{C_2R_4}x_1 - \frac{1}{C_2R_5}x_2 + \frac{1}{10C_2R_6}x_1x_3 \\ \dot{x}_3 &= -\frac{1}{C_3R_7}x_3 - \frac{1}{10C_3R_8}x_1x_2 \end{aligned}\right\} \quad (18)$$

In system (18), the variables x_1, x_2, and x_3 are the outcomes of the integrators U1A, U2A, U3A. The circuit components have been selected as: $R_1 = R_2 = R_3 = R_8 = 40$ kΩ, $R_5 = R_7 = 400$ KΩ, $R_4 = 20$ KΩ, $R_6 = 1.33$ KΩ, $R_9 = R_{10} = R_{11} = R_{12} = 100$ KΩ, $C_1 = C_2 = C_3 = 1$ nF. The supplies of all active devices are ±15 Volt. The obtained results are presented in **Figures 5** (a) - (c), which show the phase portraits of the chaotic attractor in x_1-x_2, x_2-x_3 and x_1-x_3 planes, respectively. Numerical simulations (**see Figure 1**) are similar with the circuital ones (**see Figure 5**).

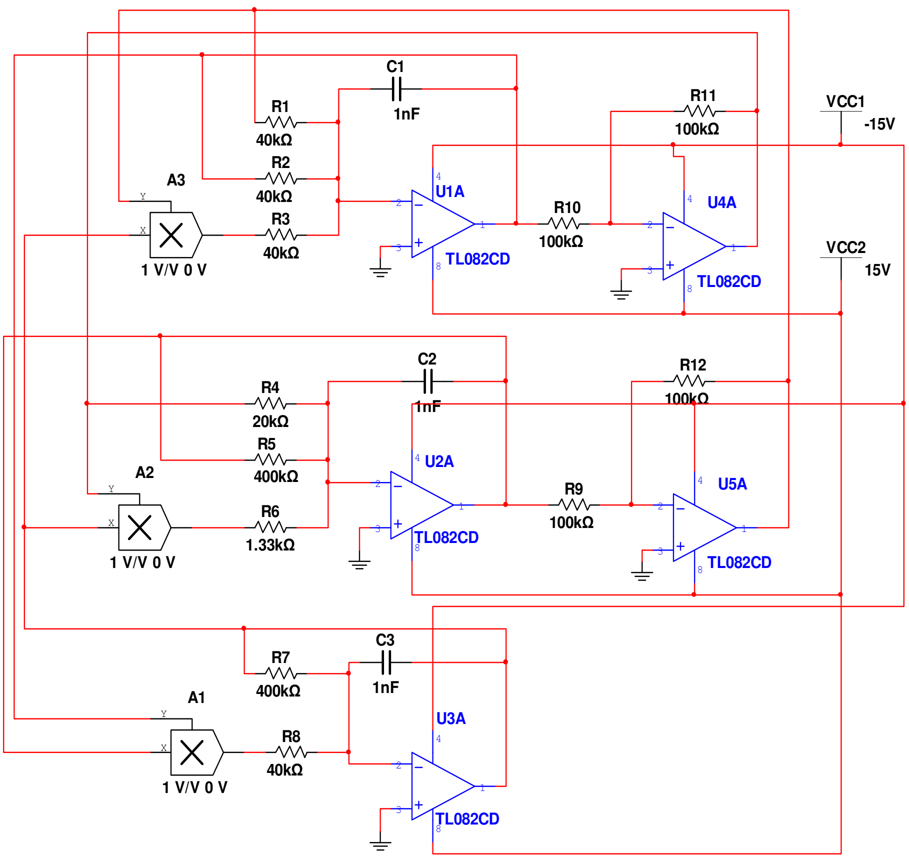
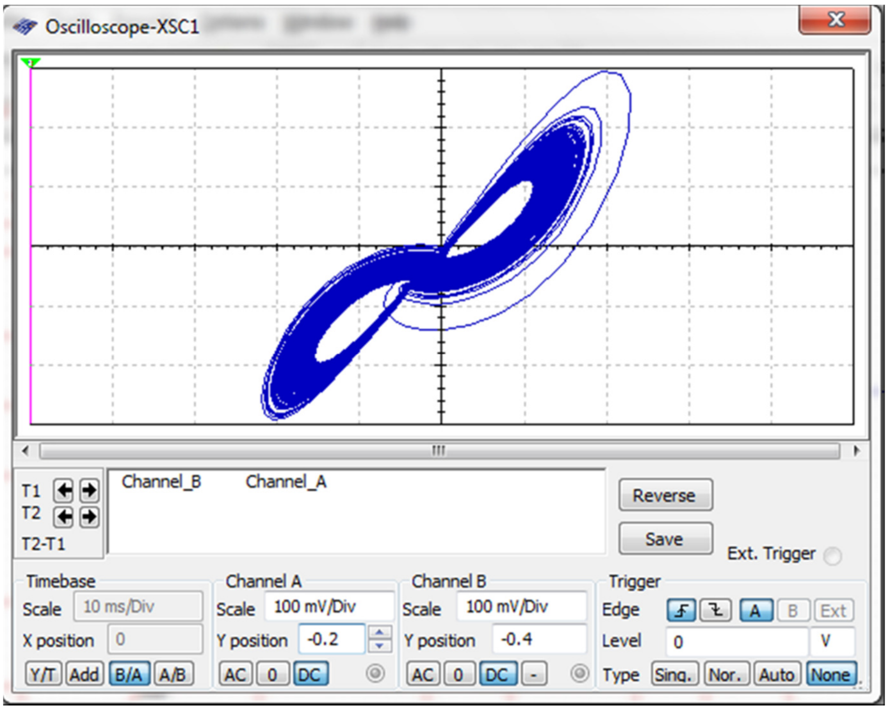
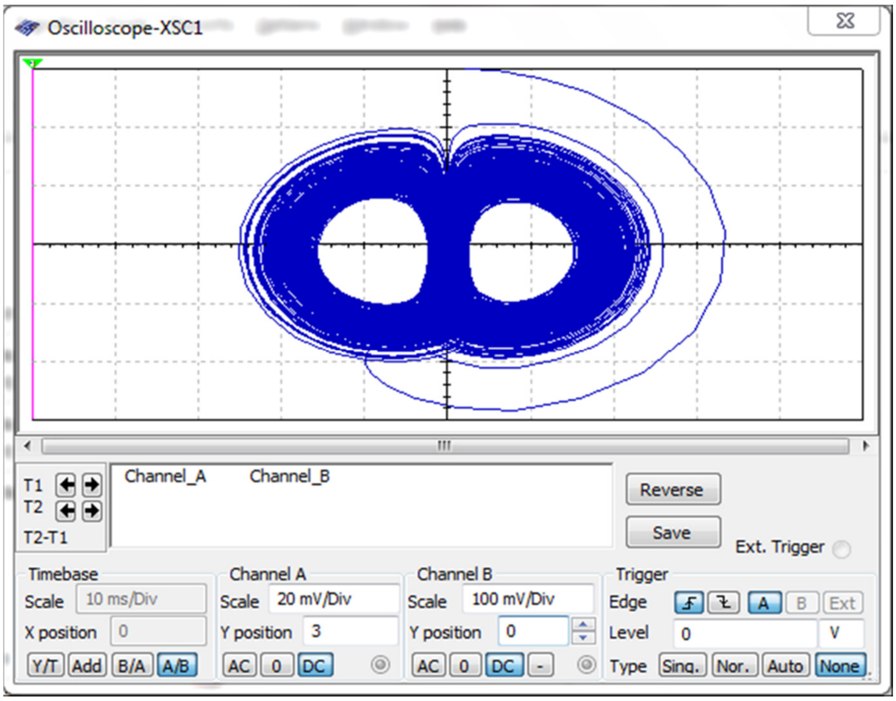


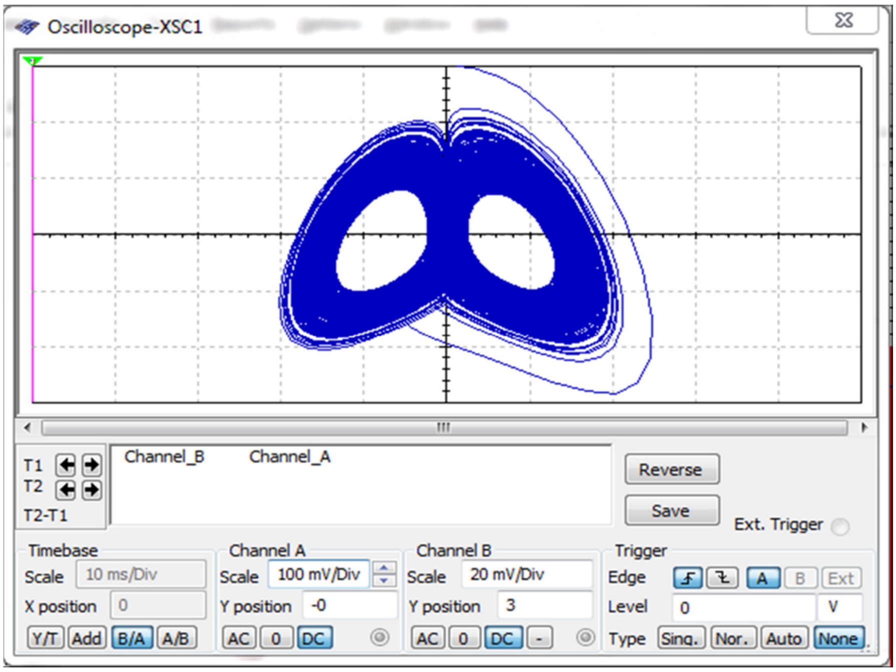
Figure 4 Circuit design for new two-scroll chaotic system (1) by MultiSIM


Oscilloscope-XSC1
T1
T2
T2-T1
Channel_B
Channel_A
Reverse
Save
Ext. Trigger
Timebase
Scale 10 ms/Div
X position 0
Y/T Add B/A A/B
Channel A
Scale 100 mV/Div
Y position -0.2
AC 0 DC
Channel B
Scale 100 mV/Div
Y position -0.4
AC 0 DC -
Trigger
Edge A B Ext
Level 0 V
Type Sing. Nor. Auto None

(a)


Oscilloscope-XSC1
T1
T2
T2-T1
Channel_A
Channel_B
Reverse
Save
Ext. Trigger
Timebase
Scale 10 ms/Div
X position 0
Y/T Add B/A A/B
Channel A
Scale 20 mV/Div
Y position 3
AC 0 DC
Channel B
Scale 100 mV/Div
Y position 0
AC 0 DC -
Trigger
Edge A B Ext
Level 0 V
Type Sing. Nor. Auto None

(b)



(c)

Figure 5 The phase portraits of new two-scroll chaotic system (1) observed on the oscilloscope in different planes (a) x_1-x_2, (b) x_2-$x_3$plane and (c) x_1-x_3 plane by MultiSIM

5. Conclusions

This work described a new two-scroll chaotic system with three quadratic nonlinearities. First, the qualitative properties of the new two-scroll chaotic system are detailed. Dynamical behaviors of the new two-scroll chaotic system with three quadratic nonlinearities are investigated through equilibrium points, projections of chaotic attractors, Lyapunov exponents and Kaplan–Yorke dimension. In addition, the adaptive control scheme of the new two-scroll chaotic system is shown via adaptive control approach. Furthermore, an electronic circuit realization of the new two-scroll chaotic system using the electronic simulation package MultiSIM confirmed the feasibility of the theoretical model.

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