

Time prediction of failure a type of lamps by using general composite hazard rate model

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Abstract. This paper discusses the basic survival model estimates to obtain the average predictive value of lamp failure time. This estimate is for the parametric model, General Composite Hazard Level Model. The random time variable model used is the exponential distribution model, as the basis, which has a constant hazard function. In this case, we discuss an example of survival model estimation for a composite hazard function, using an exponential model as its basis. To estimate this model is done by estimating model parameters, through the construction of survival function and empirical cumulative function. The model obtained, will then be used to predict the average failure time of the model, for the type of lamp. By grouping the data into several intervals and the average value of failure at each interval, then calculate the average failure time of a model based on each interval, the p value obtained from the test result is 0.3296.

1. Introduction

This model is the application of the theory for the estimation of the basic survival model of the data, as a result of development [6]. In the development grouped into two groups, namely parametric model and non parametric model [1]. Which each group has its own roles and functions. Models with exponential distributions are usually often selected for a random variable model of failure time [2]. [3],[4]. In the case here presented model examples which are simple models of composite hazard rate model and composite hazard rate model [5], [6]. The main purpose of this study was to obtain an estimate value for survival function, cumulative hazard function, along with the following derivatives. This appraisal is done for S and λ and then the result, through the regression model using the least squares method, is used to obtain estimates for λ , β , and γ . The problem discussed in this paper involves data on the timing of failure of a material type. In which case for model simulation used data with sample size of 49 pieces.

2. Related Concepts and Results

The simplest model of the composite hazard rate model is the exponential piecewise model. The exponential piecewise model is defined by the Hazard Rate Function of function as follows [5], [7].

$$\lambda(t) = \begin{cases} \lambda_1 & : 0 \leq t < y \\ \lambda_2 & : t \geq y \end{cases} \quad 1$$

The survival function is:



$$\lambda(t) = \begin{cases} e^{-t\lambda_1} & 0 \leq t < y \\ e^{-y\lambda_1} e^{-(t-y)\lambda_2} & t \geq y \end{cases} \tag{2}$$

And its density function is

$$f(t) = \begin{cases} \lambda_1 e^{-t\lambda_1} & 0 \leq t < y \\ \lambda_2 e^{-y\lambda_1} e^{-(t-y)\lambda_2} & t \geq y \end{cases} \tag{3}$$

Each segment of the composite hazard rate model has a constant hazard ratio. The simple case can be seen in the following figure, in which case the hazard increases according to the duration of time/age.

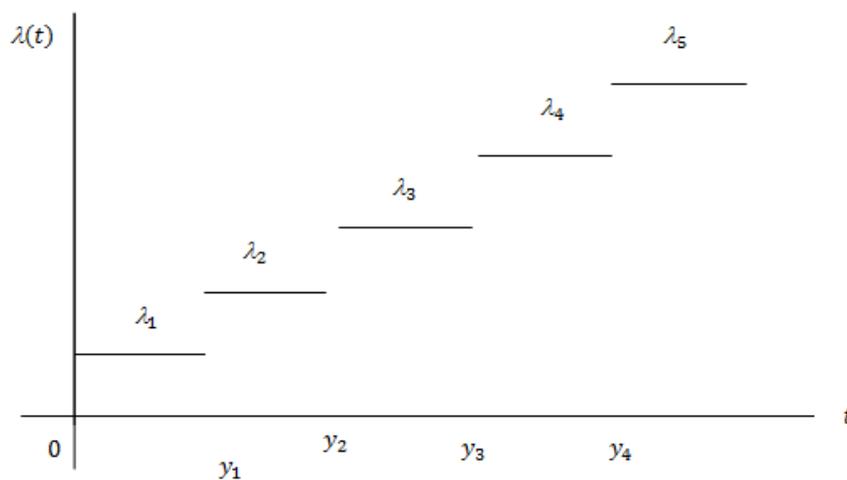


Figure 1. Hazard Rate function

In the discussion it will be presented the data in advance, in which case the model parameters will be estimated, the data in the form of long lifetime of energy-saving lamps. A sample of size $n = 49$, which is the lifespan of energy-saving lamps. The failure time, in hours, is presented in the following table by observation.

Table1. Failure time (in hours)

No	Time	No	Time	No	Time								
1	1051	8	4006	15	5905	22	8108	29	10205	36	11608	43	14110
2	1337	9	4012	16	5956	23	8546	30	10396	37	11745	44	12296
3	1389	10	4063	17	6068	24	8666	31	10861	38	11762	45	15395
4	1921	11	4921	18	6121	25	8831	32	11026	39	11895	46	16179
5	1942	12	5445	19	6473	26	9106	33	11214	40	12044	47	17092
6	2322	13	5620	20	7501	27	9711	34	11362	41	13520	48	17568
7	3629	14	5817	21	7886	28	9806	35	11604	42	13670	49	17568

To estimate the parameters of the model, the first is to establish an empirical survival function based on the observed data, $S^0(t)$ and empirical CHF, i.e. $A^0(t) = -\ln S^0(t)$, the results are presented in Table 2.

Table 2. Data on Estimated Results

t	$S^0(t)$	$A^0(t)$
1000	1.000	0.000
2000	0.898	0.108
3000	0.876	0.131
4000	0.857	0.154
5000	0.776	0.254
6000	0.673	0.395
7000	0.612	0.491
8000	0.571	0.560
9000	0.490	0.714
10000	0.429	0.847
11000	0.367	1.001
12000	0.204	1.589
13000	0.184	1.695
14000	0.143	1.946
15000	0.102	2.282
16000	0.082	2.506
17000	0.061	2.973
18000	0.000	∞

Using the minimum area criterion, the estimated value of the optimum for y is $\hat{y} = 3400$. Based on the value of these estimates, the values of λ and β are estimated by the least squares method of the regression model

$$c = \lambda x_1 + \beta x_2 \tag{4}$$

where

$$c = A^0(t), x_1 = t, \text{ and } x_2 = \frac{1}{3}(t - y)^3 \cdot I(t)$$

For this data, $\hat{\lambda} = 0.0490$ and $\hat{\beta} = 0.0038$, while the data matching test can be performed using Chi-square test, by grouping the data into several intervals and the mean value of failure time at each interval, for then calculate the average failure time of the model based on each interval. The results are presented in Table 3.

Table 3. Data test results match test

Failure time (in thousands)	Number of observations of failure time	Average time of failure
0.0 – 1.5	3	3.53
1.5 – 3.5	3	4.31
3.5 – 5.5	6	4.34
5.5 – 7.5	7	5.94
7.5 – 9.5	7	8.04
9.5 – 11.5	8	8.83
11.5 – 13.5	6	7.30
13.5 – 15.5	5	4.34
15.5 -	4	2.37

3. Conclusion

Based on the above discussion results obtained $\hat{\lambda} = 0.0490$ and $\beta = 0.0038$, while the data matching test was performed using Chi-square test, by grouping the data into several intervals and the mean value of failure time at each interval, for then calculate the average failure time of the model based on each interval, with the p value obtained from the test result is 0.3296.

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