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Analysis of concrete beams using applied element method

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Abstract. The Applied Element Method (AEM) is a displacement based method of structural analysis. Some of its features are similar to that of Finite Element Method (FEM). In AEM, the structure is analysed by dividing it into several elements similar to FEM. But, in AEM, elements are connected by springs instead of nodes as in the case of FEM. In this paper, background to AEM is discussed and necessary equations are derived. For illustrating the application of AEM, it has been used to analyse plain concrete beam of fixed support condition. The analysis is limited to the analysis of 2-dimensional structures. It was found that the number of springs has no much influence on the results. AEM could predict deflection and reactions with reasonable degree of accuracy.

1. Introduction

The Applied Element Method was developed by Kimiro Meguro and Hatem Tagel-Din [1] during their research at University of Tokyo. Based on AEM, Applied Science International (ASI), has developed a software called 'Extreme Loading for Structures', using which the behavior of structures under extreme loads such as earthquakes, hurricanes, bomb blasts and other disasters can be studied.

AEM is a numerical method of structural analysis similar to Finite Element Method. In AEM, elements are connected by a number of normal and shear springs to carry normal and shear stress respectively. Springs represent a certain volume of the elements; their properties, stresses and strains. They serve as connectors between elements. These springs can be used to model non-linear behaviour of concrete. A spring is detatched if the stress in it exceeds the allowable limit. By tracing the location of such springs, crack pattern can be found out. Therefore, behaviour of structure and crack propagation can be studied at all stages of loading. Also, simple modeling, relatively small computational time and high accuracy of results are possible with AEM [1] and [2].

2. Background to AEM

In AEM, the structure is discretised into a group of elements. The elements are connected by a set of normal and shear springs, which are on the side of the elements. Normal springs carry normal stress where as shear spring transfer shear stress from one element to the other. Figure 1 shows how a structure is modeled in AEM. Every 2D element has 3 degrees of freedom. The translation and rotation along these degrees of freedom produce stresses in springs.

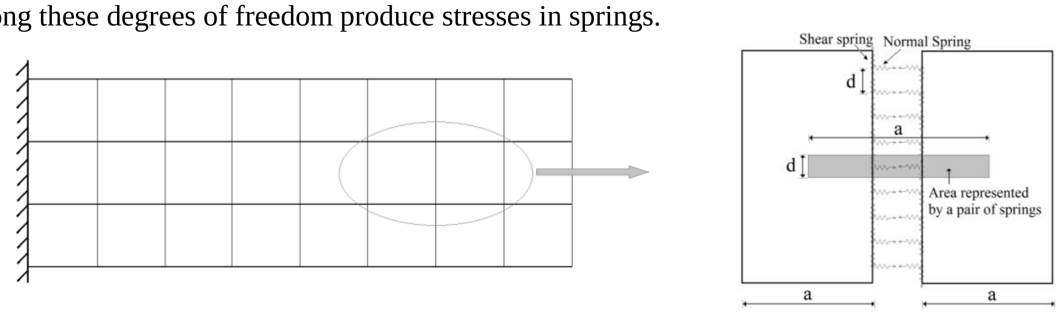

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Figure 1. Discretisation and distribution of springs in AEM.

Determination of crack pattern is one of the capabilities of AEM. The stress in every spring can be calculated from the displacement vector. When the stress exceeds the allowable limit, the spring is removed and a crack is assumed to occur at that region. The analysis is repeated with the new stiffness matrix without the contribution of the removed spring. Thus the crack points are determined and crack propagation can be captured.

2.1. Formulation of Stiffness Matrix of 2D-Element

With AEM, the structure is treated as an assembly of elements. Any two elements are connected by pairs of normal and shear springs. Springs carry stresses and strains of a certain portion of the elements considered.

The spring stiffness is determined as follows:

$$\text{Stiffness, } K = \frac{\text{Force}}{\text{Displacement}} = \frac{\frac{\text{Stress}}{\text{Strain}} \times \text{Area}}{\text{Initial Dimension}}$$

$$\text{Stiffness of normal spring, } K_n = \frac{E\,d\,t}{a} \tag{1}$$

$$\text{Stiffness of normal spring, } K_s = \frac{G\,d\,t}{a} \tag{2}$$

where,

d is the distance between springs, t is the thickness of the element, a is the c/c distance between elements, E is the modulus of elasticity and G is the modulus of rigidity of the material.

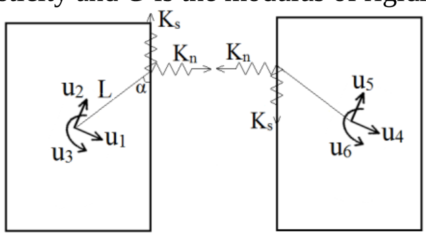


Figure 2. Typical rectangular elements and their degrees of freedom.

Each element has 3 degrees of freedom - u_1, u_2 and u_3 (as shown in figure 2). Hence a stiffness matrix of size 6 × 6 will be obtained for every set of springs. The global stiffness matrix K_G, is determined by assembling the stiffness matrix of all springs used in the structure.

2.2. Formulation of Stiffness Matrix by Physical Approach

The necessary details for the formulation of stiffness matrix is given in this section. For more details, reference [3] can be referred. First the stiffness matrix (K') is computed for the displacements in the local system (represented by u') shown in figure 3. The components of local stiffness matrix are calculated by applying unit displacement along one degree of freedom one at a time and determining forces corresponding to other degrees of freedom while the other degrees of freedom are restrained. equation (3) gives the local stiffness matrix of the spring (K').

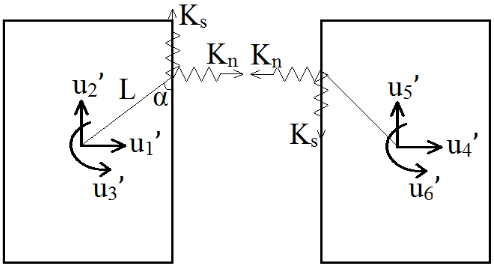


Figure 3. Local degrees of freedom.

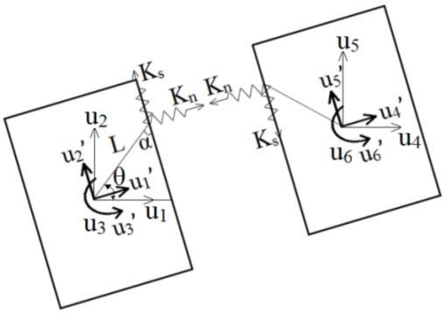


Figure 4. Global and Local degrees of freedom.

$$K' = \begin{pmatrix} K_n & 0 & -K_nL\cos\alpha & -K_n & 0 & K_nL\cos\alpha \\ 0 & K_s & K_sL\sin\alpha & 0 & -K_s & K_sL\sin\alpha \\ -K_nL\cos\alpha & K_sL\sin\alpha & \begin{matrix} K_nL^2\cos^2\alpha \\ +K_sL^2\sin^2\alpha \end{matrix} & K_nL\cos\alpha & -K_sL\sin\alpha & \begin{matrix} -K_nL^2\cos^2\alpha \\ +K_sL^2\sin^2\alpha \end{matrix} \\ -K_n & 0 & K_nL\cos\alpha & K_n & 0 & -K_nL\cos\alpha \\ 0 & -K_s & -K_sL\sin\alpha & 0 & K_s & -K_sL\sin\alpha \\ K_nL\cos\alpha & K_sL\sin\alpha & \begin{matrix} -K_nL^2\cos^2\alpha \\ +K_sL^2\sin^2\alpha \end{matrix} & -K_nL\cos\alpha & -K_sL\sin\alpha & \begin{matrix} K_nL^2\cos^2\alpha \\ +K_sL^2\sin^2\alpha \end{matrix} \end{pmatrix} \quad (3)$$

The local stiffness matrix is transformed into stiffness matrix of the spring corresponding to the global coordinates using the transformation matrix (T) determined as per figure 4. Equation (4) gives the transformation matrix.

$$T = \begin{pmatrix} \cos Ø & \sin Ø & 0 & 0 & 0 & 0 \\ -\sin Ø & \cos Ø & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos Ø & \sin Ø & 0 \\ 0 & 0 & 0 & -\sin Ø & \cos Ø & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad (4)$$

where, $Ø = \theta + \alpha - 90$

Now, Stiffness Matrix of the spring, $K = T^T K' T$ (5)

The stiffness matrix thus obtained is for a single set of spring. The sum of the stiffness matrix of all the springs in each face will give the stiffness matrix for the corresponding degrees of freedom. The stiffness matrices are then assembled to form the global stiffness matrix K_G.

The governing equation is:

$$[K_G][\Delta] = [F] \quad (6)$$

where, Δ is the displacement vector and F is the force vector

3. Analysis of Plain Concrete Beam

Plain concrete beams is analysed in two different ways. In the first case, beam is divided along the length direction only. The beam is divided along both length and depth direction in the second case.

3.1 Analysis of beams by discretising along the length direction

In this approach, the beam is divided into '*n*' elements. The degrees of freedom are assigned to all the elements as shown in figure 5. The stiffness of normal spring and shear spring are determined using Eqs. (1) and (2). The 6×6 stiffness matrix (K_1, K_2,....) of all springs are calculated as given in equation (3). Summing up all the stiffness matrices, the total stiffness matrix (K) can be obtained. This matrix corresponds to the degrees of freedom u_1, u_2, u_3, u_4, u_5 and u_6. Similarly the total stiffness matrices of all the connected elements are determined.

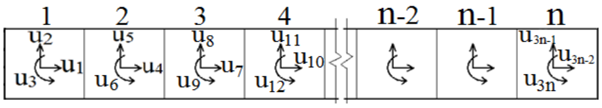


Figure 5. Elements divided along the length direction.

The global stiffness matrix is obtained by assembling the individual stiffness matrices. The global stiffness matrix is rearranged so that the degrees of freedom corresponding to known displacements are placed at left end. The unknown displacements and forces can be determined using equation (7).

$$\begin{pmatrix} K_{AA} & K_{AB} \\ K_{BA} & K_{BB} \end{pmatrix} \begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix} = \begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix} \tag{7}$$

where, U_1 is the known displacement vector, U_2 is the unknown displacement vector, P_1 is the unknown force vector and P_2 is the applied force vector.

3.1.1 Validation. A MATLAB code was developed to get the deflection, support reaction and bending moments of plain concrete beams. The coding is validated by analysing a fixed beam of span 3 m and cross-section 200 × 400 mm. The beam is shown in figure 6. The material's cube strength is 25 N/mm^2. The results obtained are shown in figure 7.

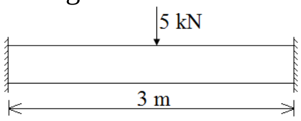
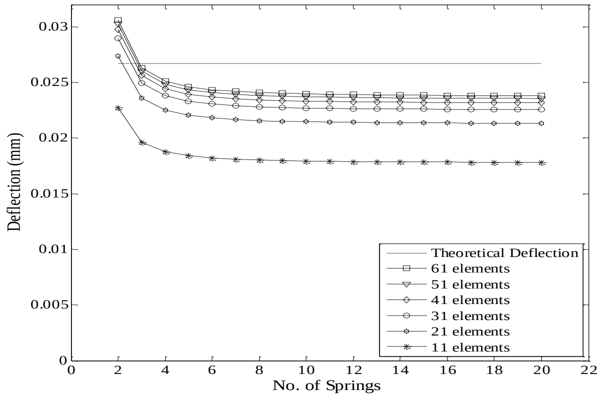


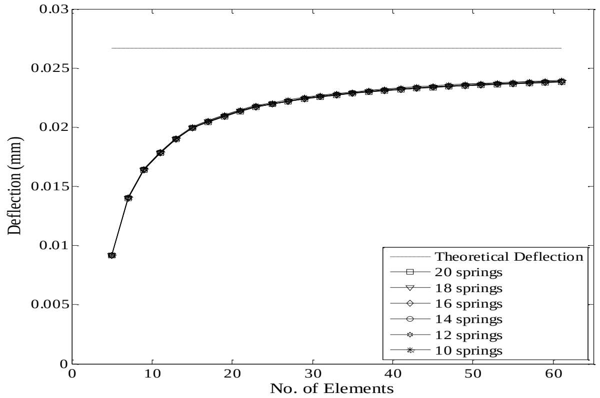
Figure 6. Fixed beam considered for analysis.

Theoretical Deflection (at 400 mm away from point load) = $\dfrac{Px^2(3l\text{-}4x)}{48EI}+\dfrac{3Px}{4AG}$ = 0.0267 mm

Theoretical Reaction = 2500 N Theoretical End Moment = 1.875 × 10^6 Nmm



(a)



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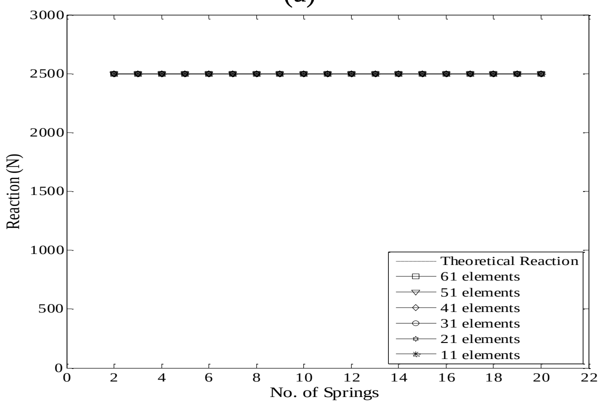

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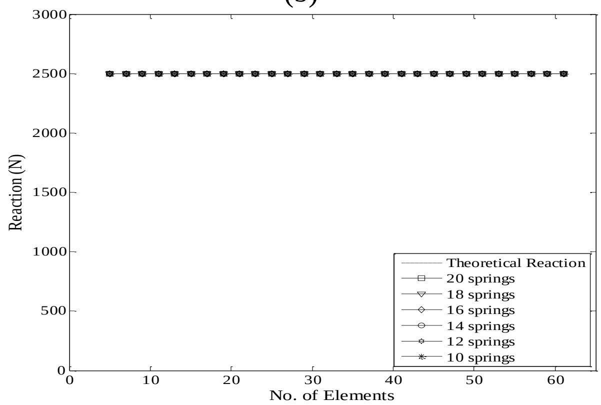
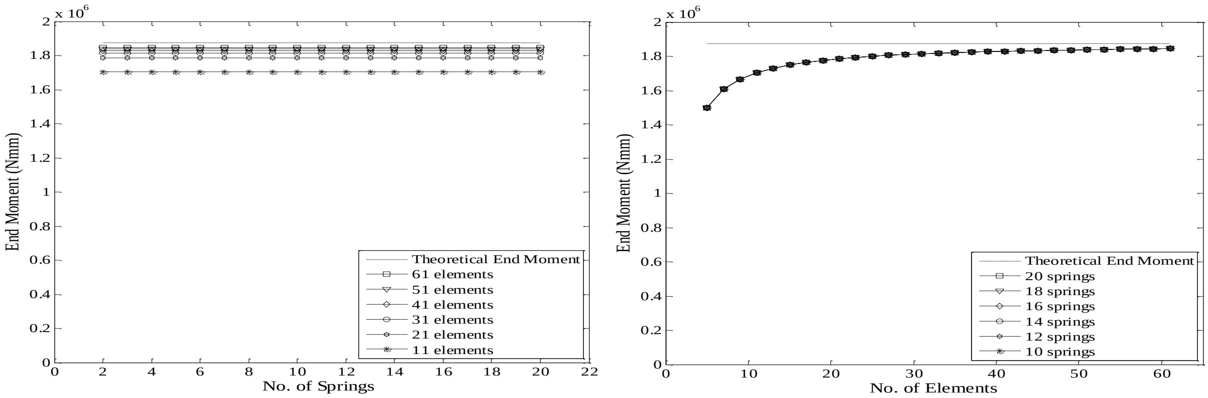


(d)



(e) (f)

Figure 7. Results in AEM (1D discretisation). (a) Deflection vs Number of springs; (b) Deflection vs Number of elements; (c) Reaction vs Number of springs; (d) Reaction vs Number of elements; (e) End Moment vs Number of springs; (f) End Moment vs Number of elements.

From figure 7 it is observed that for a given number of springs, deflection improved with increase in number of elements. It is observed that the computed deflection converges as the number of spring increases. Beyond 5 springs, no much improvement in result is obtained. So, it is appropriate to increase the number of elements rather than increasing the number of springs. The support reaction is accurately determined even with lesser number of elements and springs. The end moment and central moment converged to the theoretical value as the number of elements is increased, irrespective of the number of springs used.

3.2. Analysis of Beams by discretising along the length and depth direction

The accuracy of the result was studied by discretising the continuum in two directions. Discretisation of a beam into 4 elements in length direction and 3 elements in depth direction is shown in figure 8. When a beam is divided in two directions, stiffness of springs on all faces of the element should be considered as shown in figure 9.

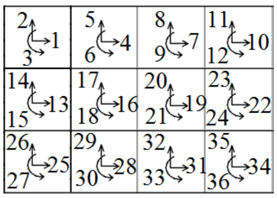


Figure 8. 2D Discretisation.

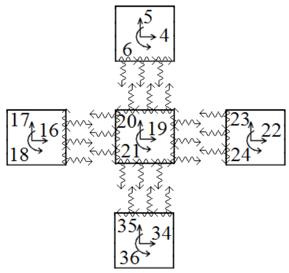


Figure 9. Spring on all faces.

As in the case of ID discretisation, the global stiffness matrix has to be reassembled to place the degrees of freedom with known displacement at the top left corner. Then the unknown displacements and forces can be determined using equation (7).

3.2.1. Validation. A MATLAB code was developed to determine deflection, support reaction and bending moments by discretising along both length and depth direction. The fixed beam considered in the previous section is studied. The elements are connected using 20 pairs of connecting springs on all its faces. The results are shown in figure 10.

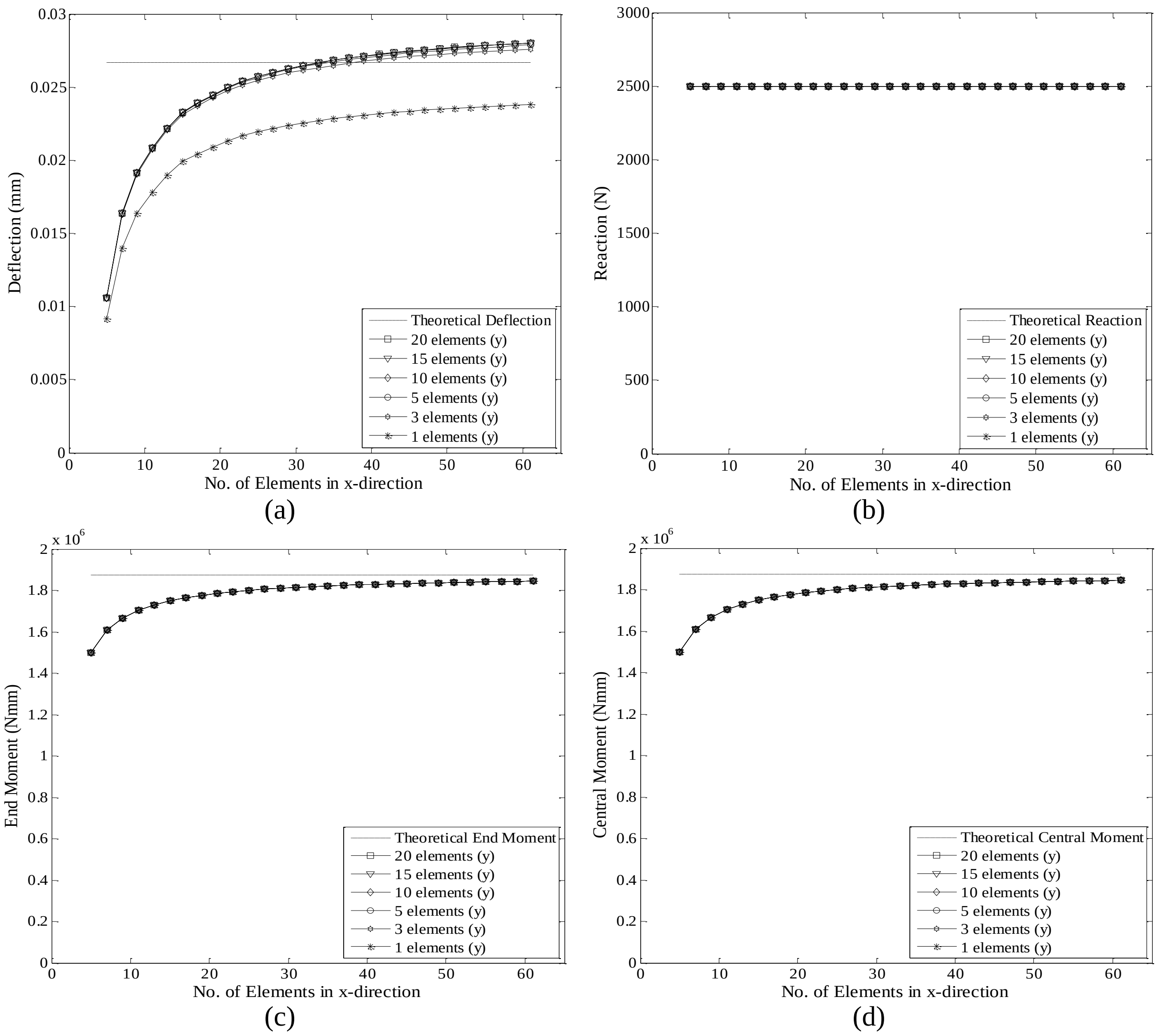


Figure 10. Results in AEM (2D discretisation). (a) Deflection vs Number of elements in x-direction; (b) Reaction vs Number of elements in x-direction; (c) End Moment vs Number of elements in x-direction; (d) Central Moment vs Number of elements in x-direction.

Figure 10 shows that when a beam is discretised in length direction alone, the deflection determined converges towards the theoretical value. But when the number of elements in depth direction is more than one, the deflection determined converges to a value higher than the theoretical deflection. But the number of elements along y-direction does not affect support reaction and bending moments. As in the case of discretisation along length direction alone, the support reaction is accurately determined. Also, the end moment and central moment converged to the theoretical value as the number of elements are increased irrespective of the number of springs used.

4. Conclusions

In this paper, AEM was used to analyse plain concrete beams of different support conditions. The following conclusions were derived from the study:

1. The results obtained using AEM became more accurate with increase in the number of elements. For a given number of elements the computed deflection converged when 5 or more springs were used.

2. The end moment and central moment converged to the exact value as the number of elements increased irrespective of the number of springs used.
3. When the beam was divided in two directions, the variation of number of springs and elements along the depth direction did not affect the support reaction and bending moment. The deflection also converged to a reasonably good value.

5. References

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[2] Meguro K and Tagel-Din H Applied Element Method for Structural Analysis: Theory and Application for Linear Materials 2000 *Int. J. of the Japan Society of Civil Engineers* **17** pp 31-45.

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[4] Gohel V, Patel P V and Joshi D 2013 Analysis of Frame using Applied Element Method *Procedia Engineering* pp 176-83.