IOP Conf. Series: Materials Science and Engineering **330** (2018) 012037 doi:10.1088/1757-899X/330/1/012037

Thermal analysis of smart composite laminated angle-ply using higher order shear deformation theory with zig zag function

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Abstract. The objective of the work is to develop the higher order theory for piezoelectric composite laminated plates with zigzag function and to determine the thermal characteristics of piezoelectric laminated plate with zig zag function for different aspect ratios (a/h), thickness ratios (z/h) and voltage and also to evaluate electric potential function by solving second order differential equation satisfying electric boundary conditions along the thickness direction of piezoelectric layer. The related functions and derivations for equation of motion are obtained using the dynamic version of the principle of virtual work or Hamilton's principle. The solutions are obtained by using Navier's stokes method for anti-symmetric angle-ply with specific type of simply supported boundary conditions. Computer programs have been developed for realistic prediction of stresses and deflections for various sides to thickness ratios (a/h) and voltages.

1. Introduction

In this work, actuator is coupled to the top of the composite laminated plate to achieve its thermal characteristics in non-dimensional form. When a load is applied to the piezoelectric material that causes deformation to take place which in turn alters the neutralized state and the desired configuration of the deformed shape can be achieved by the actuator. Piezoelectric materials have the ability to provide desired transformation from mechanical to electric energy and vice versa. Based on these characteristics piezoelectric materials can be used as actuators. To improve the accuracy of the prediction of plate deformation, a higher order shear deformation theory (HSDT) is used.

An improved simple higher-order shear deformation theory of laminated composite plates is developed. The theory has same dependent variables as in the first-order shear deformation theory which accounts for parabolic distribution of the diagonal shear strains through the thickness of the plate and diagonal shear stresses continuity across each layer interface. The present theory predicts the deflections and stresses more accurately when compared to simple higher-order theories and gives a much better approximation to the behaviour of laminated plates. Plate structures are major load carrying elements in structural mechanics, both in aeronautics, on land and in naval engineering. Such plates are often subjected to significant in plane compression forces and or shear loading. Various plate theories are available to describe the behaviour of such plates. Depending on the plate geometry and material properties, it is of interest to utilize one plate theory over another. Since in the middle of 19^{th} century there has been an on-going research and development of plate theories. This research has resulted mainly three categories in the field of plate theories:

- Classical Plate theory (CPT) or Kirchhoff Plate theory, suitable for thin plates with thickness to width ratio less than 1/10, on neglecting shear effects.
- Mindlin plate theory or first order shear deformation theory (FSDT) suitable for thick plates

with thickness to width ratio more than 1/10, on considering shear effects.

- Higher order shear deformation plate theories (HSDT) can represent the kinematics better than first order shear deformation theory (FSDT) and are especially suitable for composite plates, on including shear effects.

2. Formulation of HSDT with zig- zag function

A laminated plate of $0 \le x \le a$; $0 \le y \le b$ and $-h/2 \le z \le h/2$ is considered. The displacement components u, v and w along x, y, z and thickness directions, at any point in the plate are expanded in terms of thickness coordinate. The plate considered in this work is a 2D plate, the displacement field w (x, y, z, t) along the plate thickness is kept constant. In this work, in plane displacements are expanded as cubic functions of the thickness coordinate. The displacement field is expressed as:

$$\left.\begin{aligned} u(x,y,z) &= u_o(x,y) + z\theta_x(x,y) + z^2u_o^*(x,y) + z^3\theta_x^*(x,y) + \theta_k s_1(x,y) \\ v(x,y,z) &= v_o(x,y) + z\theta_y(x,y) + z^2v_o^*(x,y) + z^3\theta_y^*(x,y) + \theta_k s_2(x,y) \\ w(x,y,z) &= w_o(x,y) \end{aligned}\right\} \quad \ldots\ldots (1)$$

Where

θ_x, θ_y are rotations about x and y axis at midplane. u_o^*, v_o^*, θ_x^*, θ_y^* are the corresponding higher–order deformation terms defined at the midplane.

θ_k is the Zig-Zag function, defined as:

$$\theta_k = 2(-1)^k \frac{z_k}{h_k}$$

z_k is the local transverse coordinate with its origin at the centre of the k^{th} layer and h_k is the corresponding layer thickness.

The Zig-Zag function is piecewise linear with values of –1 and 1 alternately at the different interfaces. This function improves slope discontinuities at the interfaces of the smart laminated composite plates.

The strain components are:

$$\begin{aligned} \varepsilon_x &= \varepsilon_{x0} + zk_{sx} + z^2\varepsilon_{x0}^* + z^3k_x^* \\ \varepsilon_y &= \varepsilon_{y0} + zk_{sy} + z^2\varepsilon_{y0}^* + z^3k_y^* \\ \varepsilon_z &= 0 \\ \gamma_{xy} &= \varepsilon_{xy0} + zk_{sxy} + z^2\varepsilon_{xy0}^* + z^3k_{xy}^* \\ \gamma_{yz} &= \phi_{sy} + z\varepsilon_{yz0} + z^2\phi_y^* \\ \gamma_{xz} &= \phi_{sx} + z\varepsilon_{xz0} + z^2\phi_x^* \end{aligned} \quad \ldots\ldots\ldots (2)$$

3. Laminate constitutive equations

$$\{\sigma\} = [C]\{\varepsilon\} - [e]\{E\}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}^L = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} \\ Q_{12} & Q_{22} & Q_{23} \\ Q_{13} & Q_{23} & Q_{33} \end{bmatrix}^L \begin{Bmatrix} \in_x - \alpha_x \Delta T \\ \in_y - \alpha_y \Delta T \\ \gamma_{xy} - \alpha_{xy} \Delta T \end{Bmatrix}^L - \begin{bmatrix} 0 & 0 & e_{31} \\ 0 & 0 & e_{32} \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \frac{-\partial \xi(x,y,z)}{\partial x} \\ \frac{-\partial \xi(x,y,z)}{\partial y} \\ \frac{-\partial \xi(x,y,z)}{\partial z} \end{Bmatrix} \quad \ldots\ldots. (3)$$

$$\begin{Bmatrix} \tau_{yz} \\ \tau_{xz} \end{Bmatrix}^L = \begin{bmatrix} Q_{44} & Q_{45} \\ Q_{45} & Q_{55} \end{bmatrix}^L \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}^L - \begin{bmatrix} 0 & e_{24} & 0 \\ e_{15} & 0 & 0 \end{bmatrix} \begin{Bmatrix} \frac{-\partial \xi(x,y,z)}{\partial x} \\ \frac{-\partial \xi(x,y,z)}{\partial y} \\ \frac{-\partial \xi(x,y,z)}{\partial z} \end{Bmatrix}$$

$$E_x = \frac{-\partial \xi\,(x,y,z)}{\partial x}\,,\; E_y = \frac{-\partial \xi\,(x,y,z)}{\partial y},\; E_z = \frac{-\partial \xi\,(x,y,z)}{\partial z},$$

$\xi(x, y, z)$ is the electro static potential, α_x, α_y, α_{xy} are the transformed thermal coefficients of expansion. $\Delta T = T\,(x, y, z, t)$ is the temperature increment from the reference.
The governing equations of displacement model are derived using the principle of virtual work or Hamilton's principle.

$$\int_0^T (\delta U + \delta V - \delta K)\, dt = 0 \quad \ldots\ldots (4)$$

On substituting for δU, δV and δK equations in the virtual work statement in Eq. (4) and integrating through the thickness of the laminate, it is obtained as:

$$\begin{aligned}
&\int_0^T \{ \int [N_x \delta\varepsilon_{x0} + M_x \delta k_{sx} + N_x{}^* \delta\varepsilon_{x0}{}^* + M_x{}^* \delta k_x{}^* + N_y \delta\varepsilon_{y0} + M_y\, \delta k_{sy} + \\
&N_y{}^* \delta\varepsilon_{y0}{}^* + M_y{}^* \delta k_y{}^* + N_{xy} \delta\varepsilon_{xy0} + M_{xy} \delta k_{sxy} \\
&N_{xy}{}^* \delta\varepsilon_{xy0}{}^* + M_{xy}{}^* \delta k_{xy}{}^* + Q_x\, \delta\phi_{sx} + S_x \delta\varepsilon_{xz0} + \\
&\mathrm{Q_x}{}^* \delta\phi_x{}^* + Q_y\, \delta\phi_{sy} + S_y \delta\varepsilon_{yz0} + \mathrm{Q_y}{}^* \delta\phi_y{}^* - q\delta\dot{w}_0 - \\
&I_1(\dot{u}_0 \delta\dot{u}_0 + \dot{v}_0 \delta\dot{v}_0 + \dot{w}_0 \delta\dot{w}_0) - \\
&\mathrm{I}_2(\dot{\theta}_x \delta\dot{u}_0 + \dot{\theta}_y \delta\dot{v}_0 + \dot{u}_0 \delta\dot{\theta}_x + \dot{v}_0 \delta\dot{\theta}_y) - \\
&\mathrm{I}_3(\dot{u}_0 \delta\dot{u}_0^* + \dot{v}_0 \delta\dot{v}_0^* + \dot{\theta}_x \delta\dot{\theta}_x + \dot{\theta}_y \delta\dot{\theta}_y + \dot{u}_0^* \delta\dot{u}_0 + \dot{v}_0^* \delta\dot{v}_0) - \\
&\mathrm{I}_4(\dot{u}_0 \delta\dot{\theta}_0^* + \dot{\theta}_x \delta\dot{u}_0^* + \dot{u}_0^* \delta\dot{\theta}_x + \dot{\theta}_0^* \delta\dot{u}_0 + \dot{\theta}_y^* \delta\dot{v}_0 + \dot{v}_0^* \delta\dot{\theta}_y + \dot{\theta}_y \delta\dot{v}_0^* + \dot{v}_0 \delta\dot{\theta}_0^*) - \\
&\mathrm{I}_5(\dot{\theta}_x^* \delta\dot{\theta}_x + \dot{u}_0^* \delta\dot{u}_0^* + \dot{\theta}_x \delta\dot{\theta}_x^* + \dot{\theta}_y^* \delta\dot{\theta}_y + \dot{v}_0^* \delta\dot{v}_0^* + \dot{\theta}_y \delta\dot{\theta}_y^*) - \\
&\mathrm{I}_6(\dot{\theta}_x^* \delta\dot{u}_0^* + \dot{u}_0^* \delta\dot{\theta}_x^* + \dot{\theta}_y^* \delta\dot{v}_0^* + \dot{v}_0^* \delta\dot{\theta}_y^*) - \mathrm{I}_7(\dot{\theta}_x^* \delta\dot{\theta}_x^* + \dot{\theta}_y^* \delta\dot{\theta}_y^*) - \\
&RI_2((\dot{u}_0 \delta\ \dot{s}_1 + \dot{v}_0 \delta\dot{s}_2 + \dot{s}_2 \delta\dot{v}_0 + s_1 \delta\dot{u}_0) - \\
&RI_3(\dot{\theta}_x \delta\dot{s}_1 + \dot{\theta}_y \dot{s}_2 + \dot{s}_2 \delta\dot{\theta}_y + \dot{s}_1\ \delta\dot{\theta}_x) - \\
&RI_4(\dot{u}_0{}^* \delta\dot{s}_2 + \dot{v}_0^* \delta\dot{s}_2 + \dot{s}_2 \delta\dot{v}_0^* + \dot{s}_1 \delta\dot{u}_0^*) - \\
&RI_5(\dot{\theta}_0^* \delta\dot{s}_1 + \dot{\theta}_0^* \delta\dot{s}_2 + \dot{s}_2 \delta\dot{\theta}_0^* + s^*{}_1 \delta\ \dot{\theta}_0^*) - \qquad \ldots\ldots (5) \\
&RI_3(\dot{s}_1 \delta\dot{s}_1 + \dot{s}_2 \delta\dot{s}_2)] dx\ dy\} dt = 0
\end{aligned}$$

From the above equation for total potential energy and by equating the coefficients of each of virtual displacements $\delta u_o \delta v_0$, δw_0, $\delta\theta_x$, $\delta\theta_y$, δu_0^*, δv_0^*, $\delta\theta_x^*$, $\delta\theta_y^*$, δs_1, δs_2 by equating to zero, the equations of motion are obtained.

The virtual work statement shown in Eq (4), integrating through the thickness of laminate, the in-plane, transverse force and moment resultant relations in the form of matrix obtained as:

$$\begin{Bmatrix} N \\ N^* \\ -- \\ M \\ M^* \\ -- \\ Q \\ Q^* \end{Bmatrix} = \begin{bmatrix} A & | & B & | & 0 \\ \overline{B^t} & | & \overline{D_b} & | & \overline{0} \\ \overline{0} & | & 0 & | & \overline{D_s} \end{bmatrix} \begin{Bmatrix} \varepsilon_0 \\ \varepsilon_0^* \\ -- \\ K_s \\ K^* \\ -- \\ \phi_s \\ \phi^* \end{Bmatrix} - \begin{Bmatrix} N^T \\ N^{*T} \\ -- \\ M^T \\ M^{*T} \\ -- \\ 0 \\ 0 \end{Bmatrix} - \begin{Bmatrix} N^{PZ} \\ N^{*PZ} \\ -- \\ M^{PZ} \\ M^{*PZ} \\ -- \\ Q^{PZ} \\ Q^{*PZ} \end{Bmatrix} \quad \ldots\ldots. (6)$$

For homogenous laminates, the equation of motion in terms of displacements is expressed as:

$$A_{11}\frac{\partial^2 u_0}{\partial x^2} + A_{12}\frac{\partial^2 v_0}{\partial x\,\partial y} + A_{13}\left(\frac{\partial^2 u_0}{\partial x\,\partial y} + \frac{\partial^2 v_0}{\partial x^2}\right) + A_{14}\left(\frac{\partial^2 u_0^*}{\partial x^2}\right) + A_{15}\left(\frac{\partial^2 v_0}{\partial x\partial y}\right) + A_{16}\left(\frac{\partial^2 u_0^*}{\partial x\partial y} + \frac{\partial^2 v_0^*}{\partial x^2}\right) +$$
$$B_{11}\left(\frac{\partial^2\theta_x}{\partial x^2} + R\frac{\partial^2 s_1}{\partial x^2}\right) + B_{12}\left(\frac{\partial^2\theta_y}{\partial x\partial y} + R\frac{\partial^2 s_2}{\partial x\partial y}\right) + B_{13}\left(\frac{\partial^2\theta_x}{\partial x\partial y} + \frac{\partial^2\theta_y}{\partial x^2} + R\left(\frac{\partial^2 s_1}{\partial x\partial y} + \frac{\partial^2 s_2}{\partial x^2}\right)\right) + B_{14}\frac{\partial^2\theta_x^*}{\partial x^2} +$$
$$B_{15}\frac{\partial^2\theta_y^*}{\partial x\partial y} + B_{16}\left(\frac{\partial^2\theta_x^*}{\partial x\partial y} + \frac{\partial^2\theta_y^*}{\partial x^2}\right) + A_{31}\frac{\partial^2 u_0}{\partial x\partial y} + A_{32}\frac{\partial^2 v_0}{\partial y^2} + A_{33}\left(\frac{\partial^2 u_0}{\partial y^2} + \frac{\partial^2 v_0}{\partial x\partial y}\right) + A_{34}\left(\frac{\partial^2 u_0^*}{\partial x\partial y}\right) +$$
$$A_{35}\left(\frac{\partial^2 v_0^*}{\partial y^2}\right) + A_{36}\left(\frac{\partial^2 u_0^*}{\partial y^2} + \frac{\partial^2 v_0^*}{\partial x\partial y}\right) + B_{31}\left(\frac{\partial^2\theta_x}{\partial x\partial y} + R\frac{\partial^2 s_1}{\partial x\partial y}\right) + B_{32}\left(\frac{\partial^2\theta_y}{\partial y^2} + R\frac{\partial^2 s_2}{\partial y^2}\right) +$$
$$B_{33}\left(\frac{\partial^2\theta_x}{\partial y^2} + \frac{\partial^2\theta_y}{\partial x\partial y} + R\left(\frac{\partial^2 s_1}{\partial y^2} + \frac{\partial^2 s_2}{\partial x\partial y}\right)\right) + B_{34}\frac{\partial^2\theta_x^*}{\partial x\partial y} + B_{35}\frac{\partial^2\theta_y^*}{\partial y^2} + B_{36}\left(\frac{\partial^2\theta_x^*}{\partial y^2} + \frac{\partial^2\theta_y^*}{\partial x\partial y}\right)$$
$$-\left(\frac{\partial N_x^T}{\partial x} + \frac{\partial N_y^T}{\partial y}\right) - \left(\frac{\partial \mathrm{N}_{\mathrm{X}}^{\mathrm{PZ}}}{\partial \mathrm{x}} - \frac{\partial N_{XY}^{PZ}}{\partial y}\right) = I_1\frac{\partial^2 u_0}{\partial t^2} + I_2\left(\frac{\partial^2\theta_x}{\partial t^2} + R\frac{\partial^2 s_1}{\partial t^2}\right)\frac{\partial^2\theta_x}{\partial t^2} + I_3\frac{\partial^2 u_0^*}{\partial t^2} + I_4\frac{\partial^2\theta_x^*}{\partial t^2} \quad \ldots\ldots (7)$$

The Navier's solutions of simply supported (SS) anti symmetric angle ply laminated plates are considered. The SS boundary conditions are:

At edges x = 0 and x = a

$u_0=0, w_0=0, \theta_y=0, N_{xy}=0, M_x=0, u_0^*=0, \theta_y^*=0, M_x^*=0, N_{xy}^*=0, S_1=0$ .....(8.1)

At edges y = 0 and y = b

$v_0=0, w_0=0, \theta_x=0, N_{xy}=0, M_y=0, v_0^*=0, \theta_x^*=0, M_y^*=0, N_{xy}^*=0, S_2=0$ ....(8.2)

Boundary conditions are satisfied by following expansions.

$$u_0(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}U_{mn}(t)\sin\alpha x\cos\beta y$$

$$v_0(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}V_{mn}(t)\cos\alpha x\sin\beta y$$

$$w_0(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}W_{mn}(t)\sin\alpha x\sin\beta y$$

$$\theta_x(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}X_{mn}(t)\cos\alpha x\sin\beta y$$

$$\theta_y(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}Y_{mn}(t)\sin\alpha x\cos\beta y$$

$$u_0^*(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}U^*_{mn}(t)\sin\alpha x\cos\beta y$$

$$V_o^*(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}V^*_{mn}(t)\cos\alpha x\sin\beta y$$

$$\theta_x^*(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}X^*_{mn}(t)\cos\alpha x\sin\beta y$$

$$\theta_y^*(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}Y^*_{mn}(t)\sin\alpha x\cos\beta y$$

$$S_1(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}S_{1mn}(t)\sin\alpha x\cos\beta y$$

$$S_2(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}S_{2mn}(t)\cos\alpha x\sin\beta y$$

The coefficients U_{mn}, V_{mn}, W_{mn}, X_{mn}, Y_{mn}, Z_{mn}, U^*_{mn}, V^*_{mn}, X^*_{mn}, Y^*_{mn}, S_{1mn} and $S_{2mn}$of the Navier's solutions are calculated by rewriting Equation in the matrix form as:

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} & S_{17} & S_{18} & S_{19} & S_{10} & S_{11} \\ S_{12} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} & S_{27} & S_{28} & S_{29} & S_{210} & S_{211} \\ S_{31} & S_{32} & S_{33} & S_{34} & S_{35} & S_{36} & S_{37} & S_{38} & S_{39} & S_{310} & S_{311} \\ S_{41} & S_{42} & S_{43} & S_{44} & S_{45} & S_{46} & S_{47} & S_{48} & S_{49} & S_{410} & S_{411} \\ S_{51} & S_{52} & S_{53} & S_{54} & S_{55} & S_{56} & S_{57} & S_{58} & S_{59} & S_{510} & S_{511} \\ S_{61} & S_{62} & S_{63} & S_{64} & S_{65} & S_{66} & S_{67} & S_{68} & S_{69} & S_{610} & S_{611} \\ S_{71} & S_{72} & S_{73} & S_{74} & S_{75} & S_{76} & S_{77} & S_{78} & S_{79} & S_{710} & S_{711} \\ S_{81} & S_{82} & S_{83} & S_{84} & S_{85} & S_{86} & S_{87} & S_{88} & S_{89} & S_{810} & S_{811} \\ S_{91} & S_{92} & S_{93} & S_{94} & S_{95} & S_{96} & S_{97} & S_{98} & S_{99} & S_{910} & S_{911} \\ S_{101} & S_{102} & S_{103} & S_{104} & S_{105} & S_{106} & S_{107} & S_{108} & S_{109} & S_{1010} & S_{1011} \\ S_{111} & S_{112} & S_{113} & S_{114} & S_{115} & S_{116} & S_{117} & S_{118} & S_{119} & S_{1110} & S_{1111} \end{bmatrix} \begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \\ X_{mn} \\ Y_{mn} \\ U^{\bullet}{}_{mn} \\ V^{\bullet}{}_{mn} \\ X^{\bullet}{}_{mn} \\ Y^{\bullet}{}_{mn} \\ S_{1mn} \\ S_{2mn} \end{Bmatrix} +$$

$$\begin{bmatrix} m_{11} & 0 & 0 & m_{14} & 0 & m_{16} & 0 & m_{18} & 0 & m_{110} & 0 \\ 0 & m_{22} & 0 & 0 & m_{25} & 0 & m_{27} & 0 & m_{29} & 0 & m_{211} \\ 0 & 0 & m_{33} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ m_{14} & 0 & 0 & m_{44} & 0 & m_{46} & 0 & m_{48} & 0 & m_{410} & 0 \\ 0 & m_{52} & 0 & 0 & m_{55} & 0 & m_{57} & 0 & m_{59} & 0 & m_{511} \\ m_{61} & 0 & 0 & m_{64} & 0 & m_{66} & 0 & m_{68} & 0 & m_{610} & 0 \\ 0 & m_{72} & 0 & 0 & m_{75} & 0 & m_{77} & 0 & m_{79} & 0 & m_{711} \\ m_{81} & 0 & 0 & m_{84} & 0 & m_{86} & 0 & m_{88} & 0 & m_{810} & 0 \\ 0 & m_{92} & 0 & 0 & m_{95} & 0 & m_{97} & 0 & m_{99} & 0 & m_{911} \\ m_{101} & 0 & 0 & m_{104} & 0 & m_{106} & 0 & m_{108} & 0 & m_{1010} & 0 \\ 0 & m_{112} & 0 & 0 & m_{115} & 0 & m_{117} & 0 & m_{119} & 0 & m_{1111} \end{bmatrix} \begin{Bmatrix} \ddot{U}_{mn} \\ \ddot{V}_{mn} \\ \ddot{W}_{mn} \\ \ddot{X}_{mn} \\ \ddot{Y}_{mn} \\ \ddot{U}^{*}{}_{mn} \\ \ddot{V}^{*}{}_{mn} \\ \ddot{X}^{*}{}_{mn} \\ \ddot{Y}^{*}{}_{mn} \\ \ddot{S}_{1mn} \\ \ddot{S}_{2mn} \end{Bmatrix} =$$

$$\begin{Bmatrix} 0 \\ 0 \\ T_{mn} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} - \begin{Bmatrix} N^{1} \\ N^{2} \\ 0 \\ M^{1} \\ M^{2} \\ N^{*1} \\ N^{*2} \\ M^{*1} \\ M^{*2} \\ S_1^1 \\ S_2^2 \end{Bmatrix} - V_t \begin{Bmatrix} v_{t1} \\ v_{t2} \\ v_{t3} \\ v_{t4} \\ v_{t5} \\ v_{t6} \\ v_{t7} \\ v_{t8} \\ v_{t9} \\ v_{t10} \\ v_{t11} \end{Bmatrix} \qquad \text{….. (9)}$$

The displacements at the mid plane will be defined to satisfy the boundary conditions in Eq. (8.1&8.2).These displacements will be substituted in governing equations to obtain the equations in terms of A, B, D parameters. The obtained equations will be solved to find the behavior of the laminated composite plates. On substitution of Eq. (4) in governing equations of motion in displacements (7), it is obtained as:

$$\begin{aligned}&\left(A_{11}\alpha^2+A_{33}\beta^2\right)U_{mn}+\left(\mathrm{A}_{12}+A_{33}\right)\mathrm{V}_{mn}\,\alpha\beta+\left(\mathrm{B}_{13}+B_{31}\right)X_{mn}\alpha\beta+\\&\left(\mathrm{B}_{13}\alpha^2+B_{33}\beta^2\right)Y_{mn}\,\alpha\beta+\left(A_{14}\alpha^2+A_{36}\beta^2\right)U^*_{mn}+\left(A_{15}+A_{36}\right)V^*_{mn}\,\alpha\beta+\\&\left(B_{16}+B_{34}\right)X^*_{mn}\alpha\beta+\left(B_{16}\alpha^2+B_{35}\beta^2\right)Y^*_{mn}\,\alpha\beta+R\left(\mathrm{B}_{11}\alpha^2+B_{33}\beta^2\right)S_{1mn}+\\&R\left(\mathrm{B}_{12}+B_{33}\right)S_{2mn}\,\alpha\beta+\left(N_x^{pz}+N_{xy}^{pz}\right)\\&=\left(I_1\ddot{U}_{mn}+I_2\left(\ddot{X}_{mn}+RS_{1mn}\right)+I_3\ddot{U}^*_{mn}+\mathrm{I}_4\,\ddot{\mathrm{X}}^*_{mn}\right)-N^1\end{aligned} \quad ..... (10)$$

4. Results and discussions

The material properties used for each orthotropic layer are

Elastic layer (Carbon Epoxy)
E_1/E_2= 6.6 G_{12}/E_2= 7.5 G_{23}/E_2= 0.36 $E_2=E_3$= 3.3 N/ m^2 $\mu_{12}=\mu_{23}=\mu_{13}=0.3$
α_1 =2 × 10^{-6} $^0C^{-1}$, α_2 =1.125 × 10^{-3} $^0C^{-1}$, α_3 =1.125 × 10^{-3} $^0C^{-1}$

PFRC Layer:
C_{11} = 24.1Gpa, $C_{12} = C_{21}$ = 7.2Gpa, $C_{13} = C_{31}$ = 4.76Gpa, $C_{22} = C_{23}$ =7.2Gpa, C_{44} =1.2Gpa, $C_{55} = C_{66}$ =2.5Gpa, e_{31}= -6.76C/m^2, $g_{11}=g_{22}$ = 0.037E-9C/V m, g_{33}=10.64E - 9 C/V m
The center deflections and stresses are presented here in non-dimensional form using the following multipliers:

$$m_6 = e_2\alpha_1 t_0\ ,\ m_7 = \frac{a_1^2 t\ \alpha_1 t_0}{t^2}$$

From plot 1 it is noticed that deflection is maximum for 2 layers for a side to thickness ratio of 10. The refraction of the smart composite laminated plate tend to decreases as the side of the piezoelectric actuators increases and the effect of coupling on refractions is quite significant for aspect ratio less than 3.From plot 2 it is noticed that diagonal shear stress is observed maximum for 2 layers as a function of side to thickness ratio. The effect of diagonal shear stress deformation and coupling is quite significant for all values of side thickness ratio a/h < 5. From plot 3 it is noticed that normal stress is maximum for 4 layers as a function of thickness coordinate. The effect of coupling is to decrease the stress with increase in thickness coordinate, hence the plot satisfies.

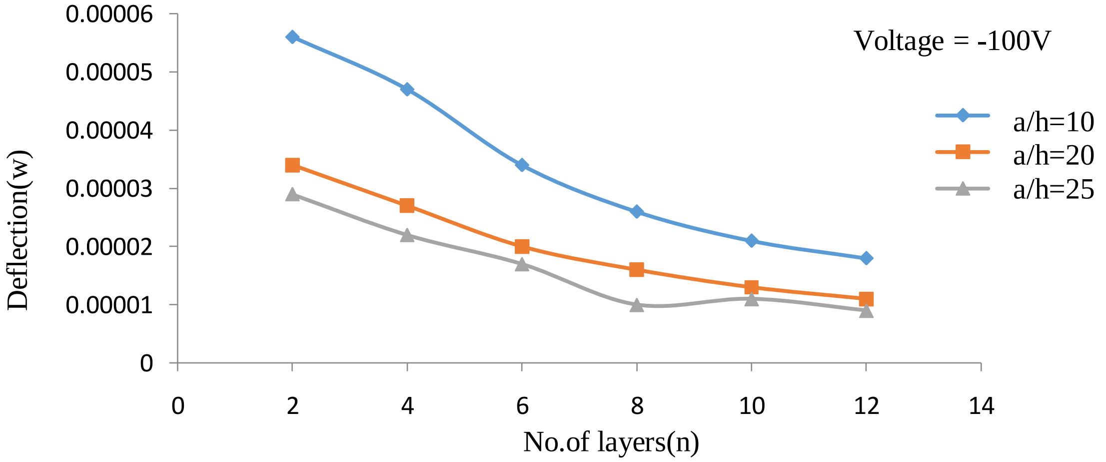

Fig.1.Non dimensionalised deflection (w) vs no.of layers for a simply supported angle ply piezoelectric composite laminated plate.

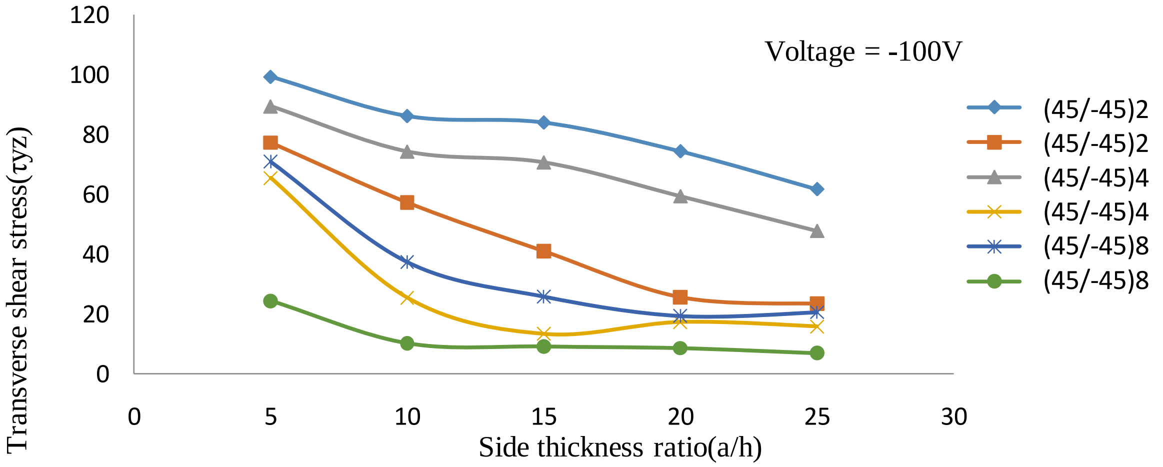

Fig.2 Non dimensionalized side thickness ratio(a/h) vs transverse shear stress(τyz) for a simply supported angle ply piezoelectric composite laminated plate.

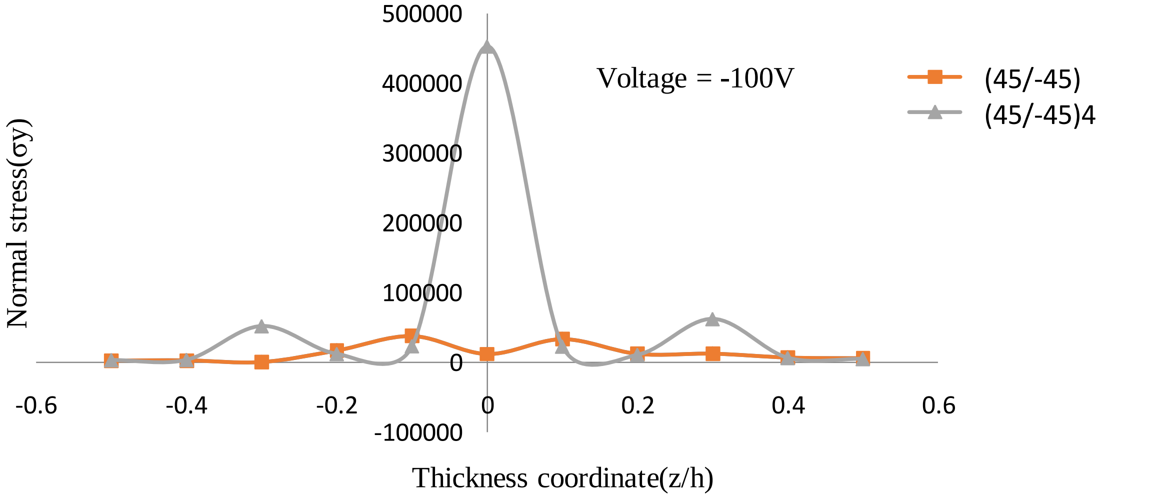

Fig.3.Non dimensionalized thickness coordinated(z/h) vs normal stress(σy) for a simply supported angle ply piezoelectric composite laminated plate.

5. Conclusion

Analytical procedure is developed for thermal analysis of smart composite laminated plates subjected to electromechanical loading. It is concluded that, the transverse displacement varies non-linearly for plate subjected to temperature gradient than for the plates subjected to mechanical loads. The use of a Zig-Zag function is more effective than a discrete layer approach of approximating the displacement variations over the thickness of each layer separately.

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