

Geometric Model of Induction Heating Process of Iron-Based Sintered Materials

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Abstract. The article studies the issue of building multivariable dependences based on the experimental data. A constructive method for solving the issue is presented in the form of equations of $(n-1)$ – surface compartments of the extended Euclidean space $E+n$. The dimension of space is taken to be equal to the sum of the number of parameters and factors of the model of the system being studied. The basis for building multivariable dependencies is the generalized approach to n -space used for the surface compartments of 3D space. The surface is designed on the basis of the kinematic method, moving one geometric object along a certain trajectory. The proposed approach simplifies the process aimed at building the multifactorial empirical dependencies which describe the process being investigated.

1. Introduction

Powder metallurgy is one of the methods aimed to improve the efficiency of modern production, which makes it possible to obtain products of relatively low cost from the materials with predetermined properties. Heat treatment is considered as the most effective method to influence the performance characteristics of structural items from low-alloyed iron-carbon powder materials [1].

However, nowadays, most issues arising from the application of the heat treatment method of products from sintered materials, from both scientific and technological points of view, require further investigation and research. A great number of technological process parameters and difficulties related to the relationship description between them lead to the need to solve the issue of analysis and synthesis of the process using trendy methods of geometric (mathematical) modeling.

The article presents and implements an approach of creating a mathematical model of the induction heating process (IHP) and making items of “bushing” type from sintered materials based on iron, in the form of a hypersurface compartment of the factor-parametric space.

2. Materials and methods

Traditionally, the IHP mathematical model is built on the basis of a nonlinear, interconnected system of Maxwell and Fourier equations with the necessary system of boundary conditions added to it. The practice has shown that none of the existing models of induction heating process of metal products can be used to the full extent to obtain information on the process of product heating from sintered materials, as well as they cannot be used to create the ASPTT PM information base. A significant shortcom-



ing of these models is that they consider thermophysical properties of materials unchanged throughout the heating interval, which is not applicable for porous materials [2].

The methods aimed to build and create geometric models for the induction heating of products from sintered powder materials of structural purpose are the objects of this study. To build the IHP geometric model, which would adequately describe the process, methods based on analytical, differential and descriptive multidimensional geometry, elements of differential and integral calculus as well as applied geometry and computer graphics were used. To test the efficiency of the model, bushings were used made of ZhGr1D1.5 material with the porosity of 15% according to GOST 26802-86. Geometric parameters of the bushing were as follows: outer diameter amounted to 20 mm; inner diameter amounted to 10 mm; height of the bushing equaled to 15 mm. For the heating purposes, the HF generator VCHG2-100 / 0.066 was used, which is part of the machine for the production of sintered products. Parameters of the heating machine were as follows: 100 kW is the nominal fixed power on the inductor; 66000 Hz is the frequency of supply current. The heating temperature was measured by an optical pyrometer PCHD 131-05 and the heating time by a stopwatch. The inductor power varied within the range of $0.1 \div 0.8$ from the maximum point.

From the point of view of geometry, a mathematical model presented in the form of $F(x_1, x_2, \dots, x_n)$ is considered as a certain geometric object. This assumption allows us to theoretically represent IHP as a hypersurface of the factor-parametric space (Fig. 1) and more precisely in the form of plane-parallel surface transfer [3, 4]:

$$y(x_1, x_2, \dots, x_{n-1}) = v(x_1, x_2, \dots, x_k) + u(x_{n-1}, x_{n-2}, \dots, x_k) - y_0,$$

where, $y = v(x_1, x_2, \dots, x_k)$ is generatrix equation v ;

$y = u(x_{n-1}, x_{n-2}, \dots, x_k)$ is the equation of direction u ;

y_0 is the coordinate of a surface intersection point on the y_0 axis.

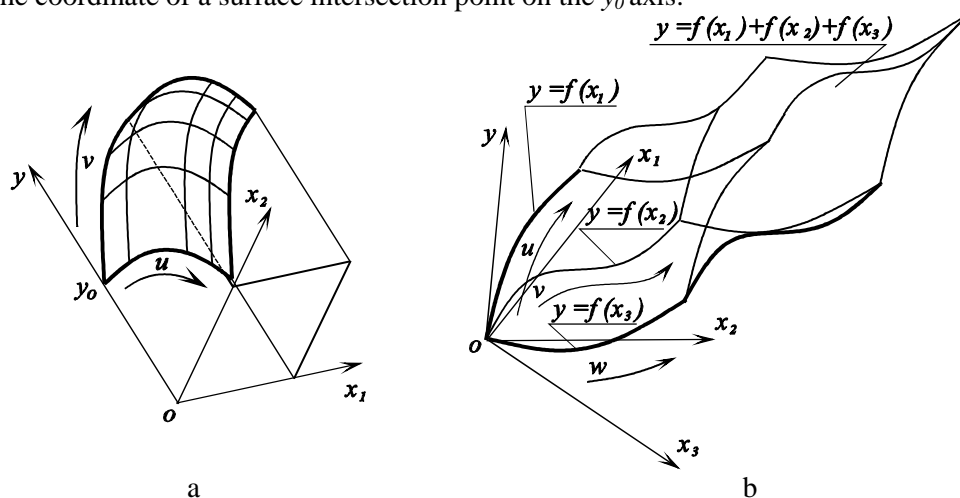


Figure 1. Hypersurfaces of plane-parallel transfer: a - three-dimensional space, b - four-dimensional space.

3. IHP Model Creation

Basing on the theoretical suppositions, the mathematical model of IHP products should be built on the basis of a nonlinear, interrelated system of Maxwell and Fourier equations with a necessary system of boundary conditions added to it [5]:

$$\begin{aligned} \operatorname{rot} \bar{E} &= -\frac{\partial \bar{B}}{\partial \tau}; & \operatorname{div} \bar{B} &= 0; & \operatorname{div} \bar{E} &= 0; \\ c(t) \gamma \frac{\partial t}{\partial \tau} - \operatorname{div}(\lambda(t) \operatorname{grad} t) + c(t) \gamma(t) \bar{V} \operatorname{grad} t &= -\operatorname{div}[\bar{E} \cdot \bar{H}] \end{aligned} \quad (1)$$

where, $\vec{H}, \vec{B}, \vec{E}$ are the vectors of intensity of magnetic and electric fields and of the magnetic induction; σ, c, γ are the specific values of electrical conductivity, heat capacity and density of material being heated; λ is the coefficient of thermal conductivity; \vec{V} is the speed vector of heated body; t is the temperature field; τ is the time.

In the case of sintered materials, it should be borne in mind that electromagnetic and thermophysical characteristics in the process of heating vary according to other laws rather than for compact materials. The greatest influence on these characteristics is caused by carbon and the porosity of material.

To create the model, the experimental dependence of resistivity ρ on the heating temperature of $T^{\circ}\text{C}$ was applied, which is represented by the following dependence:

$$\rho = 111.6785 - 110.7229e^{(-151.3911T^{-1.4606})} \quad (2)$$

The dependence of magnetic permeability μ on the frequency of current-carrying inductor f and magnetic field strength H was determined as:

$$\mu(H, f) = \left(a + \frac{b}{H}\right) \frac{c + f}{f + d} \quad (3)$$

where, $a = 28.5513$; $b = -5765.1743$; $c = 487.6289$; $d = 192.8$.

To determine numerical value λ , the experimental dependence of the thermal conductivity on the heating temperature of material $T^{\circ}\text{C}$ was also applied:

$$\lambda = 12.8590 e^{\left(-\frac{346.3552}{T}\right)} \quad (4)$$

Taking into account the technological features of heating and the assumptions made, the “real base model” will take different form compared to the original one:

$$\frac{\partial^2 T(P, \tau)}{\partial \tau^2} + A \frac{\partial T(P, \tau)}{\partial \tau} + B \frac{\partial T(P, \tau)}{\partial P} = f(P, \tau) \quad (5)$$

where, T is the temperature of heated bushing; τ is the heating time; P is the relative input power; A and B are the coefficients that take into account the thermophysical and electromagnetic characteristics of materials.

The experiment has shown that the right-hand side of the above-mentioned equation can be written as:

$$f(P, \tau) = a - b/P + c * (e - d * \tau)^2 \quad (6)$$

The solution of the initial differential equation can be shown as:

$$T(P, \tau) = a_{12} + b * \ln(P) + c * \tau + d * \tau^2 + g * \tau^3 \quad (7)$$

This equation was accepted as a basic model of the induction heating process of the component parts of “bushing” type.

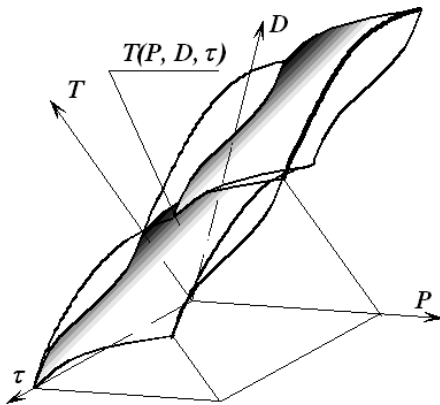


Figure 2. Model of induction heating process of bushings

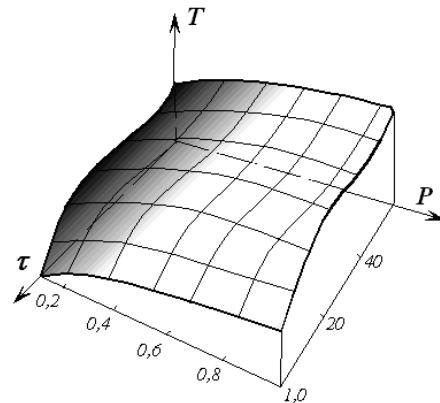


Figure 3. Two-dimensional section of hypersurface $T(P, D, \tau)$

For heating at a constant temperature and taking into account the inconsistencies of values of thermal conductivity: $\lambda = 9.1837 \cdot e^{-245.3552/T}$,
 heating capacity: $C_{y0} = 97.232338 \cdot (6.0946107 - e^{-0.0043747803 \cdot T})$,
 specific electrical resistivity: $\rho(t) = 111.6785 - 110.7229 \cdot e(-151.3911 \cdot t^{-1.4606})$,
 and the depth of current penetration into the material:

$$\xi(\mu, \omega, \rho) = \sqrt{\frac{2}{\mu \omega / \rho}},$$

the average value of useful power is shown as:

$$Pt = G \cdot C_{y0} \cdot T_{cp} / \tau \quad (8)$$

where, G is the mass of the blank part in kg; τ is the heating time taking into account the inertia; C_{y0} is the heating capacity; T_{cp} is the average temperature over the cross section.

In parallel with the IHP theoretical model, a statistical model was constructed (from experimental data).

Based on the average values obtained as a result of the experiment (10 repeated experiments were made at each control point), using the Table of 3D Curve, a package a statistical model was obtained.

Table 1. Temperature dependence of the heating temperature on the inductor and heating time

Power - 0.1P _{max}											
T°C	1	2	3	4	5	6	7	8	9	10	τ _{cp}
20	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
170	12.5	7.1	7.0	6.0	5.0	8.0	7.0	10.0	9.0	7.4	7.4
320	15.8	16.7	15.0	13.5	15.8	12.6	15.0	24.0	14.0	15.8	15.8
800	33.0	30.5	34.9	40.0	33.0	36.4	37.1	48.0	46.3	33.0	33.0
970	42.0	57.5	57.0	42.0	44.3	41.0	47.0	66.0	40.0	44.2	44.3
Power - 0.2P _{max}											
20	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
170	3.0	3.0	2.0	3.0	2.5	3.0	2.0	2.0	2.0	2.3	2.2
320	5.0	4.7	4.5	4.5	4.5	5.3	4.9	4.5	5.0	4.8	4.7
800	9.8	8.1	11.3	12.5	9.4	5.9	8.1	7.5	11.8	8.2	8.1
970	12.0	11.0	12.0	14.0	10.0	12.5	10.0	15.0	13.0	12.2	12.1
970	42.0	57.5	57.0	42.0	44.3	41.0	47.0	66.0	40.0	44.2	44.3
Power - 0.3P _{max}											

20	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
170	2.0	2.0	2.0	1.5	1.5	1.5	2.0	2.0	2.0	1.8	1.8
320	3.5	3.1	3.5	3.2	3.0	3.0	3.0	2.8	3.5	3.1	3.2
800	5.5	5.5	5.5	4.5	5.5	5.5	6.5	5.5	5.5	5.5	5.5
970	7.0	6.6	6.0	6.5	7.0	6.0	7.0	7.0	6.0	6.5	6.6

The equation below was chosen out of 130 other equations offered by the program. The criterion was the correspondence of the sections of the empirical surface with the approximation curves (see Fig. 4).

The equation of the modeling surface is represented as follows:

$$T = a + b/n \cdot P + c/nP^2 + d \ln t + e(\ln t)^2 + f(\ln t)^3, \quad (9)$$

where $a = 482.068434$; $b = -224.116991$; $c = 11.109037$; $d = 546.914566$; $e = -39.068289$; $f = -0.057704$.

The equation of modeling surface describes the process under study, which is confirmed by the following statistical characteristics: coefficient of determination $r^2 = 0.7850830161$; Adj $r^2 = 0.7290177159$; standard error FitStdErr = 190.145955; Fisher's criterion F-value = 17.534205109.

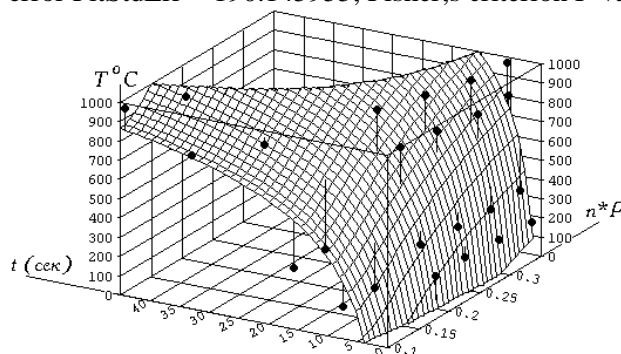


Figure 4. The experimental dependence of the heating temperature of bushing (T) on the heating time (t) and the power on the inductor ($n \cdot P$, $n = 0, \dots, 1$)

The experimental verification showed that the discrepancy predicted from the model and the experimental values of useful power does not exceed 10%.

4. Summary

The proposed method aimed at the development of complex multidimensional dependencies, based on the possibilities of synthetic multidimensional geometry, as well as on the generalized method of constructing hypersurfaces of extended Euclidean space as an abstract model of real factorial-parametric makes it possible to obtain models adequately describing the real processes. It has to be mentioned that with this approach it becomes possible to make approximate dependencies that completely satisfy theoretical concepts.

The IHP model of bushing heating made from sintered materials on the basis of iron helped to achieve significant energy savings during the technological process design.

The kinematic method of constructing hypersurfaces according to predefined conditions, greatly simplifies derivation of the form of approximating dependence for the entire range of technological parameters and system factors during the experiments. At the same time, the volume and difficulties related to calculations when processing the statistical data are sharply reduced due to the application of one-dimensional space objects. It is very convenient in terms of solving many issues related to the building of processes models of factor-parametric space.

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