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Mathematical modelling of powder material motion and transportation in high-temperature flow core during plasma coatings application

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Abstract. A problem of mathematical modelling of powder material motion and transportation in gas thermal flow core has been addressed. Undertaken studies indicate significant impact on dynamics of motion of sprayed particles of phenomenological law for drag coefficient and accounting momentum loss of a plasma jet upon acceleration of these particles and their diameter. It is determined that at great dispersion of spraying particles, they reach detail surface at different velocity and significant particles separation takes place at spraying spot. According to the results of mathematical modelling, requirements for admissible dispersion of diameters of particles used for spraying have been formulated. Research has also allowed reducing separation of particles at the spraying spot due to the selection of the method of powder feed to the anode channel of the plasma torch.

1. Introduction

It is known [1-7,9] that coatings produced using plasma gas thermal method allow significant increase in serviceability of various engineering product parts. Protective properties of plasma coatings and their quality are determined by physico-chemical characteristics of powder materials and part materials, characteristics of process tools, jigs and fixtures, kinematic configuration and spraying modes. According to a large body of research [1-14], deformational, thermal and topo-chemical phenomena upon interaction of powder particles and surface also influence these properties. Taking into account that kinetics of these phenomena depends on such physical parameters as velocity, temperature and heat content of spraying particles, it is natural that there are quite a large number of publications [1-14] dedicated to determination of estimation of these parameters. Undertaken analysis indicates that the main difficulty in modelling of these processes is a correct setting of the mathematical model itself and selection of boundary conditions in accordance with peculiarities of plasma gas thermal spraying. These features of modelling of plasma gas-thermal spraying influencing significantly analyzed parameters, but being reflected not fully, or too schematically, or incorrectly in cited publications, are the following.

Sprayed powder material with quite large dimension dispersion is introduced into high-temperature section of plasma jet core in the direction close to its axis normal. That results in a situation where particles of different diameters have different motion trajectories and different residence time in high-temperature section, and therefore their velocities and temperatures differ as well. Accordingly stream of particles separated in space depending on their dimensions, velocities and temperatures falls down to condensation surface. However, there are no data on quantitative estimation of this phenomenon and its impact on coatings quality in cited literature sources.

While accelerating solid particle, the plasma jet is losing part of its momentum. This results in necessity of taking into account impact of powder consumption on dynamic characteristics of gas flow. Necessity of taking into account this effect and estimation of its significant impact is given in [2-4]. However, in cited publications on theoretical models of particle motion, this effect is not taken into consideration. More than that, the approach used in [2-4] leads to incorrect physical result, analysis of which will be carried out hereinafter.

Equations of particle motion dynamics in a gas stream are based on phenomenological application [2-8] of accelerating force and a basic experimentally determinable parameter – particle drag coefficient C_x:

$$F_x = 0.5\rho_g C_x S_m \left(U_x - V_x\right)^2, \tag{1}$$

where ρ_g is flow gas density; U_x is *x*-dimension gas flow velocity; V_x is particle velocity along the direction of flow motion; $S_m = \pi D^2/4$ is particle frontal area; *D* is particle reference length.

It is deduced from experiments [2-3,8,12] that the drag coefficient of irregularly shaped but rounded particles without projecting points at the stage of steady flow slip is determined by Reynolds number $\mathrm{Re} = (U - V)D/\nu$, where *v* is flow gas kinematic viscosity. At small Re ($\mathrm{Re} \ll 1$), particle drug coefficient C_x coincides with theoretical value (Stokes formula). With *Re* rising up to the level of $\mathrm{Re} \approx 5 \cdot 10^3$, this coefficient decreases smoothly in accordance with slower law [12]. In terms of diverse variation, range *Re* is approximated by various power law dependences [2] of form $C_x = 24/\mathrm{Re} + C_0/(\mathrm{Re})^k$, where C_0, *k* are non-dimensional parameters of particle drug. In accordance with the results of experimental data processing, drag coefficient within the range of $\mathrm{Re} = 1...500$ with the inaccuracy of less than 2% can be equated as [2,8]: $C_x = 24/\mathrm{Re} + 4/(\mathrm{Re})^{1/3}$. Taking into account the fact that in this situation the variation range is $\mathrm{Re} = 1...30$, this binary relation could be traced to simpler monomial relation:

$$C_x = C_0/(\mathrm{Re})^k, \tag{2}$$

where parameters C_0 = 24.4 and *k* = 0.793 can be in error of experimental data deviation within the range of less than 4%. Nevertheless, in most cited publications, e.g. [2-4], upon derivation of analytic expressions for particle velocity either Stokes flow regime is used unreasonably, resulting in more than double increase of drug coefficient, or flow regime with $C_x = \mathrm{const}$, occurring in the event of $\mathrm{Re} \gg 1$, or monomial relations (2) with other parameter values are used without estimated accuracy of such relations use.

Consequently, undertaken analysis of studies in the area of plasma gas thermal jet spaying particles of motion dynamics has shown, on the one hand, importance of such studies for production of qualified coatings, and on the other hand, the range of problems either not studied yet or studied not to the fullest extent or not exactly correct despite availability of quite great amount of publications on this problem. This article contains only part of studies connected with particle motion dynamics in the area of plasma jet core with due regard to stated process peculiarities which predetermine novelty of set task of mathematical modelling.

2. Mathematical modelling task statement

Mathematical modelling task statement includes following. Accelerated subsonic axial-symmetric gas thermal plasma flow goes through plasma torch anode cylindrical channel to the environment (Figure 1). When making theoretical description of plasma torch plasma jets with decent accuracy for coating manufacturing technology following model representations are used.

Three typical sections are distinguished in plasma jet: initial, transition and main. Initial section reckoned from anode nozzle exit section (Figure 1 item BB_1), it includes jet core (Figure 1 item BEB_1), mixing area (Figure 1 items CBE and C_1B_1E) and steady flow area (Figure 1 items DBE and D_1B_1E).

It is deduced from experiments that high precision flow parameters have constant values in the plasma torch anode channel and in jet core in the steady plasma flow area, with the exception of the thin area of the boundary layer inside the plasma torch anode cylindrical channel.

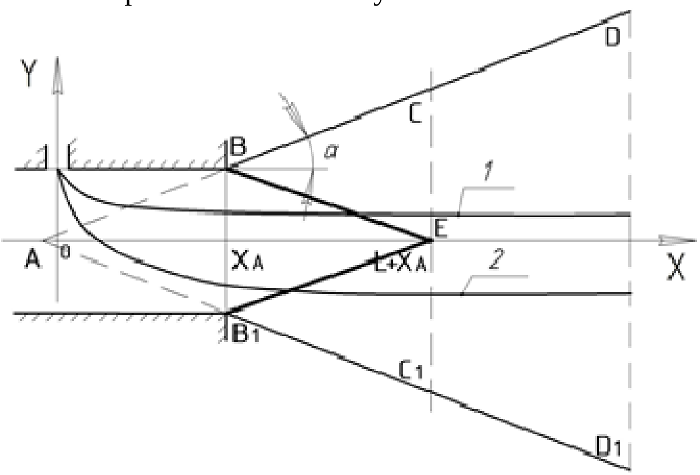

Figure 1. Schematic diagram of calculation of plasma jet parameters and motion trajectory (1,2) of spraying particles

Powder material particles of near-spherical shape are introduced into the plasma torch anode cylindrical channel along the *y* axis. Affected by gas flow, particles having initial velocity along the *y* axis accelerate and move along certain trajectories shown in Figure 1 by lines 1 and 2. Let us consider that selection of the result of the powder consumption mode in the situation when particles do not collide in the flow and, therefore, motion equations are individual for each particle; particle crushing does not occur, and material sublimation and erosion from their surface is insignificant. In addition, let us consider that accelerating force component in direction perpendicular to flow axis is remarkably lower than of component along axis; and the particle is affected only by accelerating force (1) along *x* axis (Figure 1), applied to its center of mass

3. Mathematical modelling of powder particles motion in the plasma jet core

Given the assumptions are made, an equation of motion of an individual particle introduced into the plasma flow in accordance with the diagram shown in Figure 1 can be written as:

$$m\left(\mathrm{d}V_x/\mathrm{d}t\right)=0.5\rho_g C_x S_m\left(U_x-V_x\right)^2;\, V_y=V_{y,0};\; V_x(0)=0\,, \tag{3}$$

where $m=\pi\rho D^3/6$ – mass of a particle of powder material; ρ – density of a particle of powder material; V_x, V_y – velocity components of the center of mass of a particle; $V_{y,0}$ – velocity component of a particle when leaving the transport channel and entering the anode channel of plasma torch.

Due to the fact that by accelerating a particle, gas flow loses part of its momentum along the *x*-axis according to the law of conservation of momentum, one gets:

$$\mathrm{d}\left(mV_x\right)=-\mathrm{d}\left(m_g U_x\right), \tag{4}$$

where m_g – mass of gas per particle. In (4) the losses of momentum for gas friction and internal surface friction on the anode channel of plasma torch are neglected, as well as gas pressure change in the "immersed" gas thermal jet.

When solving the system of equations (3-4) and finding velocities and trajectories of particle motion, two areas should be distinguished. The first area is the area where thermal jet parameters U_0, ρ_g, ν prior to the spraying particles introduction are constant (the area of the plasma torch anode channel and the jet core area), and the second area is the area where these parameters are the functions of x and y coordinates (mixing zone, transition zone and steady flow zone).

Let us consider motion of particles in the first area. Introducing new parameters $\tilde{V}_x = V_x / U_0$ and $\tilde{U}_x = U_x / U_0$, and using (2), equation (3) may be written as:

$$d\tilde{V}_x = A_t \left(\tilde{U}_x - \tilde{V}_x\right)^{2-k} dt\,. \qquad (5)$$

Given that $\mathrm{d}\tilde{V}_x / \mathrm{d}t = \tilde{V}_x \mathrm{d}\tilde{V}_x / \mathrm{d}x$, equation (5) can also be represented as follows:

$$\mathrm{d}\tilde{V}_x = A_x \left[\left(\tilde{U}_x - \tilde{V}_x\right)^{2-k} / \tilde{V}_x\right] \mathrm{d}x\,; \qquad (6)$$

$$B_t = 3 \cdot C_0 \cdot \rho_g \cdot \nu^k \left(U_0\right)^{1-k} / 4 \cdot \rho \cdot D^{1+k};\; A_x = A_t / U_0\,; \qquad (7)$$

Performing integration in (4) along the x-axis within the first area from $x = 0$ $\left(U(0) = U_0; V_x(0) = 0\right)$ to a certain value of x, and noting that $m/m_g = G/G_g$, one gets:

$$U_x = U_0 - V_x G/G_g\,, \qquad (8)$$

where G and G_g are mass flow rates of powder and plasma-supporting gas, respectively.

By substituting (8) into (5) and (6) and considering that U_0, A_t and A_x are constant in this area, let us obtain a separable equation:

$$A_t\,\mathrm{d}t = \mathrm{d}\tilde{V}_x \Big/ \left(1 - \alpha_G \tilde{V}_x\right)^{2-k};\; B_x\,\mathrm{d}x = \tilde{V}_x\,\mathrm{d}\tilde{V}_x \Big/ \left(1 - \alpha_G \tilde{V}_x\right)^{2-k};\; \tilde{V}_x(0) = 0\,, \qquad (9)$$

where $\alpha_G = 1 + G/G_g$ - discharge coefficient of powder material and plasma-supporting gas.

Performing integration in (11) and (3) when $0 < k < 1$, let us obtain an equation of the particle trajectory in the area of the plasma jet core in the parametric form:

$$\alpha_G A_t t = \frac{1}{1-k}\left[\frac{1}{\left(1 - \alpha_G \tilde{V}_x\right)^{1-k}} - 1\right]; \qquad (10)$$

$$\alpha_G^2 A_x x = \frac{1}{1-k}\left[\frac{1}{\left(1 - \alpha_G \tilde{V}_x\right)^{1-k}} - 1\right] - \frac{1}{k}\left[1 - \left(1 - \alpha_G \tilde{V}_X\right)^k\right]; \qquad (11)$$

$$y = 0{,}5 d_c - V_{y,0} t\,, \qquad (12)$$

where d_c – diameter of the plasma torch anode channel.

In some papers, for example [3,4], the law of particle motion (2) with k = 0 corresponding to the case of very large Re numbers is considered. In a number of cases, the law of motion with k=1 corresponding to very small Re numbers (Stokes law) is used. Taking into account these cases, one can get a relation of $\tilde{V}_x$ on x for these laws. Having integrated in (9), let us get:

$$\alpha_G^2 A_x x = \ln\left(1 - \alpha_G \tilde{V}_x\right) + \left(1 - \alpha_G \tilde{V}_x\right)^{-1} - 1,\; k = 0\,; \qquad (13)$$

$$\alpha_G^2 A_x x = \ln\left(1 - \alpha_G \tilde{V}_x\right)^{-1} - \alpha_G \tilde{V}_x,\; k = 1\,, \qquad (14)$$

where A_x is determined by ratio (9) for $k = 0$ and $k = 1$, respectively.

The difference between these ratios (13) and (14) and the ratios given in [3-4], as well as in a number of other papers, resides in the fact that it was obtained in these papers under condition $\tilde{V}_x \ll 1$. Using this condition in (13) and (14) gives the ratio for particle velocity, which was obtained in the above-mentioned works:

$$\left(V_x\right)^2 = 2\left(U_0\right)^2 A_x x\,. \qquad (15)$$

It should be noted that the use of the same condition $\tilde{V}_x \ll 1$ (in particular, $\alpha_G \tilde{V}_x \ll 1$) gives the same ratio (15) from the general equation (11). That is, the law of acceleration of a particle at the initial stage of its acceleration, depending on the *x* coordinate, has the same form (15), however the parameter A_x (7) in these laws is different and, depending on the specific value of *k*, leads to a significant diverse influence of the particle and jet parameters on the particle velocity. Thus, from (7) it is clear that for the case of $k = 0$ the particle velocity does not depend on the plasma viscosity ν, which is physically implausible, if only because, depending on plasma temperature, this parameter can vary tenfold or more, and also significantly affects dependence of particle velocity on its diameter and plasma flow velocity U_0.

In addition, as evident from (15), in this approximation ($\tilde{V}_x \ll 1$), the particle velocity does not depend on the α_G parameter, associated with the ratio of flow rates of powder G and plasma-supporting gas G_g. In fact, this result is quite obvious. If one assumes that the particle has acquired a very small velocity increment $\tilde{V}_x \ll 1$ in the jet, then the loss of momentum by plasma jet will also be small and may not be taken into account in this approximation. Therefore, the approach used in [2-4,6], when the actual use of condition $\tilde{V}_x \ll 1$, which eliminates accounting of the loss of plasma jet momentum in this approximation, gives a ratio similar to (15), and which is subsequently used to calculate the effect of the powder flow rate on the flow rate of plasma-supporting gas and the particle velocity, is physically incorrect.

The initial velocity $V_{y,0}$ of particles is acquired at trajectory channel when they are accelerated by transport nitrogen, the flow velocity of which is $U_{tr} \ll U_0$. The velocity $V_{y,0}$ can be calculated from the equation (14), since the acceleration of particles at transport channel is carried out a priori with $\mathrm{Re} \ll 1$ and with the application of Stokes' law ($C_0 = 24$; $k = 1$ in the ratio (2)). Re-denoting in (14) the coordinate of particles motion at trajectory channel through *z*, and relative velocity through $\tilde{V}_{y,0} = V_y(z)/U_{tr}$, let us obtain:

$$\alpha_G^2 A_z z = \ln\left(1 - \alpha_G \tilde{V}_{y,0}\right)^{-1} - \alpha_G \tilde{V}_{y,0};\ A_z = \left(3 \cdot C_0 \cdot \rho_{tr} \cdot \nu_{tr}\right) / \left(4 \cdot \rho \cdot D^2 \cdot U_{tr}\right);\ C_0 = 24. \quad (16)$$

By substituting parameters of particles and gas, as well as trajectory channel length $z = z_{tr}$, into (16), let us determine initial feed velocity of the particle into anode channel of plasma torch $V_{y,0} = V_{y,0}(z = z_{tr}) = U_{tr} \tilde{V}_{y,0}$. However, in terms of process, this velocity is set by changing U_{tr} due to the change in pressure and transport gas flow rate, so that the particles with a diameter equal to the particle diameter at the maximum of their diameter distribution pass through the center of the point with coordinates $y = 0$ and $x = x_A + L$ at the plasma jet core exit (Figure 1). The fact of such an exit of particles is often established by visual observation.

4. Results and discussion

In accordance with the mathematical model developed, the velocities and trajectories of particles of $ZrO_2 - 8\%Y_2O_3$ powder material with dispersion of 10 - 80 μm in the plasma jet core were calculated (Figure 2).

Depending on the parameters of gas in the plasma jet core and particles diameter, parameters such as trajectory, particle residence time in the core, and exit velocity at the core vary widely. Upon motion in the core of plasma jet, flow separation occurs, in this connection small particles appear at the periphery, and larger particles appear in the core at the core exit (Figure 2). At this stage, dispersion of trajectories is not critical, however the motion of each particle along its trajectory at great distances will lead to an increase in dispersion of trajectories and formation of zones on condensing surface that are structurally non-uniform. Significant differences in velocities of particles and their residence time in the plasma jet core should be noted. Thus, velocity at the core exit for the particles with a diameter of 80 μm is almost 4.8 times lower than for the particles with a diameter of 10 μm. Accordingly, 10-micron particles reside in the core 5.4 times less than 80-micron particles. This substantial change in particles parameters was obtained not only by taking into account the influence of their diameters on the nature of motion in the jet core, but also by taking into account the change in velocity $V_{y,0}$ at the powder input channel by transport gas. Taking into account this influence, makes the difference in velocities $V_{y,0}$ at the stage of introducing a particle into the nozzle of plasma torch almost 8 times for particles of different diameters.

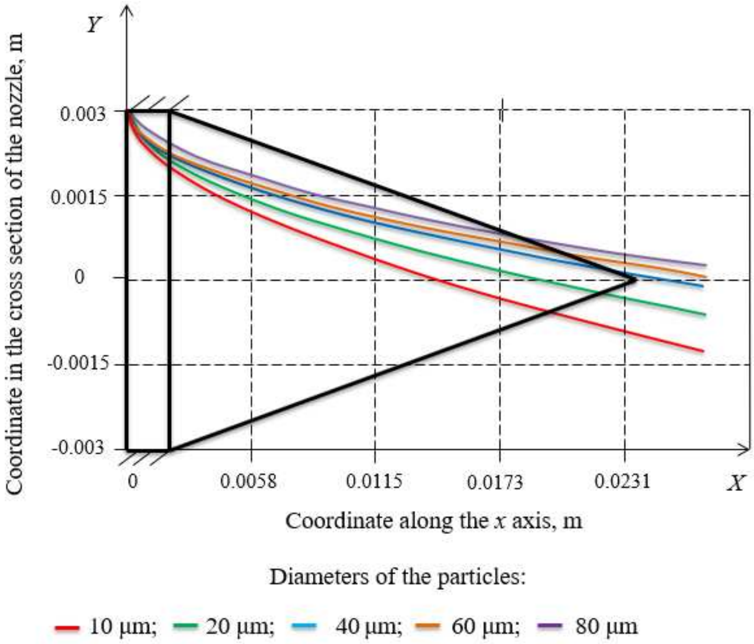

Figure 2. Trajectories of particles of $ZrO_2 – 8\%Y_2O_3$ powder material in the plasma jet core, depending on their diameter.

5. Conclusions

Undertaken studies showed significant impact of the phenomenological law for the drag coefficient and accounting of loss of plasma jet momentum upon acceleration of these particles and their diameter on motion dynamics of spraying particles. It is determined that at great dispersion of diameters of spraying particles they reach the detail surface at different velocities and significant particles separation at the spraying spot. According to the results of mathematical modelling, undertaken studies allowed laying down requirements for permissible dispersion of diameters of particles used for spraying, and reducing particles separation at the spraying spot due to selection of powder feed to the anode channel.

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