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Simulation of random road microprofile based on specified correlation function

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Abstract. The paper aims to develop a numerical simulation method and an algorithm for a random microprofile of special roads based on the specified correlation function. The paper used methods of correlation, spectrum and numerical analysis. It proves that the transfer function of the generating filter for known expressions of spectrum input and output filter characteristics can be calculated using a theorem on nonnegative and fractional rational factorization and integral transformation. The model of the random function equivalent of the real road surface microprofile enables us to assess springing system parameters and identify ranges of variations.

1. Introduction

When assessing springing and vibration isolation system efficiency for designed and modernized wheeled vehicles, one deals with the issue of random road microprofile simulation.

Using high-performance computers, one can create input impacts equivalent of the impacts of special testing roads by creating a bank of ordinates of road microprofiles by means of geodetic surveying of areas of a certain length.

However, when studying and developing springing and vibration isolation systems, in particular when introducing new models of suspension and tyre elements, it is necessary to use mathematical simulation of a random road microprofile equivalent of the test road microprofile.

A relevant software-based approach is an efficient tool for studying dynamic systems of the car moving in different modes [2, 5].

2. Simulation of a random function equivalent of a real road surface microprofile

Random road microprofile simulation involves determining an impulse transition function of a generating filter which would transform digital ‘white’ noise into a stationary process with pre-set characteristics (for example, with the following correlation function):

$$\rho(\Delta l) = A_1 e^{-\alpha_1|\Delta l|} + A_2 e^{-\alpha_2|\Delta l|} \cos \beta|\Delta l|, \qquad (1)$$

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where ρ (Δl) is the standardized correlation function; A_1, A_2 are coefficients of impact ($A_1 + A_2 = 1$); α_1, α_2 are coefficients of random process irregularity; β is the frequency of the periodic component; Δl is the interval of length correlation.

The analytical form of the correlation function is commonly used and representative for approximation of empirical values of correlation functions of road microprofiles [1].

The scheme of the generating filter is as follows:

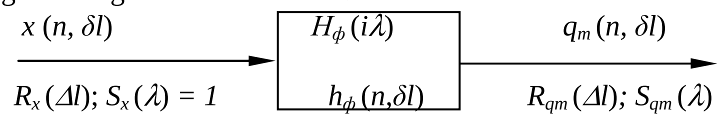


Figure 1. A scheme of the generating filter

where x(n, δl), R_x (Δl), S_x (λ) are 'white' noise, its correlation function and spectral density; $H_\phi(i\lambda)$, $h_\phi(n,\delta l)$ are transfer and impulse functions of the filter; q_m (n, δl), R_{qm} (Δl), S_{qm} (λ) are simulated signal, its correlation function and spectral density; λ is wavelength frequency of the microprofile; δl is the simulation step; n is the number of simulation points (n = 0…ND).

The simulation algorithm for target process q_m involves several stages: 'white' noise simulation, construction of the transfer function of the filter, construction of the impulse transfer function, generation of the target process and accuracy assessment for a simulation method.

'White' noise with single intensity (signal dispersion D_x =1) is simulated by formula [1]:

$$x\,(n,\delta l) = \eta + \frac{1}{20 \cdot 12}(\eta^3 - 3\eta), \qquad (2)$$

where $\eta = \sum (a_i - 0{,}5)$; a_i are independent random variables equally distributed over the range of 0…1.

The transfer function of the filter can be presented by the Fourier integral connecting spectral density and a correlation function of the target process (random microprofile of the road):

$$S_q(\lambda) = \frac{1}{\pi}\int_{-\infty}^{\infty} R_q(\Delta l) e^{-i\lambda\Delta l} d(\Delta l), \qquad (3)$$

where $R_q(\Delta l)=D_q p(\Delta l)$; D_q is the dispersion of road microprofile ordinates;

It can also be presented based on the Wiener-Khinchin theorem which connects spectral densities of input and output signals through the squared absolute value of the transfer function:

$$S_q(\lambda) = \left|H_\phi(i\lambda)\right|^2 S_x(\lambda) \qquad (4)$$

Using a factorization method for the spectral density, one can develop the following expression of the transfer function of the filter:

$$H_\phi(i\lambda) = k_0 \frac{[(\lambda + b_2) - i\alpha_2][(\lambda + b_2) + i\alpha_2]}{(\lambda + \alpha_1)[(\lambda + \alpha_2) - i\beta_2][(\lambda + \alpha_2) + i\beta_2]} \qquad (5)$$

where k_0, a_2, b_2 are real functions of parameters of the target microprofile (D_q, A_1, A_2, a_1, a_2, β).

The impulse transfer function of the filter can be presented by the Laplace integral:

$$h_\phi(l) = \frac{1}{\pi}\int_0^{\infty} H_\phi(i\lambda) e^{\lambda l} dl \qquad (6)$$

For the discrete argument and function $H_\phi(i\lambda)$ which is fraction rational with all different roots, one has:

$$h_{\phi}(n,\,\delta l)=\sum_{j=1}^{p}\frac{A_s(\lambda_j)}{B'_p(\lambda_j)}e^{\lambda_j n\delta l}, \tag{7}$$

where $A_s(\lambda_j)$, $B'_p(\lambda_j)$ are polynomial nominator and denominator expressions for function (5) for each pole λ_j of the transfer function; s is the number of roots; p is the number of poles.

If one inserts expressions for poles λ_j for all intermediate transformations, equation (7) takes the form:

$$h_{\phi}(n,\delta l)=k_0\left[\frac{A_1}{B_1}e^{-\alpha_1 n\delta l}+2e^{-\alpha_2 n\delta l}(\gamma_0\cos\beta n\delta l-\mu_0\sin\beta n\delta l)\right] \tag{8}$$

where A_1, B_1, γ_0, μ_0 are real functions of parameters of the target microprofile.

Ordinates of the target microprofile can be presented by the convolution integral:

$$q(n,\delta l)=\int_0^{\infty}h_{\phi}(n,\delta l)\cdot x(l-n\delta l)d(n\delta l)\,. \tag{9}$$

At small simulation step δl, it can be replaced by the sum:

$$q(n,\delta l)=\sqrt{\delta l}\sum_{k=0}^{n}h[k\delta l]\cdot x[n-k] \tag{10}$$

where n=0…ND; k is the current value of the simulation step, ND is the limited value of simulation points.

The correlation function of the simulated process is calculated by formula:

$$R_{qm}(n,\delta l)=\frac{1}{ND-n}\sum_{m=1}^{ND-n}q_m\cdot q_{m+n} \tag{11}$$

When assessing simulation (factorization) accuracy, the coefficient of multiple correlation between correlation functions of simulated and experimental microprofiles is used:

$$r^2=\frac{\sum_{i=1}^{k}X_iY_i\sum_{i=1}^{k}X_iY_i}{\sum_{i=1}^{k}X_i^2\sum_{i=1}^{k}Y_i^2}, \tag{12}$$

where $X_i\equiv R_{qmi}$ is the correlation function of the simulated microprofile,

$Y_i\equiv R_{qi}$ is the correlation function of the experimental microprofile.

Based on that algorithm (see Fig. 20), the authors developed software, and numerical simulation was applied for five special roads of the auto polygon of the Research Centre for Testing and Engineering of Motor Vehicles (Dmitrov) [3, 4].

Figure 3 shows simulation results for the cobble road.

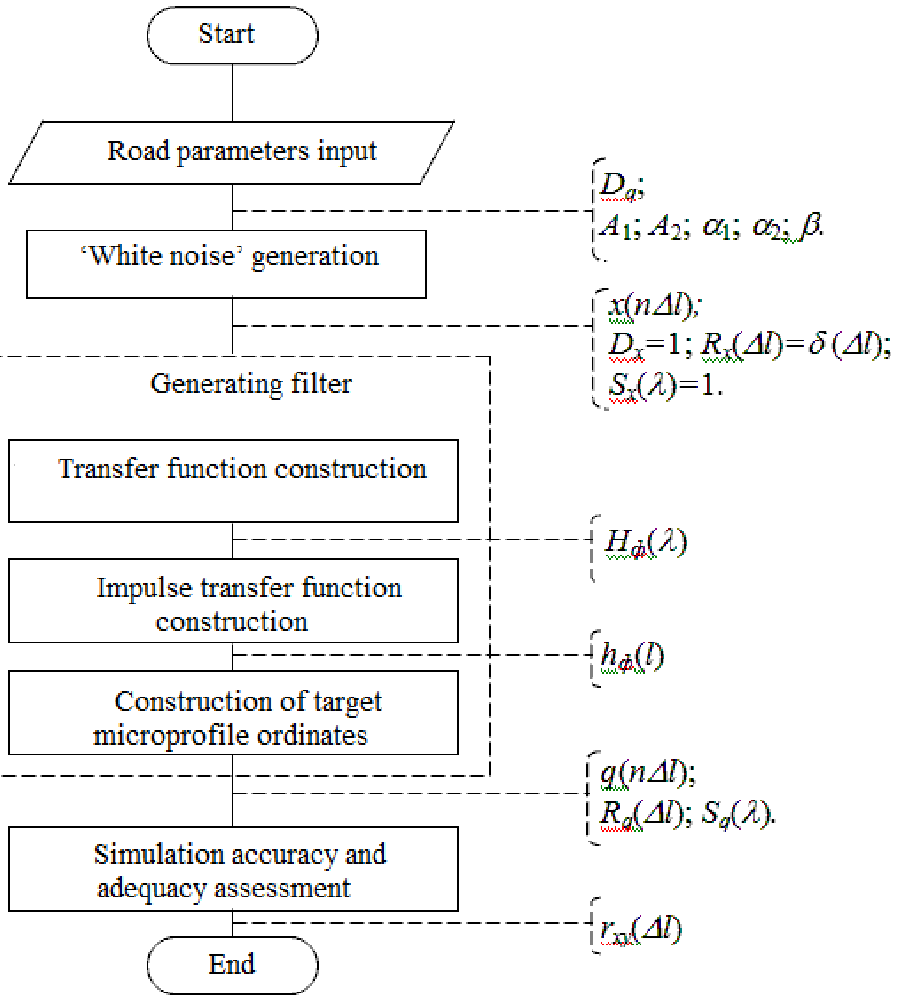
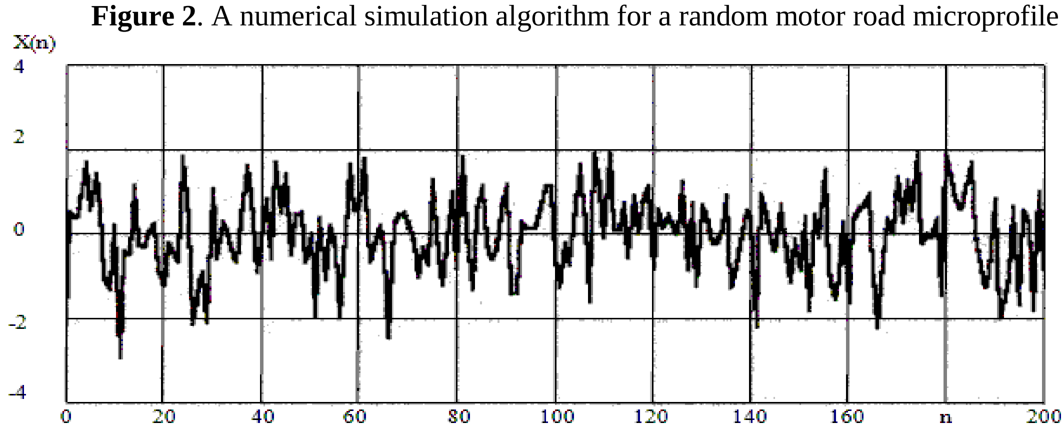
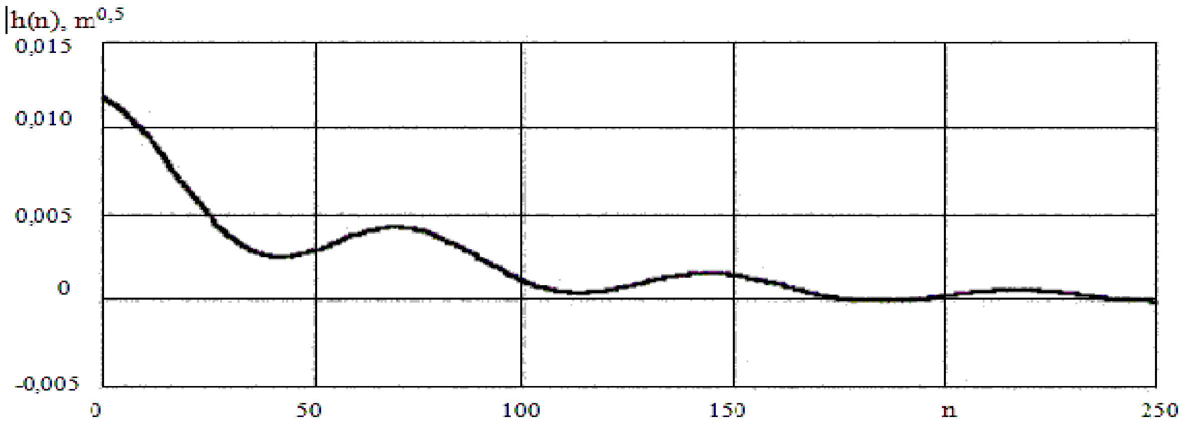


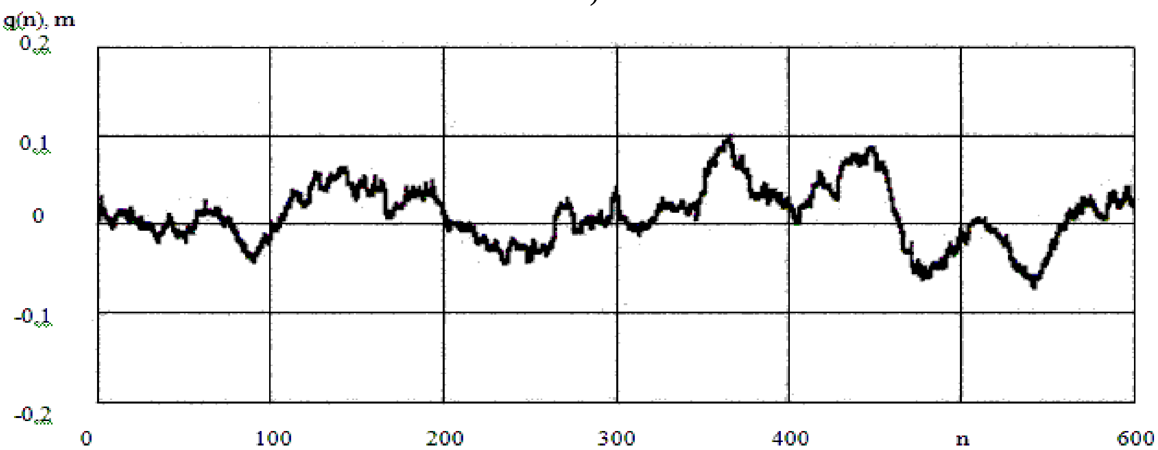
Figure 2. A numerical simulation algorithm for a random motor road microprofile



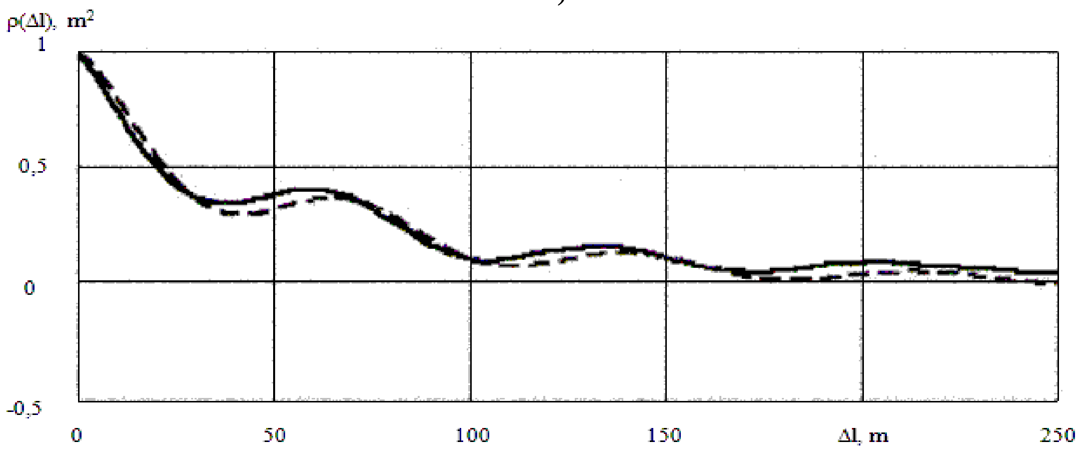
a)


|h(n), m^0,5
0,015
0,010
0,005
0
-0,005
0
50
100
150
n
250

b)


q(n), m
0,2
0,1
0
-0,1
-0,2
0
100
200
300
400
n
600

c)


ρ(Δl), m²
1
0,5
0
-0,5
0
50
100
150
Δl, m
250

d)

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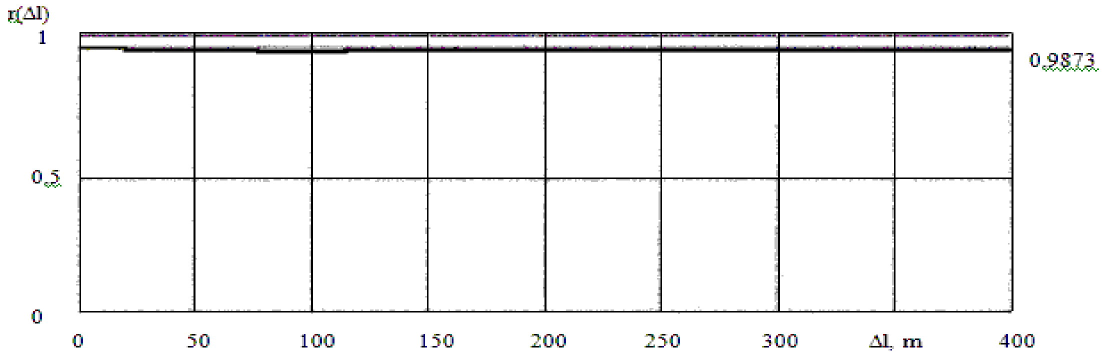

e)

Figure 3. Calculation results for a random microprofile of a test road of the auto polygon: a - random stationary function ‘white’ noise; b - impulse transfer function of the filter; b - simulated random stationary process; d - correlation functions: - - - of the simulated microprofile; --- of the experimental microprofile; e - dependence of the multiple correlation coefficient on the correlation interval.

3. Conclusion

Analysis of the diagrams and correlation functions enables us to conclude that:

1) factorization method (as opposed to the recurrent correlation method) allows us to simulate random processes equivalent of a random microprofile of the real road;

2) correlation functions of simulated and real processes agree in the initial section; coordinates of the first X-intersection of the functions coincide; amplitudes and periods of periodical components of functions and consequent points of X-intersection coincide;

3) values of multiple correlation coefficients vary over the range of $1 \leq r \leq 0{,}95$ for a significant range of the correlation interval which proves convergence of processes;

4) developed simulation algorithm, software and simulation results for five roads of the auto polygon of the Research Centre for Testing and Engineering of Motor Vehicles can be used for studying random impacts on car control systems and identifying all frequency and spectral systems response characteristics as well as for assessing degree of non-linearity of systems’ elements.

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