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Mathematical model of polyethylene pipe bending stress state

Anatoly Serebrennikov, Daniil Serebrennikov

Tyumen Industrial University, 38, Volodarskogo St., Tyumen, 625001, Russia

E-mail: serebrennikovaa@tyuiu.ru

Abstract. Introduction of new machines and new technologies of polyethylene pipeline installation is usually based on the polyethylene pipe flexibility. It is necessary that existing bending stresses do not lead to an irreversible polyethylene pipe deformation and to violation of its strength characteristics. Derivation of the mathematical model which allows calculating analytically the bending stress level of polyethylene pipes with consideration of nonlinear characteristics is presented below. All analytical calculations made with the mathematical model are experimentally proved and confirmed.

1. Introduction

Polyethylene pipes are getting more and more widespread in engineering structures lines.

They are not only lighter and cheaper than metal ones, but also are more flexible and resistant to physical impacts. Introduction of new machines and new technologies of polyethylene pipeline installation is usually based on the polyethylene pipe flexibility [1].

However, it is very important to ensure that existing bending stress does not lead to the irreversible polyethylene pipe deformation and to violation of its strength characteristics.

As a result, it can affect operating performance of the pipeline (including its reliability and operational life).

2. Current status review

One of the ways to reduce polyethylene pipelines installation costs is employment of special mechanisms operating on the basis of cable-laying machines or drainpipe-laying machines [2, 3].

The specifics of operation of these machines is that small diameter polyethylene pipes (less than 110 mm) are transported from a manufacturer to a pipeline installation site in spools.

A spool is installed on a pipe-laying tractor which allows laying a pipeline at a required depth without making and backfilling trenches (see figure 1). As a result, the pipeline construction prime cost is significantly reduced.

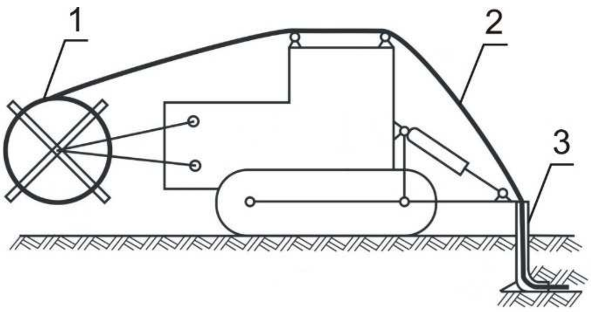

Figure 1. Scheme of the pipe-laying tractor
1 – spool,
2 – polyethylene pipeline,
3 – working tool

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The practical use of this technique requires scientific justification in order to ensure guaranteed retention of the polyethylene pipes' strength characteristics as they are exposed to several types of stress during laying.

It has been proved that the maximum level of stress occurs at the curvilinear surface when the pipe passes through the pipe-guide unit of the working tool [4]. Due to a relatively small bend radius in this area, the material of the pipe is shortly exposed to elastic and plastic deformations. It is necessary to ensure that these impacts do not lead to any irreversible deformations. So, the research of stress-strain state should be conducted in order to retain strength characteristics of the pipes.

3. Principal Conditions

Since the length of the pipe is much greater than its diameter, the fundamentals of the curved rods and thin-wall pipes theory can be used to describe its stress-strain state. In this case, the nonlinear characteristics of the material should be considered [5, 6].

Differential Area (dA) of the pipe cross-section (Figure 2) is described by the following formula:

$$dA = \delta \cdot r \cdot d\alpha;\ y = r \sin \alpha \tag{1}$$

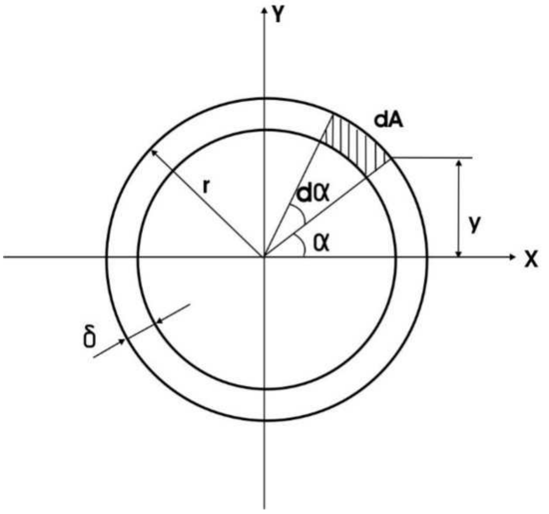

Figure 2. Polyethylene pipe cross-section

The value of internal forces (Figure 3) in the cross-section of a bended pipe should be calculated by integrating stress over area:

$$N_s = \int_A \sigma_s dA;\ M_s = \int_A \sigma_s y dA. \tag{2}$$

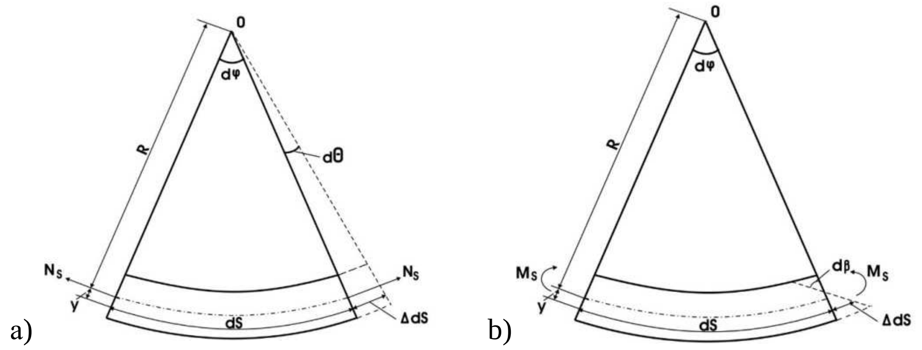

Figure 3. Deformation of the bended pipe section exposed to a) tensoin (N_s); b) bending moment (M_s).

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The tension deformation (N_s) is characterized by rotation of the section around the curvature center at the dφ angle:

$$\varepsilon_N = \frac{\Delta dS}{dS} = \frac{d\theta(R+y)}{d\varphi(R+y)} = \frac{d\theta}{d\varphi}. \tag{3}$$

The bending moment deformation (M_s) is characterized by rotation of the section around its own axis:

$$\varepsilon_M = \frac{\Delta dS}{dS} = \frac{d\beta}{d\varphi} \cdot \frac{y}{R+y}. \tag{4}$$

The total stress of the pipe section (Figure 4) is composed of the stress that occurs during bending along the axis of curved rod σ_S (Pa) and radial stress σ_T (Pa). The radial stress occurs if there is the internal gas pumping pressure. Since the pipeline installation process is considered, the radial stress will not be taken into account.

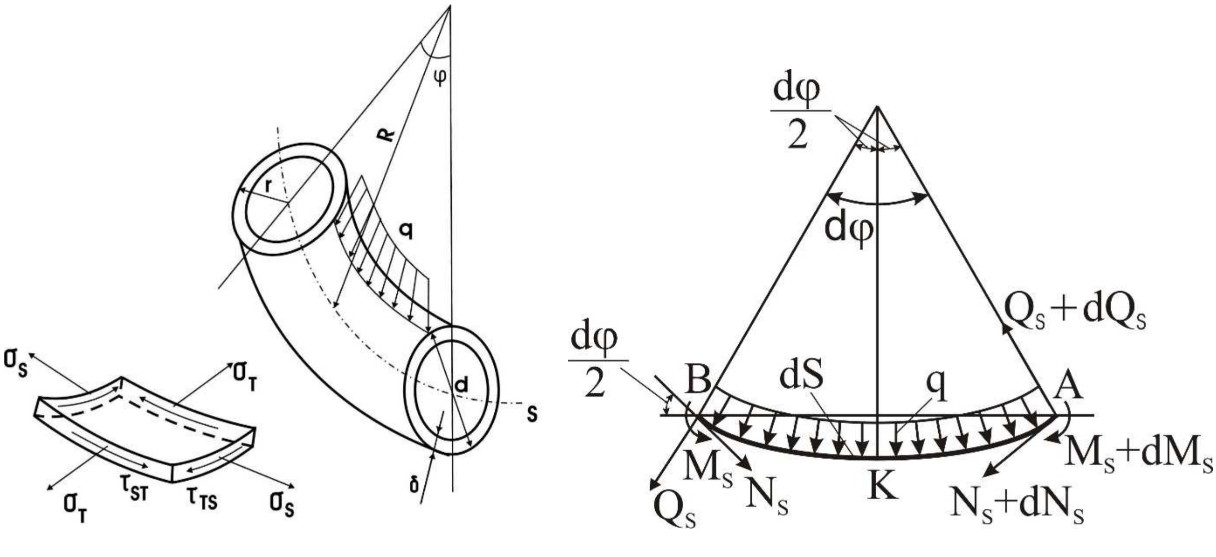

Figure 4. Stress and strains of the pipe section.

The total relative deformation of the bended pipe section from bending moment:

$$\varepsilon_s = \varepsilon_M + \varepsilon_N.$$

Stress caused by tension force and bending moment can be calculated as follows:

$$\sigma_s = E_c \varepsilon_s. \tag{5}$$

To take into account nonlinear characteristics of the material, the variable (secant) modulus (E_c) notion is used.

The variable (secant) modulus is changed depending on the material stiffness coefficient (b - Pa-2) and is calculated according to the following formula:

$$E_c = \frac{E_0}{1 + b\sigma_{in}^2}, \tag{6}$$

where E_0 is instantaneous elasticity modulus of the material (Pa) and σ_{in} is stress intensity (Pa).

4. Differential Equations of the Mathematical Model

Taking into consideration formulas (3) and (4), formula (5) can be written as:

$$\sigma_s = E_c \left[\frac{d\theta}{d\varphi} + \frac{d\beta}{d\varphi} \frac{y}{R + y} \right]. \tag{7}$$

Applying this formula to formula (2), one can get the following:

$$M_s = \int_A \sigma_s y dA = \int_A E_c \frac{d\theta}{d\varphi} y dA + \int_A E_c \frac{d\beta}{d\varphi} \frac{y}{R + y} y dA;$$

$$N_s = \int_A \sigma_s dA = \int_A E_c \frac{d\theta}{d\varphi} dA + \int_A E_c \frac{d\beta}{d\varphi} \frac{y}{R + y} dA. \tag{8}$$

Applying formula (1) to (8), one obtains the following formulas for internal force calculation:

$$N_s = \delta r \int_0^{2\pi} E_c \frac{d\theta}{d\varphi} d\alpha + \delta r^2 \int_0^{2\pi} E_c \frac{d\beta}{d\varphi} \frac{\sin\alpha d\alpha}{R + r\sin\alpha};$$

$$M_s = \delta r^2 \int_0^{2\pi} E_c \frac{d\theta}{d\varphi} \sin\alpha d\alpha + \delta r^3 \int_0^{2\pi} E_c \frac{d\beta}{d\varphi} \frac{\sin^2\alpha}{R + r\sin\alpha} d\alpha. \tag{9}$$

After introducing the notations, formulas (9) for internal force calculations are written as follows:

$$B_N = \delta r \int_0^{2\pi} E_c d\alpha; \; D_N = \delta r^2 \int_0^{2\pi} E_c \sin\alpha d\alpha;$$

$$B_M = \delta r^2 \int_0^{2\pi} E_c \frac{\sin\alpha}{R + r\sin\alpha} d\alpha; \; D_M = \delta r^3 \int_0^{2\pi} E_c \frac{\sin^2\alpha}{R + r\sin\alpha} d\alpha$$

$$N_s = B_N \frac{d\theta}{d\varphi} + B_M \frac{d\beta}{d\varphi}; \; M_s = D_N \frac{d\theta}{d\varphi} + D_M \frac{d\beta}{d\varphi}, \tag{10}$$

where B_N, B_M, D_N, D_M are characteristics of stiffness, stretching and bending in consideration of nonlinear characteristics of the material.

When considering (Figure 4) and comparing the equations of statics, one obtains the following:

$$\frac{1}{R}\frac{dN_s}{d\varphi}+\frac{1}{R^2}\frac{dM_s}{d\varphi}=0;$$
$$\frac{1}{R^2}\frac{d^2M_s}{d\varphi^2}-\frac{N_s}{R}=q. \tag{11}$$

When applying formulas (10) to the equations of statics, there is the following:

$$\frac{1}{R}\frac{d}{d\varphi}\left(B_N\frac{d\theta}{d\varphi}+B_M\frac{d\beta}{d\varphi}\right)+\frac{1}{R^2}\frac{d}{d\varphi}\left(D_N\frac{d\theta}{d\varphi}+D_M\frac{d\beta}{d\varphi}\right)=0;$$
$$\frac{1}{R^2}\frac{d^2}{d\varphi^2}\left(D_N\frac{d\theta}{d\varphi}+D_M\frac{d\beta}{d\varphi}\right)-\frac{1}{R}\left(B_N\frac{d\theta}{d\varphi}+B_M\frac{d\beta}{d\varphi}\right)=q.$$

After calculating the derivatives, the differential equation system becomes as follows:

$$\frac{1}{R}\left(\frac{dB_N}{d\varphi}\frac{d\theta}{d\varphi}+B_N\frac{d^2\theta}{d\varphi^2}+\frac{dB_M}{d\varphi}\frac{d\beta}{d\varphi}+B_M\frac{d^2\beta}{d\varphi^2}\right)+$$
$$+\frac{1}{R^2}\left(\frac{dD_N}{d\varphi}\frac{d\theta}{d\varphi}+D_N\frac{d^2\theta}{d\varphi^2}+\frac{dD_M}{d\varphi}\frac{d\beta}{d\varphi}+D_M\frac{d^2\beta}{d\varphi^2}\right)=0; \tag{12}$$

$$\frac{1}{R^2}\left(\frac{d^2D_N}{d\varphi^2}\frac{d\theta}{d\varphi}+2\frac{dD_N}{d\varphi}\frac{d^2\theta}{d\varphi^2}+D_N\frac{d^3\theta}{d\varphi^3}+\right.$$
$$\left.+\frac{d^2D_M}{d\varphi^2}\frac{d\beta}{d\varphi}+2\frac{dD_M}{d\varphi}\frac{d^2\beta}{d\varphi^2}+D_M\frac{d^3\beta}{d\varphi^3}\right)-$$
$$-\frac{1}{R}\left(B_N\frac{d\theta}{d\varphi}+B_M\frac{d\beta}{d\varphi}\right)=q. \tag{13}$$

5. Discussion and Results

Solution of the differential equations system (12, 13) should be attended by consideration of the limiting conditions in relation to variables (θ) and (β). The section of the pipe passing through the pipe-guide unit (Figure 1) of the working tool is described as a curved rod with two sliding joints on its edges. The derivatives of unknown functions (θ) and (β) are third derivatives and their solution requires implementation of six boundary conditions.

It is necessary to consider that if the stiffness along the length of the bended pipe is regular and the material is deformed in the elastic linear range, the secant modulus (E_c) takes on a constant value equal to the elasticity modulus (E_0). In this case, equation system (12, 13) is simplified to the following form:

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$$\frac{1}{R^2} D_M \frac{d^3\beta}{d\varphi^3} - \frac{1}{R} B_N \frac{d\theta}{d\varphi} = q;$$

$$\frac{1}{R} B_N \frac{d^2\theta}{d\varphi^2} + \frac{1}{R^2} D_M \frac{d^2\beta}{d\varphi^2} = 0.$$

In order to evaluate the changes of the stress and strain state of the pipe, the finite differences method has been used. This method is successfully applied if the differential equation of the task is known.

The method essence is to substitute the differential equation of the task with an algebraic equation system with the use of approximate expressions for the derivatives of the required function. The values of derivatives (θ) and (β) for all points of the computational scheme have been calculated and defined by the method [7].

Accurate solution of the equation system (12 and 13) can be done using Navier equation. In this case, to solve derivatives (θ), (β) and (q) should be written in Fourier series:

$$\beta = \sum \beta_{mn} \sin(2\varphi); \theta = \sum \theta_{mn} \sin(2\varphi); q = \sum q_{mn} \cos(2\varphi).$$

The only particular case of a possible solution using a precise method is the case when the pipe is hinged at both ends. In this case, the values of boundary conditions are equal to 0 (φ1 = φ2 = 0, δ1 = δ2 = 0, $\gamma_1 = \gamma_2 = 0$).

The legitimacy of analytical estimation, defining the values of stress depending on the bend radius, is proved experimentally for polyethylene pipes of different diameters.

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