IOP Conf. Series: Materials Science and Engineering **327** (2018) 022080 doi:10.1088/1757-899X/327/2/022080

Solving cross-disciplinary problems by mathematical modelling

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Abstract. The article deals with the creation of a human tibia 3D model by means of "Autodesk Revit-2016" PC based on tomogram data. The model was imported into "Lira-SAPR2013 R4" software system. To assess the possibility of education and the nature of bone fracture (and their visualization), the Finite Element Analysis (FEA) method was used. The geometric parameters of the BBK model corresponded to the physical parameters of the individual. The compact plate different thickness is modeled by rigidity properties of the finite elements in accordance with the parameters on the roentgenogram. The BBK model included parameters of the outer compact plate and the spongy substance having a more developed structure of the epiphysic region. In the "Lira-SAPR2013 R4" software system, mathematical modeling of the traumatic effect was carried out and the analysis of the stress-strain state of the finite element model of the tibia was made to assess fracture conditions.

1. Introduction

Modern computer and 3D modelling software [1] makes it possible to build up 3D objects of any complexity, to go into the process of biological tissue deterioration. Such studies can be done for biological materials with physical characteristics of a particular individual and in any specified conditions. All this allows one to objectively analyze an unlimited number of injury options within the framework set by a forensic investigator or a court of legal expertise [2-11].

In 2016, the Federal State Budgetary Educational Institution of Higher Education Samara State Technical University became a flagship university of Samara region and started to actively work in the field of cross-disciplinary problems [12-14]. One of these tasks is to build a 3D tibia model, as well as mathematical modeling and analysis of the stress and strain state of the finite element model of a tibia under traumatic effects to assess the conditions under which a tibia fracture might occur [2-4].

2. Materials and methods

To assess a possibility of a bone fracture and its nature (and its visualization), the method of Finite Element Analysis (FEA) can be used. This method is widely used to solve mechanics problem of deformable solids and in other areas of physics.

To build up a model of a tibia, the authors used tomography data (DICOM files) and tibia X-ray diffraction pattern. The Tibia 3D model was made by Autodesk Revit-2016 PC and is shown in Figure 1.

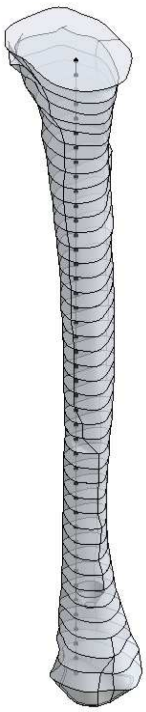
3. Mathematical modeling and analysis of the stress and strain state of the finite element model of a tibia under traumatic effects to assess the conditions under which a tibia fracture might occur

The Tibia model of a particular individual was formed according to the following characteristics, maintaining the proportions of the outer contour of the bone and the position of its spongy material: The length of the tibia – 42.48 cm, the weight of the shin – 4.0 kg, the thickness of a compact plate in the proximal epiphysis ranges from 0.28 to 0.34 cm, its thickness in the body area, at the border of the upper and the middle part – from 0.22 to 0.69 cm.

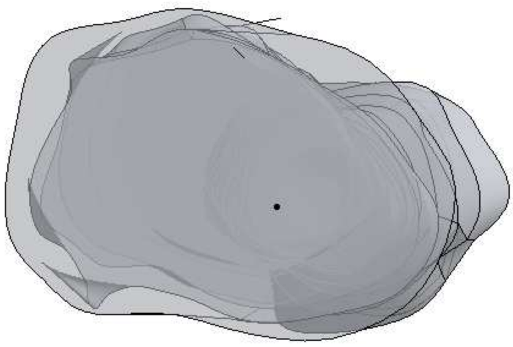
Mathematical modelling of traumatic effects and analysis of the stress and strain state of the finite element model of a tibia bone to assess conditions under which a tibia fracture might occur was performed by the "Lira-SAPR2013 R4" software system. The spatial finite element tibia model was built using triangular and quadrangular plate finite elements (of types 42 and 44). The model is shown in Figure 2.

Geometric parameters of the model matched physical parameters of a certain individual. The compact plate thickness varies vertically in different parts of the model. This difference was simulated by the finite elements stiffness property according to the parameters in the X-ray diffraction pattern. The tibia model included parameters of an external compact plate and a spongy material with a more advanced epiphyses structure. The elastic module of the compact plate was 18100 MPa, of the spongy matter – 90 MPa; Poisson coefficient – 0.27. According to the authors' references, the compact plate tension breaking strength ranges from 20 to 125 MPa. Its compression breaking strength ranges from 40 to 180 MPA. The values of the parameters are taken from statistical data available in special literature [15], with account of the predicted age reduction of the bone strength (25% of tibia strength of a grown-up healthy person aged 25-30): 90 MPa for tension, 140 MPA for compression.

The rigidity distribution of the finite element model in "Lira-SAPR 2013R4" PC is shown in Figure 3 according to the accepted types of rigidity given in Table 1.



a)



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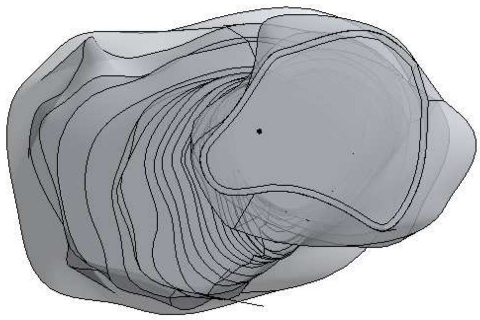

c)

Figure 1. Tibia model in "Autodesk Revit-2016" PC: (a) 3D view; (b) top view; (c) bottom view

When simulating traumatic effects in "Lira-SAPR 2013R4" PC, we analyzed a case when a person rested upon his left straightened leg, his body reflexed, forming an angle in the hip; with his left foot tightly fixed. When a person holds a shovel by its spade handle in both hands and makes a sweeping motion, his left leg and body are twisted counterclockwise.

In "Lira-SAPR 2013R4" PC the finite element model of the shin is fixed in the way as follows: the movement along X, Y, Z axes is restricted, rotation about Z axis is restricted, there is a joint about X and Y axes that simulate the ankle joint work. In order to simulate the support of one leg for a 84-kg person, a force along the tibia axis was exerted. To do that, the rest of the body was remodeled in the upper part of the left shin with core elements, with a total mass of 82 kg (the mass of a person (84 kg) – the mass of the shin (4 kg) + the mass of a shovel (2 kg) = 82 kg). The design of the upper part of the shin ensured that the shin could deviate from the vertical by 15 degrees posterior when the torso was bent forward in the hip joint. The left shin is rotated by two pairs of forces, with a value of 0.3 tN with the shoulder of 0.025 m at the top of the shin. The torque is equal to 30 kg/m.

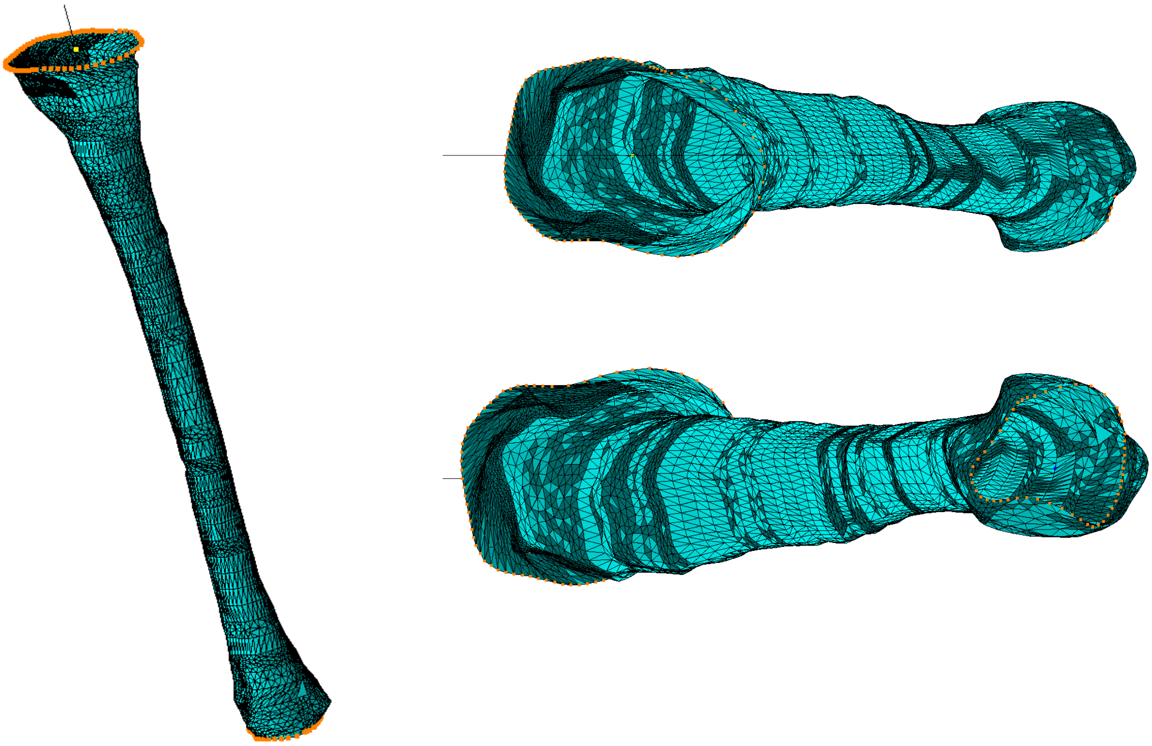

Figure 2. Tibia finite element model in "Lira-SAPR 2013R4" PC: (a) 3D view; (b) top view; (c) bottom view

Table 1. Tibia finite element model rigidity

Rigidity type	Name	Parameters (sections-(cm) rigidity – (t, m) weight – (t, m)
1	Plate H 1	E=9177,V=0.27,H=1,Ro=0.4
2	Plate H 0.15	E=1.38e+006,V=0.27,H=0.15,Ro=1.8
3	Plate H 0.2	E=1.845e+006,V=0.27,H=0.2,Ro=1.8
4	Plate H 0.3	E=1.845e+006,V=0.27,H=0.3,Ro=1.8
5	Plate H 0.4	E=1.845e+006,V=0.27,H=0.4,Ro=1.8
6	Plate H 0.5	E=1.845e+006,V=0.27,H=0.5,Ro=1.8
7	Plate H 0.6	E=1.845e+006,V=0.27,H=0.2,Ro=1.8

8	Plate H 0.7	E=1.845e+006,V=0.27,H=0.7,Ro=1.8
9	Plate H 0.8	E=1.845e+006,V=0.27,H=0.8,Ro=1.8
10	Plate H 0.35	E=1.845e+006,V=0.27,H=0.35,Ro=1.8

Tension distribution and deformation characteristics are given as isofields in Figure 7. Tension values are shown as a color scale. Thus, pressure tension is shown in blue and maximum ("critical") tensile stresses under which the material might be destroyed, shown in red.

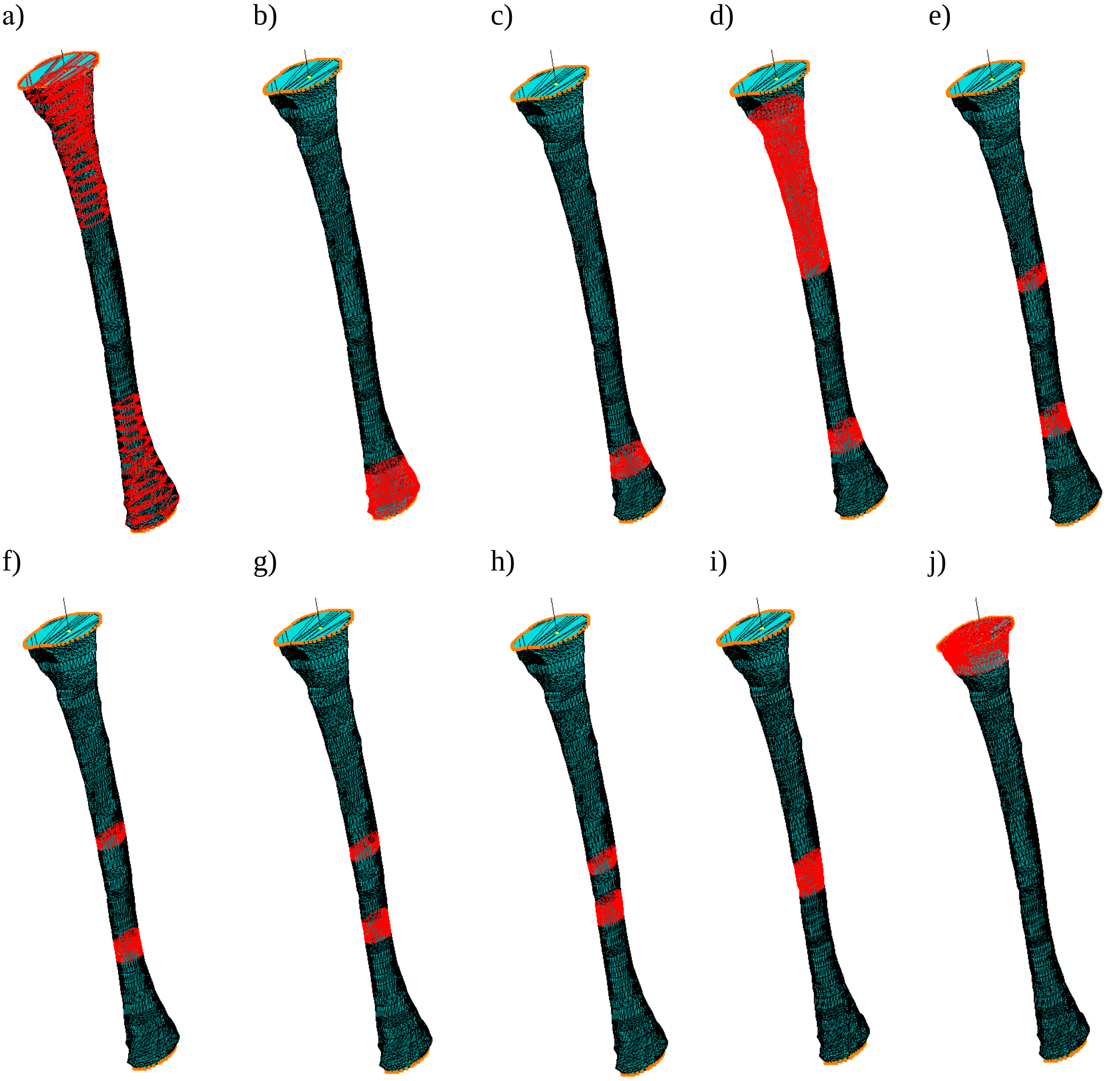

Figure 3. Tibia finite element model rigidity in "Lira-SAPR 2013R4" PC: (a) Rigidity Type – 1; (b) Rigidity Type – 2; (c) Rigidity Type – 3; (d) Rigidity Type – 4; (e) Rigidity Type – 5; (f) Rigidity Type – 6; (g) Rigidity Type – 7; (h) Rigidity Type – 8; (i) Rigidity Type – 9; (j) Rigidity Type – 10.

At different loading steps, the applied load causes maximum tensile stresses in the tibia bone at the edge of the upper and middle third of the shin on the rear-exterior surface.

In the upper third of the shin along its rear surface, there are concentrations of tensile stresses between which compression areas can be seen. It raises a possibility that a piece of a bone might break off.

At 100% of the full load, maximum tensile stresses are concentrated at the upper and middle-third of the shin. The tension value (90 MPa) reaches the tension breaking strength limit. A bone fracture is formed at this stage.

In order to compare the results of the finite element model tests and to assess the possibility of achieving the tensile strength limit of the tibia, the following physical calculations were performed during the rotating load:

Source data for calculations are: body height – 172 cm, body weight P = 82 kg. The weight of the shovel is 2 kg. The length of the shovel is 1.13 m. the rotation radius is R = 1 m.

According to the tomography, the thickness of the compact plate in the proximal head of the tibia is from 2.8 mm to 3.4 mm. The thickness of the compact plate in the tibia at the border of its top and middle thirds ranges from 2.2 mm to 6.9 mm. The shape of the tibia section at the upper and middle-third border is shown in Figure 4. The polar section modulus for a complex section shown in Figure 5 is found using the Autodesk AutoCAD software system.

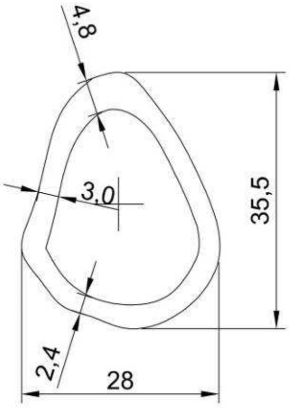

Figure 4. Shin section at the border of its upper and middle parts

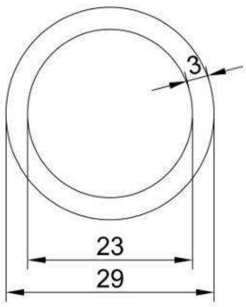

Figure 5. Reduced section of the calculation section

To confirm the calculation by the classic variant of calculating a rotary shaft [16], the section of the shin was made equal to a circular hollow shaft with a similar polar section modulus. The values of the polar section modulus relative to the rotation of the transformed section of a hollow shaft are calculated by the following formulas:

$$I_p = \frac{\pi \cdot R_2^4}{2}\left(1-\xi^4\right) = \frac{\pi \cdot d_2^4}{32}\left(1-\xi^4\right); \tag{1}$$

$$W_p = \frac{\pi \cdot R_2^3}{2}\left(1-\xi^4\right) = \frac{\pi \cdot d_2^3}{16}\left(1-\xi^4\right); \tag{2}$$

$$\xi = \frac{R_1}{R_2} = \frac{d_1}{d_2}; \tag{3}$$

$$R_1 = \xi \cdot R_2 \tag{4}$$

For the transformed section of a hollow shaft with an outside diameter of 29 mm and an internal diameter of 23 mm, let us get the moment of resistance as seen below:

$$W_p = \frac{3.14 \cdot 29^3}{16}\left(1-\left(\frac{23}{29}\right)^4\right) = 2892\,\text{mm}^2$$

Under torsion of the shaft, the highest normal stresses (main stretch and main compression) operate on the surface of the shaft at an angle of 45° to the generatrix (see Figure 6). They are equal in absolute value to the maximum shearing stress in the shaft cross sections: |σ1| = |σ2| = Tmax.

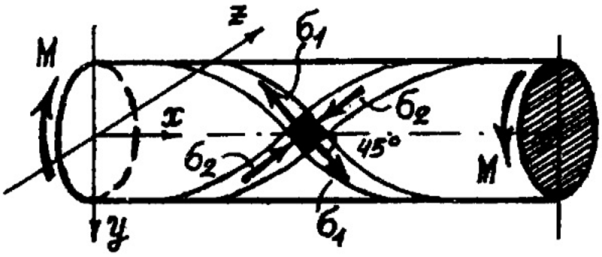

Figure 6. Main tension

The strength of the torsional shaft is calculated by the maximum shearing stress on the surface of the shaft.

The strength condition of a hollow shaft in torsion is as follows:

$$T_{max} = M_{rot} / W_p \leq [T], \quad (5)$$

where M_{rot} is the torque; W_p is the polar section modulus; [T] is the highest permissible shearing stress (torsional strength).

The strength of the bone at torsion for 25-35 years is 105.4 MPa. This rate is reduced to 90 MPa [15] with age.

Torque M_{rot} is caused by the rotation of the body while swinging hands with a shovel with m = 2 kg. The mass of a human hand is up to 5% of the whole mass of the body, i.e. (82 * 0.05) = 4.1 kg. Let us take a mass of 3 kg for the calculation. The rotation radius of a shovel is r = 1 m.

The sum of the forces on the body while making a sweeping motion of the arms with a shovel equals to the moment of inertia of the body at the angular acceleration:

$$M_{rot} = I \cdot \varepsilon. \quad (6)$$

Moment of inertia in relation to the axis of rotation is:

$$I = m \cdot r^2, \quad (7)$$

where m is the weight of the shovel (2 + 3 = 5 kg); r is a radius of rotation which is equal to 1 m.

In total:

$$I = m \cdot r^2 = 5 \cdot 1^2 = kg \cdot m^2$$

Angular acceleration ε is defined by the formula:

$$\varepsilon = \frac{\omega}{t}, \quad (8)$$

where ω is an angular velocity, t is a stop time or an interaction time.

Interaction time t is taken as 0.1 c. Angular velocity ω = 6.28 radians (i.e. 1 turnover per 1 sec).

Total angular acceleration equals:

$$\varepsilon = \frac{\omega}{t} = \frac{6.28}{0.1} = 62.8 \text{ rad/sec}$$

Total torque equals:

$$Mk_{rot} = I \cdot \varepsilon = 5 \cdot 62{,}8 = 314 \text{ H*m} \ \ (\text{or } 31 \text{ kg*m})$$

By substituting the torque value in the strength equation, there is:

$$T_{max} = M_{rot} / W_p = 314 \text{ H} \cdot \text{m} / 2{,}89 \cdot 10^6 \cdot \text{m}^3 = 108{,}6 \cdot 10^6 \text{ Pa} = 108{,}6 \text{ MPa}.$$

This value is greater than the limit of 90 MPa, i.e. the strength condition is not fulfilled and the bone is destroyed along the lines of the main stresses shown in Figure 6, (i.e. spiral lines of the fracture).

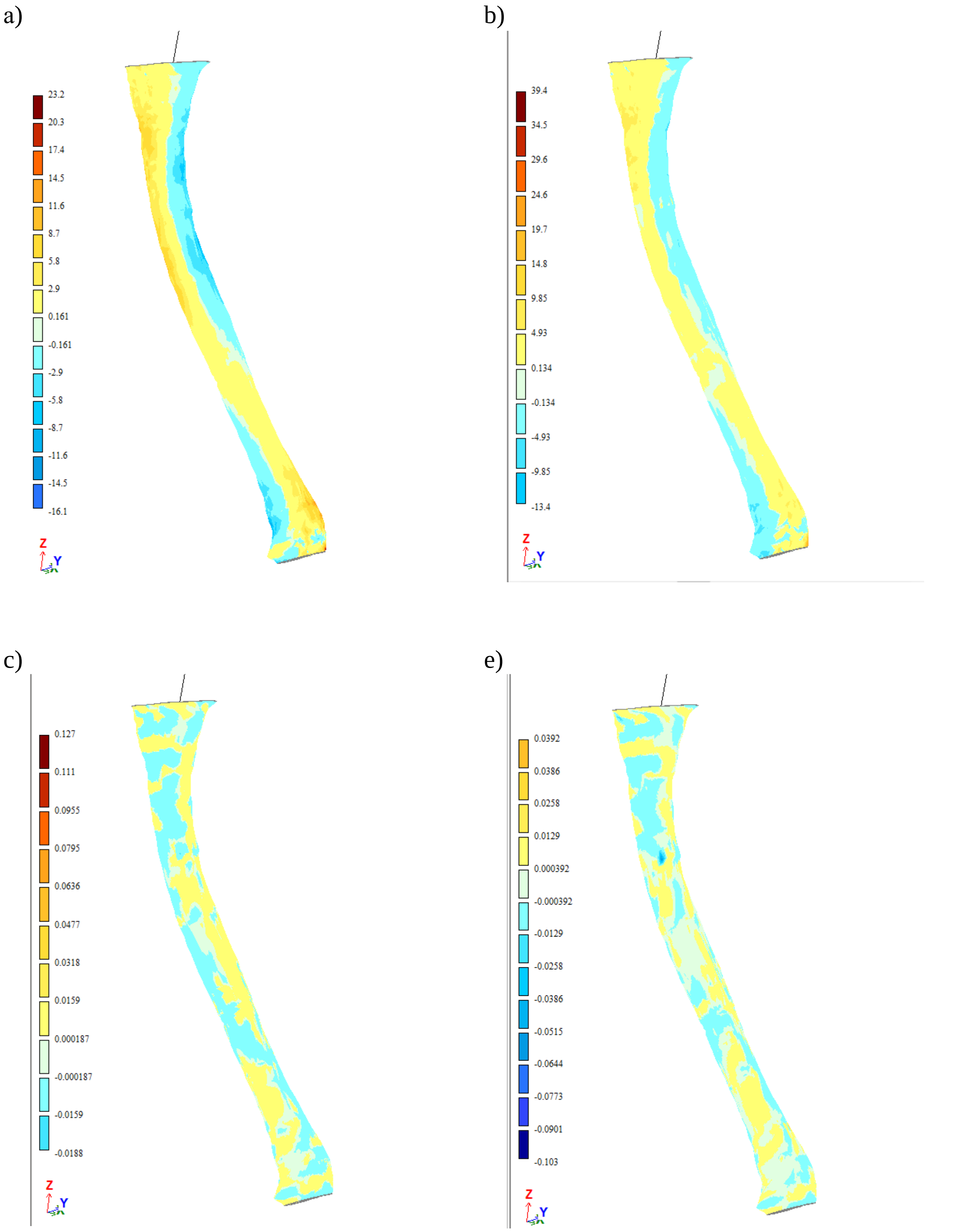

Figure 7. Tension isofields in the finite element model based on the results of the calculation in "Lyra-SAPR 2013R4" PC: (a) tension in N_x ; (b) tension in N_y; (c) tension in M_x; (d) tension in M_y.

4. Conclusion

The simulation of the FEM described in the paper made it possible to perform an analysis of the stress and strain state under traumatic effects and to study the mechanism of tibia fracture formation in the following conditions: the tibia was in a combined stress state, with applied load on the axis of the bone and its torsion clockwise (in the outer direction) in the top third part with a foot fixed.

The obtained results of experimental FEM modelling, applied in the field of construction mechanics, can be also applied in the practical work of forensic experts while they try to determine the place of the traumatic force as well as its nature in the conditions of bones complex stress when tibia is injured.

The tests results of the finite element model of the tibia are consistent with the laws of classical mechanics and resistance of materials, and are related to the research results obtained in forensic medicine.

References

[1] Selyaev V P 2017 *Information technology in architecture and construction: II all-Russia science and practical seminar "BIM technology ALLPLAN in architecture and civil engineering".* (Saransk: Mordovskiy university)

[2] Leonov S V, Pinchuk P V, Krupin K N and Panfilov D A 2017 *Forensic examination*. **2(60)** 11-13

[3] Leonov S V, Pinchuk P V, Krupin K N and Panfilov D A 2017 *Medical examination and law*. **1** 24-28

[4] Pinchuk P V, Krupin K N, Panfilov D A 2016 *Medical examination and law*. **6** 42-46

[5] Leonov S V, Vlasuk I V, Krupin K N 2013 *Forensic examinations.* **6(56)** 14-16

[6] Kislov M A, Leonov S V 2016 *System analysis and management in biomedical systems*. **3(15)** 521-525

[7] Leonov S V, Butuzova U P, Finkelstein V T 2013 *Medical examination and law*. **1** 29-32

[8] Kislov M A, Klevno V A 2016 *Medical examination and law*. **1** 31-34

[9] Gusarov A A, Makarov I U, Emelin V V, Fetisov V A 2017 *Medical examination and law*. **4** 13-18.

[10] Adamec J, Jelen K, Kubovy P, Lopot F, Schuller E. J Forensic Sci. 2010. doi:10.1111/j.1556-4029.2010.01445.x.

[11] Mo F, Arnoux PJ, Cesari D, Masson C. Saf Sci. 2014. doi:10.1016/j.ssci.2013.07.024.

[12] Panfilov D A, Goryncev V O 2017 Courses for the design of buildings and structures using the ALLPLAN 2016 PC *Information technology in architecture and civil engineering: II all-Russia science and practical seminar "BIM technology ALLPLAN in architecture and civil engineering"* (Saransk: Mordovskiy university)

[13] Cherkasov R A, Eremicheva O U, Makarova N V Interdisciplinary project teams of SamGTU and their collboration with special economic zones of the region *Food Industry and Catering: Current status and development prospects. 1st Russian scientific and practical conference with international participation collection of materials*

[14] Kostikova E V, Starygina G T, Kolesnikov S A 2017 Teaching engineering drawing at a construction university on the basis of interdisciplinary links *Traditions and innovations in architecture and civil engineering. Architecture and design. The collection of articles*. (Samara State Technical University. Samara)

[15] Knets I V, Pfaford G O , Saulgozis U Zh 1980 *Deformation and destruction of solid biological tissues* (Riga: Zinatne)

[16] Gorshkov A G, Troshin V N , Shalashilin V I 2008 *Material Resistance: textbook*