Mathematical model of process of loading of easy-deformable object on gas buffer layer

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Abstract. The paper deals with the process of loading an easily deformed object onto a gas buffer layer. Equations for finding the stiffness and damping coefficients are obtained. The described mathematical model allows designing the technological equipment for manufactures where gentle impact on object in the form of a disc is required.

1. Introduction

According to classical aerodynamics, at some distance h_1 from the hole, the gas velocity at the axis of the jet will be equal to the gas velocity at the outlet of the hole. This can be explained by the fact that at the so-called initial area of the jet, the magnitude of the pressure gradient on the axis of the jet is very small [1, 7]. However, when an obstacle is introduced into this area, the pressure distribution changes, and the statement that there is an initial plot is incorrect in this case [1]. This implies qualitative changes, while quantitative ones, with a certain probability, do not contradict the assumption that the gradient of the gas pressure in the flow on the part of the motion from h_1 to zero height is negligibly small in application to some practical problems.

2. Mathematical model

When the object approaches the gas-distributing grate of the pneumatic plant under the action of gravity, the trajectory of its motion can be conditionally divided into three sections (Figure 1) [4, 6].

In the first section, the supporting surface of the object is at a distance from the working surface of the gas-distributing grate by distance S greater than h_1 [5, 6]. The law of motion of its center of mass at boundary conditions $t=0$; t_1, V_1, $z=S$; h_1 is:

$$m\ddot{z} = -mg + W_1 , \qquad (1)$$

where m – object mass, kg;
g – acceleration of gravity, m/s^2;
W_1 – resistance force of the gas stream, N;
$\ddot{z}$ – acceleration of the product falling, m/s^2;

In the second section, the supporting surface of the object is moved from the surface of the gas-distributing grate for a distance between h_1 and h, where h is the distance at which film gas flow between the object bearing surface and the gas distribution grate occurs. In this case, the law of motion with boundary conditions $t=t_1$; t_2, $\dot{z}=V_1$; V_2, $z=h_1$; h has the form:

$$m\ddot{z} = -mg + W_2 , \qquad (2)$$

where W_2 – resistance force of the gas stream, N;
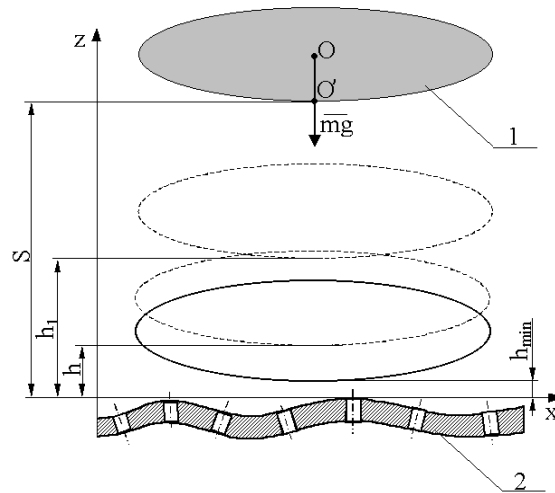

Figure 1. Loading on the gas buffer layer: 1 – object; 2 – gas-distributing grate

In the third section, the distance between the gas distribution grate and the bearing surface of the object varies from a value of h to a minimum thickness of gas buffer layer h_{min} at which the velocity of the easy-deformable object is 0.

The law of motion in the third section with boundary conditions $t= t_2$; t_3, $\dot{z} = V_2$; 0, $z=h$; h_{min} is:

$$m\ddot{z} = -mg + \int\limits_{A(x,y)} p(x, y)dA \tag{3}$$

where x, y – coordinates, m;
A – area of the bearing surface of the object, m^2;
p - total absolute pressure in the gas buffer layer corresponding to current radius r, Pa;

In the case of film gas flow between the gas distribution grate and the bearing surface of the object, the velocity distribution in the gas buffer layer has a parabolic character [2, 4].

Let us consider the aerodynamic processes occurring when buoyant jets collide with the bearing surface of an object. Let us use the basic and described earlier assumptions of the theory of gas lubrication.

In general, taking into account the assumptions, the equation describing the pressure field in the gas buffer layer can be written in the form:

$$\frac{d}{dx}\left[-\frac{z^3(x, y, t)}{6\mu}\frac{dp}{dx}\right] + \frac{d}{dy}\left[\frac{z^3(x, y, t)}{6\mu}\frac{dp}{dy}\right] = 0, \tag{4}$$

where t – time, sec;
μ - viscosity;
z – coordinates, m;

Let us consider the process of loading of a solid disc onto a gas buffer layer formed by the outflow of gas through a porous gas-distributing grate which dimensions are much larger than the dimensions of the disc.

In the first section, the falling of the object can be described as a movement in the gas stream:

$$U_{om} = \dot{z} + U , \tag{5}$$

where U_{om} – - velocity of the gas flow flowing out from the hole of the gas-distributing grate, on the bearing surface of the sphere, m/s;
U – the flow velocity of the medium in gas buffer layer, m/s;
$\dot{z}$ – falling velocity of the object, m/s;

During the movement, resistance force W_1 acts on the object:

$$W_1 = \frac{C_z}{2} \pi \rho R^2 U_{om}^2 ,$$

(6)

where c_z – coefficient of hydrodynamic resistance;

ρ - density of medium, kg/m^3;

R – radius of the object, m;

The expression for determining the maximum velocity on the axis of the jet is rewritten in the following form:

$$U = \frac{\xi U_0 \sqrt{A_p}}{z} ,$$

(7)

where U_o – velocity of the outflowing medium, m/s;

ζ – disturbance, m;

A_p – area of grate surface, m^2;

Rewriting (6) with considering (7):

$$W_1 = \frac{C_z}{2} \pi \rho R^2 \left(\dot{z} + \frac{\xi U_0 \sqrt{A_p}}{z} \right)^2 .$$

(8)

Taking into account (1.8), equation (1.1) has the form:

$$m\ddot{z} = -mg + \frac{C_z}{2} \pi \rho R^2 \left(\dot{z} + \frac{\xi U_0 \sqrt{A_p}}{z} \right)^2 .$$
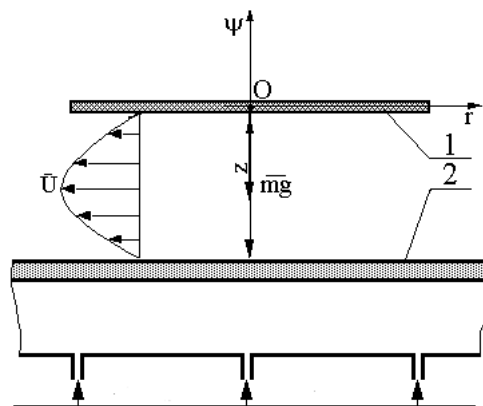
(9)

Figure 2. Object (disc) on gas buffer layer: 1 – disc; 2 – gas-distributing grate

When point O' reaches distance h, a film flow of the medium between the surface of the gas-distributing grate and the disc bearing surface forms (Figure 2). The gas velocity between the disc bearing surface and the surface of the gas-distributing grate is determined from the expression describing the Poiseuille flow in a slot with variable boundary $z = \psi(t)$:

$$U = \frac{1}{2\mu} \frac{dp}{dr} \left(\psi^2 - z\psi \right).$$

(10)

The expression for determining the flow rate of the medium in any section of the gas buffer layer can be written as:

$$Q = 2\pi r \int_0^z U d\psi = -\frac{\pi r^3}{6\mu} \frac{dp}{dr} ,$$

(11)

or

$$Q = \pi r^2 (U_0 + \dot{z}), \tag{12}$$

where ψ – coordinate axis with time-varying origin;

Equating the left sides of (11) and (12) and solving the differential equation taking into account the boundary conditions (for $\psi=0$ $p=p_0$, for $r=r$ $p=p_r$), one obtains an expression describing the field of medium pressure in the gas buffer layer:

$$p_0 - p_r = \frac{3\mu r^2 (U_0 + \dot{z})}{z^3}. \tag{13}$$

Integrating it over dA, one obtains an equation for determining the force exerted on the object from the side of the gas buffer layer:

$$\int_{A(r,\varphi)} p(r,\varphi) dA = \frac{3\pi\mu R^4}{2z^3}(U_0 + \dot{z}). \tag{14}$$

where φ – angular coordinate;

Taking into account (14), expression (2) has the following form:

$$m\ddot{z} = -mg + \frac{3\pi\mu R^4}{2z^3}(U_0 + \dot{z}). \tag{15}$$

Height h can be determined using the law of conservation of energy:

$$mgh - mgh_0 = \frac{c(h - h_0)^2}{2}. \tag{16}$$

From expression (16), one obtains:

$$h = h_0 + \frac{2mg}{c}. \tag{17}$$

The expression for determining the thickness of the gas buffer layer for stationary conditions:

$$h_0 = \sqrt[3]{\frac{Q_0\mu\left[\frac{3dR^4}{4a_w^2} + \frac{2R^3}{a_w}(2b - d)\right]}{mg}}. \tag{18}$$

After some mathematical transformations, let us obtain the following equations. Differential equation of oscillations:

$$m\ddot{\xi} + \frac{9\pi\mu R^4}{2h_0^3}\dot{\xi} + \frac{3Q_0\mu}{h_0^4}\left[\frac{3dR^4}{4a^2} + \frac{2R^3}{a_w}(2b - d)\right]\xi = 0. \tag{19}$$

Damping coefficient:

$$n = \frac{9\pi\mu R^4}{4h_0^3}. \tag{20}$$

Coefficient of rigidity of gas buffer layer is:

$$c = \frac{3Q_0\mu}{h_0^4}\left[\frac{3dR^4}{4a_w^2} + \frac{2R^3}{a_w}(2b - d)\right]. \tag{21}$$

Cyclic frequency of free undamped oscillations in the "object-gas buffer layer" system:

$$K' = \sqrt{\frac{c}{m}} = \frac{R}{h_0^2}\sqrt{3Q_0\mu\left[\frac{3dR^2}{4a_w^2} + \frac{2R}{a_w}(2b - d)\right]}. \tag{22}$$

3. Conclusion

The presented mathematical model can be used when designing the process equipment used in manufactures where gentle impact on objects in the form of a flat disk is required.

4. Acknowledgments

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