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# Shaping Cutter Original Profile for Fine-module Ratchet Teeth Cutting

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**Abstract**. The methods for determining geometric characteristics of a theoretical original profile of the cutter for cutting ratchet teeth with a module of 0.3–1.0 mm are considered in the article. Design models describing the shaping process of cutting edges of cutter teeth are developed. Systems of expressions for determining coordinates of the points of front and back edges of cutter teeth; the workpiece angles of rotation during the cutting process; the minimum cutter radius are received. The basic data when using the proposed technique are: radii of circumferences passing through cavities of cutter teeth and external cut teeth; the gradient angle and length of straight section of the front edge of a cut tooth; angles of rotation of the cutter and the workpiece at the moment of shaping.

## 1. Introduction

Freewheel mechanisms are the most loaded elements of different machine drives. They function with a significant switching frequency, which can reach 50 Hz, large transmitted torque and dynamic loads [1-8]. At present, a large number of constructive schemes of freewheel mechanisms are used in engineering. Load transmission in them can be carried out by friction forces as well as by normal forces.

Mechanisms using friction forces are characterized by insufficient load capacity and durability, since their performance depends on the value of the friction coefficient, which varies over a wide range, $f$ =0.04–0.20 [5, 8].

Mechanisms transferring the load by normal forces due to engagement of ratchet teeth are characterized by greater load capacity and reliability. However, when the ratchet engagement operates, considerable noise and shock loads occur [6, 8].

It is proposed in [9] to reduce these drawbacks by using a ratchet engagement of a special profile with a small value module, $m_t$ =0.3–1.0 mm. This kind of engagement can be used in eccentric freewheel mechanisms of non-friction type.

However, the existing design methods for cutters targeted at cutting involute teeth and teeth of other commonly used profiles [10-19] are not applicable to the cutters targeted at cutting fine-module ratchet teeth of the proposed profile due to the peculiarities of their geometry.

## 2. Profile designing of the front edge of the cutter tooth

Figure 1 shows the process of cutting teeth with a conditionally stopped cutter with the center at point $O_U$. In drawing up the design model, the following was set: $r_U$ and $r_{U1}$ are the radii of circumferences passing through dedendum and addendum of cutter teeth; $r$ and $r_1$ are the radii of the workpiece circumferences passing through the dedendum of the external and internal cut teeth ($r = r_{f1}$ and $r_1 = r_{f2}$); $\gamma_1$ is the gradient angle of the front edge of ratchet teeth; $l_t$ is the length of straight section of front edge of ratchet teeth.

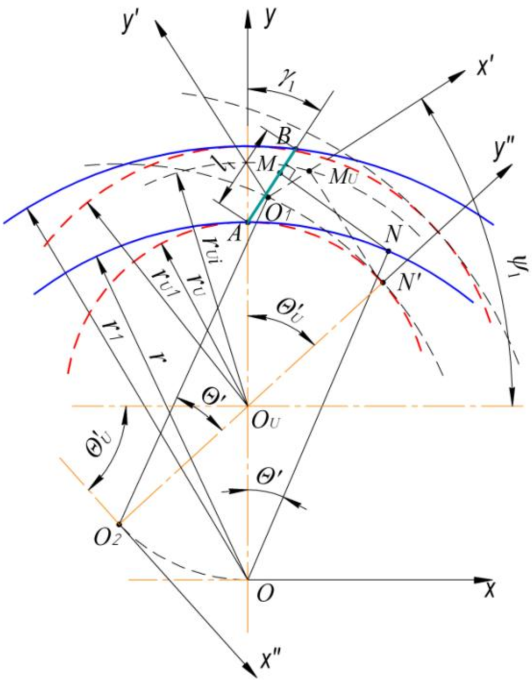


**Figure** 1. A design model for the profile shaping of the front edge of the cutter teeth.

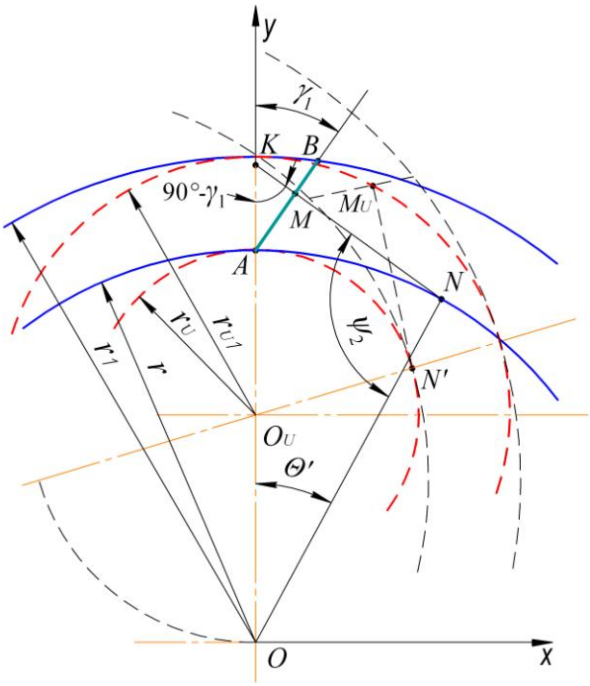


**Figure** 2. A design model for determining the workpiece angle of rotation and the cutter radius.

The workpiece circumference of radius $r$ is rolled along the cutter circumference of radius $r_U$. Shaping the edge points of the cut teeth, for example point $M$, occurs at the moment when the cross point of the perpendicular to the tooth profile and a circumference of radius $r$ (point $N$) becomes the pitch point of the machine mesh, i.e. take the position of point $N'$. This will happen after the workpiece rotates at angle $\Theta'$, while the cutter is rotating at angle $\Theta'_U = \Theta' r/r_U$. Then point $M$ will coincide with the conjugated point of the cutting edge of cutter $M_U$.

The problem of determining the profile of the front edge of cutter teeth comes down to determining the coordinates of points $M_U$ for all points $M$ of line segment $AB$, which will be solved in the following sequence.

The coordinates of point $O_1$ in system $xOy$ are defined by the system of expressions [20]:

$$\begin{cases} x_{01} = x_{02} + x''_{01}\cos(-\Theta'_U) - y''_{01}\sin(-\Theta'_U)\ ; \\ y_{01} = y_{02} + x''_{01}\sin(-\Theta'_U) + y''_{01}\cos(-\Theta'_U)\ . \end{cases} \tag{1}$$

Here $x_{02}$ and $y_{02}$ are the coordinates of point $O_2$ in system $xOy$; $x''_{01}$ and $y''_{01}$ are the coordinates of point $O_1$ in system $x''O_2y''$; $(-\Theta'_U)$ is the angle between the positive direction of axes $Ox$ and $O_2x''$.

Coordinates: $x''_{01} = -r\sin\Theta'_U$ ; $y''_{01} = r\cos\Theta'_U$ ; $x_{02} = -(r - r_U)\sin\Theta'_U$ ; $y_{02} = -(r - r_U)\cos\Theta'_U$ .

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After the substitution, the system of expressions (1) takes the form:

$$\begin{cases} x_{01} = -(r - r_U)\sin\Theta'_U + r\sin(\Theta'_U - \Theta')\ ; \\ y_{01} = -(r - r_U)\cos\Theta'_U + r\cos(\Theta'_U - \Theta')\ . \end{cases} \tag{2}$$

Coordinates of the point of the front edge of cutter tooth $M_U$ in system $xOy$ are:

$$\begin{cases} x_{R1} = x_{01} + x'\cos\psi_1 - y'\sin\psi_1\ ; \\ y_{R1} = y_{01} + x'\sin\psi_1 + y'\cos\psi_1\ . \end{cases} \tag{3}$$

Here $x'$ and $y'$ are the coordinates of point $M_U$ in system $x'O_1y'$; besides $y' = 0$; $\psi_1$ is the angle between the positive direction of axes $Ox$ and $O_1x'$, $\psi_1 = 90 - \gamma_1 + \Theta' - \Theta'_U$.

After the substitution of the system of expressions (2) into expressions (3), one obtains a system of expressions for determining the coordinates of the front edge of the cutter teeth:

$$\begin{cases} x_{R1} = -(r - r_U)\sin\Theta'_U + r\sin(\Theta'_U - \Theta') + x'\sin(\gamma_1 + \Theta'_U - \Theta')\ ; \\ y_{R1} = -(r - r_U)\cos\Theta'_U + r\cos(\Theta'_U - \Theta') + x'\cos(\gamma_1 + \Theta'_U - \Theta')\ . \end{cases} \tag{4}$$

To determine the coordinates of the profile points of the front edge of the cutter teeth, it is necessary to know the value of the workpiece angle of rotation $\Theta'$ at the moment of their shaping.

## 3. Determination of the workpiece angle of rotation

The value of the workpiece angle of rotation according to figure 2 can be determined by $\cos\Theta' = y_N/r$ or $\sin\Theta' = x_N/r$. This requires to define the coordinates of point $N$ in system $xOy$. Point $N$ lies at the intersection of line $KN$ and circumference of radius $r$, which are described by equations $y = -x\,\mathrm{tg}\gamma_1 + |OK|$ and $x^2 + y^2 = r^2$. Besides, $|OK| = r + l_t/\cos\gamma_1$.

Consequently, the equation system is valid for point $N$:

$$\begin{cases} y_N = -x_N\,\mathrm{tg}\gamma_1 + r + l_t/\cos\gamma_1\ ; \\ x_N^2 + y_N^2 = r^2\ . \end{cases} \tag{5}$$

After solving the equation system (5), one obtains:

$$\begin{cases} x_N = \cos\gamma_1\left[(r + l_t/\cos\gamma_1)\sin\gamma_1 \pm (r^2 - (r + l_t/\cos\gamma_1)^2\cos^2\gamma_1)^{1/2}\right]; \\ y_N = (r + l_t/\cos\gamma_1)\cos^2\gamma_1 \pm (r^2 - (r + l_t/\cos\gamma_1)^2\cos^2\gamma_1)^{1/2}. \end{cases} \tag{6}$$

Figure 2 shows that of two coordinate values of point $N$, the lesser are considered; i.e. in the system of expressions (6), let us put the "minus" sign before the root in the expression for determining $x_N$, and the "plus" sign – in the expression for determining $y_N$.

Finally:

$$\begin{cases} x_N = r\cos\gamma_1\sin\gamma_1 + l_t\sin\gamma_1 - \cos\gamma_1(r^2\sin^2\gamma_1 - 2rl_t\cos\gamma_1 - l_t^2)^{1/2}; \\ y_N = r\cos^2\gamma_1 + l_t\cos\gamma_1 + \sin\gamma_1(r^2\sin^2\gamma_1 - 2rl_t\cos\gamma_1 - l_t^2)^{1/2}\ . \end{cases} \tag{7}$$

## 4. Determination of the cutter radius

To manufacture a cutter, it is necessary to be able to determine its radius. The radius of circumference $r_{Ui}$, on which point $M_{Ui}$ is located (figure 1), is determined by equation

$$r_{Ui}^2 = x_i^2 + y_i^2 \tag{8}$$

One should bear in mind that extreme point $M_0$ of the straight section of cut tooth $l_t$ (figure 1) is shaped by the tip of cutter tooth, i.e. it lies on the circumference of radius $r_{U1} = r_U + H_t$.

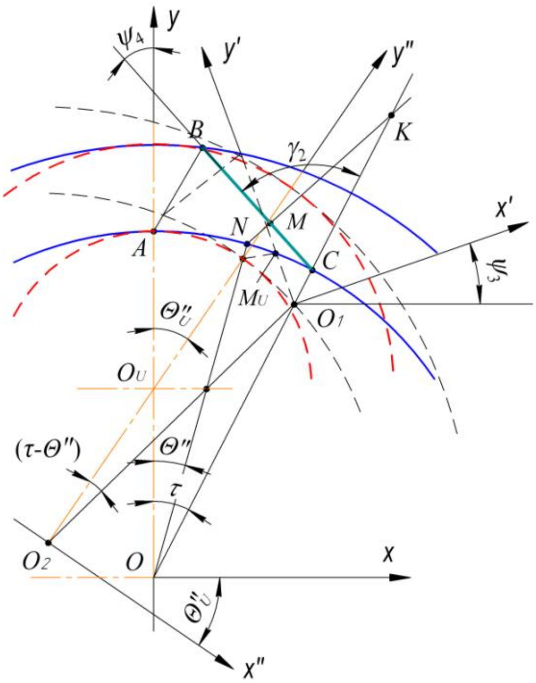


**Figure** 3. A design model for the profile shaping of the back edge of the cutter teeth.

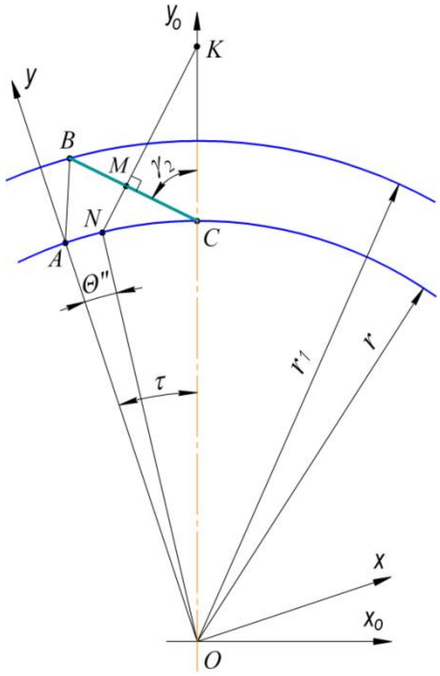


**Figure** 4. A design model for determining the workpiece angle of rotations.

From triangle $ONK$, let us determine $\psi_2 = \arcsin(\cos\gamma_1 + l_t/r)$.

After substitution of the system of expressions (4) into equation (8) and transformation with provision for $H_t = m_t$ and $x' = l_t$, let us determine the minimum radius of the cutter:

$$r_{U\min} = \frac{2r^2(1-\cos\Theta') - l_t^2}{2[m_t + r(1-\cos\Theta')] - l_t(\cos\gamma_1 + l_t/r)} \ . \tag{9}$$

## 5. Profile designing of the back edge of the cutter tooth

The problem of determining the theoretical profile of back edge of cutter teeth is identical to the previous one.

The coordinates of point $O_1$ in system $xOy$ (figure 3) are defined by the following system of expressions [20]:

$$\begin{cases} x_{01} = x_{02} + x''_{01}\cos(-\Theta''_U) - y''_{01}\sin(-\Theta''_U)\ ; \\ y_{01} = y_{02} + x''_{01}\sin(-\Theta''_U) + y''_{01}\cos(-\Theta''_U)\ . \end{cases} \tag{10}$$

where $x_{02}$ and $y_{02}$ are the coordinates of point $O_2$ in system $xOy$; $x''_{01}$ and $y''_{01}$ are the coordinates of point $O_1$ in system $x''O_2y''$; $(-\Theta''_U)$ is the angle between the positive direction of axes $Ox$ and $O_2x''$.

Coordinates $x''_{01} = r\sin(\tau - \Theta'')$; $y''_{01} = r\cos(\tau - \Theta'')$; $x_{02} = -(r - r_U)\sin\Theta''_U$; $y_{02} = -(r - r_U)\cos\Theta''_U$

After the substitution, the system of expressions (10) takes the form:

$$\begin{cases} x_{01} = r\sin(\tau + \Theta''_U - \Theta'') - (r - r_U)\sin\Theta''_U \ ; \\ y_{01} = r\cos(\tau + \Theta''_U - \Theta'') - (r - r_U)\cos\Theta''_U \ . \end{cases} \tag{11}$$

Coordinates of the point of the back edge of cutter teeth $M_U$ in system $xOy$ are:

$$\begin{cases} x_{R2} = x_{01} + x'\cos\psi_3 - y'\sin\psi_3 \ ; \\ y_{R2} = y_{01} + x'\sin\psi_3 + y'\cos\psi_3 \ . \end{cases} \tag{12}$$

where $x$ and $y$ are the coordinates of point $M_U$ in system $x'O_1y'$; besides $x' = 0$; $\psi_3$ is the angle between the positive direction of axes $Ox$ and $O_1x'$, $\psi_3 = \psi_4 + \Theta'' - \Theta''_U = \gamma_2 - \tau + \Theta'' - \Theta''_U$ .

After the substitution of the system of expressions (11), expressions (12) take the form:

$$\begin{cases} x_{R2} = r\sin(\tau + \Theta''_U - \Theta'') - (r - r_U)\sin\Theta''_U - y'\sin(\gamma_2 - \tau + \Theta'' - \Theta''_U) \ ; \\ y_{R2} = r\cos(\tau + \Theta''_U - \Theta'') - (r - r_U)\cos\Theta''_U + y'\cos(\gamma_2 - \tau + \Theta'' - \Theta''_U) \ . \end{cases} \tag{13}$$

To determine the coordinates of the profile points of the back edge of cutter teeth, it is necessary to know the value of the workpiece angle of rotation $\Theta''$ at the moment of their shaping.

## 6. Determination of the workpiece angle of rotation

The value of the workpiece angle of rotation according to figure 4 can be determined by $\cos\Theta'' = y_N/r$ or $\sin\Theta'' = x_N/r$. This requires to define the coordinates of point $N$ in system $xOy$. Point $N$ lies at the intersection of line $KN$ and circumference of radius $r$, which are described by $y = x\,\mathrm{tg}\gamma_2 + |OK|$ and $x^2 + y^2 = r^2$. Besides, $|OK| = r + y'/\cos\gamma_2$ .

Consequently, the equation system is valid for point $N$:

$$\begin{cases} y_{N0} = x_{N0}\,\mathrm{tg}\gamma_2 + r + y'/\cos\gamma_2 \ ; \\ x_{N0}^2 + y_{N0}^2 = r^2 \ . \end{cases} \tag{14}$$

After solving equation system (14), one obtains:

$$\begin{cases} x_{N0} = \cos\gamma_2[-(r + y'/\cos\gamma_2)\sin\gamma_2 \pm (r^2 - (r + y'/\cos\gamma_2)^2\cos^2\gamma_2)^{1/2}] \ ; \\ y_{N0} = \cos\gamma_2\,\mathrm{tg}\gamma_2[-(r + y'/\cos\gamma_2)\sin\gamma_2 \pm (r^2 - (r + y'/\cos\gamma_2)^2\cos^2\gamma_2)^{1/2}] + (r + y'/\cos\gamma_2) \ . \end{cases} \tag{15}$$

Figure 4 shows that of two coordinate values of point $N$, the lesser are considered; i.e. in the system of expressions (15), let us put the "plus" sign before the root.

Finally:

$$\begin{cases} x_{N0} = \cos\gamma_2[r^2\sin^2\gamma_2 - 2ry'\cos\gamma_2 - (y')^2]^{1/2} - r\cos\gamma_2\sin\gamma_2 - y'\sin\gamma_2 \ ; \\ y_{N0} = \sin\gamma_2[r^2\sin^2\gamma_2 - 2ry'\cos\gamma_2 - (y')^2]^{1/2} + r\cos^2\gamma_2 + y'\cos\gamma_2 \ . \end{cases} \tag{16}$$

The coordinates of point $N$ in system $xOy$ are:

$$\begin{cases} x_N = x_{N0}\cos\tau - y_{N0}\sin\tau \ ; \\ y_N = x_{N0}\sin\tau + y_{N0}\cos\tau \ . \end{cases} \tag{17}$$

After the substitution of the system of expressions (16) into the system of equations (17), one finally obtains:

$$\begin{cases} x_N = \cos(\gamma_2 - \tau)[r^2 \sin^2 \gamma_2 - 2ry' \cos\gamma_2 - (y')^2]^{1/2} - \sin(\gamma_2 - \tau)(r\cos\gamma_2 + y') \ ; \\ y_N = \sin(\gamma_2 - \tau)[r^2 \sin^2 \gamma_2 - 2ry' \cos\gamma_2 - (y')^2]^{1/2} + \text{ños}(\gamma_2 - \tau)(r\cos\gamma_2 + y') \ . \end{cases} \tag{18}$$

The obtained dependences (4), (9) and (13) make it possible to determine the geometric parameters necessary for cutter manufacturing.

## 7. Conclusion

The technique for determining the theoretical original profile of the cutter for cutting fine-module ratchet teeth is developed on the basis of the general methodology for shaping cylindrical surfaces using the indexing method. The obtained design models and systems of expressions allow calculating the parameters of cutting edges of cutter teeth, which provides the cutting of the ratchet teeth with the given geometric characteristics – a module value and a gradient angle, the length of the straight edge. Usage of a cutter as a cutting tool allows one to cut both external and internal ratchet teeth.

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