

Optimization of parameters of special asynchronous electric drives

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Abstract. The article considers the solution of the problem of parameters optimization of special asynchronous electric drives. The solution of the problem will allow one to project and create special asynchronous electric drives for various industries. The created types of electric drives will have optimum mass-dimensional and power parameters. It will allow one to realize and fulfill the set characteristics of management of technological processes with optimum level of expenses of electric energy, time of completing the process or other set parameters. The received decision allows one not only to solve a certain optimizing problem, but also to construct dependences between the optimized parameters of special asynchronous electric drives, for example, with the change of power, current in a winding of the stator or rotor, induction in a gap or steel of magnetic conductors and other parameters. On the constructed dependences, it is possible to choose necessary optimum values of parameters of special asynchronous electric drives and their components without carrying out repeated calculations.

1. Introduction

Optimization of parameters of any electric drive is quite a difficult and relevant task during creation and design of new devices and the equipment [1]. Creation of new approaches and adequate mathematical models allows one to solve certain optimizing problems only. Big complexity is finding dependence between the optimized parameters of [2] special asynchronous electric drives and components in case of change of power size, current in a winding of the stator or rotor, induction in a gap or steel of magnetic conductors and others [3, 4]. On the constructed dependences, it is possible to choose certain optimum values of required parameters during production control [5, 6]. Development of the theory in the field of optimization of parameters will allow one to create more technological and difficult equipment and mechanisms of various industries [7, 8]. Optimization of special asynchronous electric drives will allow one to create the equipment and mechanisms [9, 10] with the improved mass-dimensional characteristics and power indicators.

2. Solution of a task optimization of parameters of special asynchronous electric drives

For the solution of the task, let us use methods of geometrical programming, decomposition [11, 12] taking into account the results of the solution of private optimizing problems of special electric drives and components received earlier [13].

An average radius is determined by a formula:



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$$r_{av} = \frac{r_{ext} + r_{int}}{2} \quad (1)$$

where r_{ext} – external radius;

r_{int} – internal radius.

Let us enter a new parameter in the formula:

$$d = r_{ext} - r_{int} \quad (2)$$

For drawing up restrictions, let us consider further that the relation of external radius r_{ext} to its internal radius r_{int} $k = 3$, therefore:

$$d = r_{av} \quad (3)$$

Let us use expressions [13]:

$$a = \frac{2}{9} \pi r_{av} \quad (4)$$

$$l = \frac{2}{9} \pi r_{av} \quad (5)$$

where l – stator yoke thickness;

a – rotor thickness.

The condition of equality of the currents proceeding in short-circuited rings of the winding of the rotor [13]:

$$k_1 h_1 = \frac{r_{ext}}{r_{int}} k_2 h_2 \quad (6)$$

where k_1, h_1 – width and height of an external short-circuited ring of the winding of the rotor;

k_2, h_2 – width and height of an internal short-circuited ring of the winding of the rotor.

Let us receive the volume of steel of the stator and copper of the winding of the stator located in grooves taking into account (1), (2), (3) and (5):

$$V_1 = \pi \left(r_{ext}^2 - r_{int}^2 \right) (l + b) = 2\pi r_{av} dl + 2\pi r_{av} db = \frac{4}{9} \pi^2 r_{av}^3 + 2\pi r_{av}^2 b \quad (7)$$

where b – stator groove depth.

Let us write down the volume of front parts of the winding of the stator, considering that over each tooth, the number of conductors $y_{step}/2$ passes:

$$\begin{aligned}
 V_{\text{Front parts1}} &= V_{F \text{ ext1}} + V_{F \text{ int1}} = 2\pi r_{\text{ext}} \frac{y_{\text{step}} cb}{2} + 2\pi r_{\text{ext}} \frac{y_{\text{step}} cb}{2} = \\
 &= \pi y_{\text{step}} \frac{cb}{2} \frac{r_{\text{ext}} + r_{\text{int}}}{2} = \pi y_{\text{step}} \frac{cbr_{\text{av}}}{2}
 \end{aligned} \quad (8)$$

where $V_{F \text{ ext1}}, V_{F \text{ int1}}$ – external and internal volumes of front parts of the winding of the stator;

y_{step} – stator winding step;

c – groove width.

Let us write down the volume of steel of a rotor and copper of cores of a short-circuited winding of the rotor, using (1) – (4):

$$V_2 = \pi \left(r_{\text{ext}}^2 - r_{\text{int}}^2 \right) a = \frac{4}{9} \pi^2 r_{\text{av}}^3 \quad (9)$$

In one of the private optimizing tasks, the following ratios have been received:

$$k_{h1} = 0.075 \frac{z d_{cm}^2}{r_{av}} \quad \text{and} \quad d_{cm}^2 = \frac{4 \rho_c z i_2^2}{\pi^2 k_T \Delta T} \frac{r_{av}}{T} \quad (10)$$

where d_{cm} – diameter of a short-circuited core of a winding of a rotor;

ρ_c – specific resistance of a core;

z_2 – number of short-circuited cores of a winding of a rotor;

i_2 – the phase current proceeding on a short-circuited winding of a rotor;

k_T – heat-transfer coefficient;

ΔT – the difference of temperatures between the heated surface of the case and the environment.

Let us transform ratios (10) to the equation:

$$k_{h1} = 0.3 \frac{\rho_c z^2 i_2^2}{\pi^2 k_T \Delta T r_{av}^2} \quad (11)$$

The volume of short-circuited rings of a winding of a rotor is:

$$V_{\text{Front parts2}} = V_{F \text{ int2}} + V_{F \text{ ext2}} = 2\pi r_{\text{int2}} \frac{k_{h1}}{2} + 2\pi r_{\text{ext1}} \frac{k_{h1}}{2} \quad (12)$$

where $V_{F \text{ ext2}}, V_{F \text{ int2}}$ – volume of internal and external rings of a short-circuited winding of a rotor.

Let us substitute formula (2) in (12). It is necessary to substitute the received expression in (6). Taking into account (1), (11) and (3), let us transform the turned-out expression to:

$$V_{Front\ parts2} = 4\pi k \frac{h}{2} \frac{r}{2} \frac{av}{2} + 2\pi k \frac{h}{1} \frac{d}{1} = 4\pi \frac{k}{k} \frac{h}{av} r + 2\pi k \frac{h}{1} \frac{r}{av} = \frac{\rho}{\pi k} \frac{z^2 i^2}{T} \frac{2}{av} \quad (13)$$

Let us make the total volume of a component of the electric drive from (7), (8), (9) and (13):

$$\begin{aligned} \Sigma V = V_1 + V_{Front\ parts1} + V_2 + V_{Front\ parts2} &= \frac{8}{9} \pi^2 r^3 \frac{av}{av} + 2\pi r^2 \frac{b}{av} + \\ + \pi y_{step} \frac{cbr}{av} + \frac{\rho}{\pi k} \frac{z^2 i^2}{T} \frac{2}{av} &= c_1 r^3 + c_2 r^2 \frac{b}{av} + c_3 \frac{cbr}{av} + c_4 \frac{z^2 i^2}{r \frac{av}{av}}, \end{aligned} \quad (14)$$

where c_1, c_2, c_3 and c_4 – coefficients of the first, second, third and fourth pozinoms.

With the solution of private optimizing tasks, ratios between various parameters of special asynchronous electric drives and components have been received [13]. Using them, it is possible to receive restrictions for the general optimizing task.

These ratios have the following appearance:

$$\frac{m}{4\pi k} \frac{R}{T} \frac{i^2}{av} = \frac{1}{2} \quad \text{and} \quad \frac{z}{4\pi k} \frac{R}{T} \frac{i^2}{av} = \frac{1}{2} \quad (15)$$

$$P_2 = 82k \frac{z}{MX,A} \frac{i^2 R}{2} \frac{s}{2} \frac{1}{2} \frac{n}{s} \quad P_2 = \frac{1}{360} n k \frac{\pi^2 r}{av} \frac{s}{1} d^2 \left(\begin{array}{cc} B_g^2 & B_{mc}^2 \\ 2\delta \frac{g}{\mu} & - \frac{mc}{\mu} a \end{array} \right) \quad (16)$$

where m_1 – number of phases of a stator winding;

i_1 – the phase current proceeding on a stator winding;

R_1 – active resistance of one phase of a winding of the stator;

R_2 – active resistance of one phase of a winding of a rotor;

δ – size of an air gap;

μ_0 – magnetic permeability of air;

μ_a – magnetic permeability of steel;

B_g, B_{mc} – magnetic induction of a gap and a magnetic conductor;

n_1 – frequency of rotation of the field of the stator;

s – size of the sliding of a rotor;

$k_{MX,A}$ – the coefficient considering mechanical and additional losses;

$s_1 = 1 - s$ – the entered parameter;

P_2 – the power removed from a shaft.

Let us receive restriction from ratios (15):

$$\frac{m R i^2}{z R i^2} \leq 1 \quad \text{or} \quad c \frac{1}{z R i^2} \leq 1, \quad (17)$$

where c_5 – coefficient of the fifth pozinom.

Let us receive the second restriction from ratios (16), (3) and (4):

$$\frac{14760 \mu z i^2 R}{\pi^2 n s \delta B^2 r^3} + \frac{B^2 \mu \pi r}{B^2 \mu 9 \delta} \leq 1 \quad \text{or} \quad c \frac{z i^2 R}{r^3} + c \frac{r}{av} \leq 1 \quad (18)$$

where c_6, c_7 – coefficients of the sixth and seventh pozinom.

For the successful solution of an optimizing task, let us enter a ratio for depth b and width c of a groove on the stator:

$$bc = k_{11}, \quad (19)$$

where k_{11} – some set size.

Using (19), the third restriction will have an appearance:

$$k_{11} \frac{1}{bc} \leq 1 \quad \text{or} \quad c \frac{1}{bc} \leq 1, \quad (20)$$

where c_8 – coefficient of the eighth pozinom.

For the solution of a matrix of an exhibitor, let us group parameters in the form of the following variables: $r_{cp}, b/c, c, z_2^2 i_2^2, z_2/R_2$. Making an optimizing task, let us write down it taking into account the used variables.

Let us solve the following optimizing problem:

to find the minimum total volume of the special asynchronous electric drive and a component:

$$c \frac{r^3}{av} + c \frac{r^2}{av} \left(\frac{b}{c} \right) + c \frac{c^2}{av} \left(\frac{b}{c} \right) r + c \frac{\left(\frac{z_2^2 i_2^2}{2} \right)}{r av}$$

under restrictions:

$$c \frac{\begin{pmatrix} z_2 \\ R \\ 2 \end{pmatrix}}{\begin{pmatrix} z_2^2 i_2^2 \\ 2 \ 2 \end{pmatrix}} \leq 1, \quad c \frac{\begin{pmatrix} z_2^2 i_2^2 \\ 2 \ 2 \end{pmatrix}}{\begin{pmatrix} z_2 \\ R \\ 2 \end{pmatrix} r_{av}^3} + c \frac{r_{av}}{7} \leq 1, \quad c \frac{1}{8 \left(\frac{b}{c}\right)^c} \leq 1.$$

The condition of normalization of this task is presented by the equation:

$$\delta_1 + \delta_2 + \delta_3 + \delta_4 = 1; \tag{21}$$

the condition of orthogonality is defined from a matrix of an exhibitor:

$$\begin{pmatrix} r_{av} & \left(\frac{b}{c}\right)^c & \begin{pmatrix} z_2^2 i_2^2 \\ 2 \ 2 \end{pmatrix} & \begin{pmatrix} z_2 \\ R \\ 2 \end{pmatrix} \end{pmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{matrix} \begin{pmatrix} 3 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -3 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}. \tag{22}$$

Let us solve a matrix of exhibitor (22):

	R_{av}	b/c	c	$z_2^2 i_2^2$	z_2/R_2	№1	№2	№3	b_0	b_1	b_2
7	1					-3	-1	3	-1	-22	1
2		1				0	0	0	0	1	0
8			1			0	1	0	1	-1	-1/4
4				1		0	0	0	0	1	0
6					1	0	0	1	0	1	1/4
1	3					1	0	0	0	8	0
3	1		-1			0	1	0	1	-1	-1/4
5	-3				-1	0	0	1	0	1	1/4

where b_0 – normalization vector equal to the second to a column;

b_1 – the first vector of discrepancy is received from addition of the unit, the product by the first column vector by the seven and the product of a vector of normalization by minus;

b_2 – the second vector of discrepancy is received from the difference of the third vector column and a vector of the normalization increased by one fourth.

The common decision of conditions of normalization and orthogonality has an appearance:

$$\delta = b_0 + rb_1 + rb_2,$$

or

$$\left\{ \begin{array}{l} \delta_1 = 8r_1, \\ \delta_2 = r_1, \\ \delta_3 = 1 - r_1 - \frac{1}{4}r_2, \\ \delta_4 = r_1, \\ \delta_5 = r_1 + \frac{1}{4}r_2, \\ \delta_6 = r_1 + \frac{1}{4}r_2, \\ \delta_7 = -1 - 22r_1 + r_2, \\ \delta_8 = 1 - r_1 - \frac{1}{4}r_2. \end{array} \right. \quad (23)$$

where r_1, r_2 – any real numbers meeting a non-negativity condition.

On a condition of normalization (21), the sum of components, connected with a direct function, has to be equal to a unit. Using (23), let us rewrite a normalization condition:

$$8r_1 + r_1 + 1 - r_1 - \frac{1}{4}r_2 + r_1 = 1 \quad \text{or} \quad r_2 = 36r_1. \quad (24)$$

The common decision of conditions of normalization and orthogonality has to meet a condition:

$$\delta = b_0 + rb_1 + rb_2 \geq 0. \quad (25)$$

Considering condition (25), let us define intervals of change of the r_1 and r_2 variables and choose from them the corresponding values of variables. Using (24), it is necessary to investigate variables δ_3 and δ_7 , entering conditions of normalization and orthogonality:

$$\left\{ \begin{array}{l} \delta_3 = 1 - r_1 - \frac{1}{4}r_2 = 10 \left(\frac{1}{10} - r_1 \right), \\ \delta_7 = -1 - 22r_1 + r_2 = 14 \left(r_1 - \frac{1}{14} \right). \end{array} \right. \quad (26)$$

Solving the system of the equations (26), there will be the following:

$$\frac{1}{14} \leq r_1 \leq \frac{1}{10} \quad \text{and} \quad \frac{36}{14} \leq r_2 \leq \frac{36}{10}. \quad (27)$$

Let us pick up size r_1 so that conditions (27) were satisfied and:

$$c = \frac{\pi}{z_1} r_{av} \quad (28)$$

where z_1 – number of grooves on the stator.

Using the fourth approval of the first theorem of duality, let us write down the following equation:

$$\frac{2r_{av}}{y_{step}^c} = \frac{r_1 V(\delta)}{\left(1 - r_1 - \frac{1}{4} r_1\right) V(\delta)} \quad \text{or} \quad c = \frac{2}{y_{step}} \frac{\left(1 - r_1 - \frac{1}{4} r_1\right)}{r_1} r_{av} \quad (29)$$

Let us divide (28) on (29) and solve the received expression relatively r_1 , replacing r_2 with r_1 in formula (24):

$$r_1 = 0.096 \cdot \quad (30)$$

Let us substitute (30) in (24) and the following will result:

$$r_2 = 3.456 \cdot \quad (31)$$

The final condition of normalization and orthogonality, taking into account (30) and (31), has an appearance:

$$\left\{ \begin{array}{l} \delta_1 = 0.770, \\ \delta_2 = 0.096, \\ \delta_3 = 0.040, \\ \delta_4 = 0.096, \\ \delta_5 = 0.960, \\ \delta_6 = 0.960, \\ \delta_7 = 0.344, \\ \delta_8 = 0.040. \end{array} \right. \quad (32)$$

Using (32), let us execute the fourth approval of the first theorem of duality for the first, second and fifth pozinoms:

$$\frac{4\pi r_{av}}{9b} = \frac{8r_1 V(\delta)}{r_1 V(\delta)} \quad \text{and} \quad \frac{m R i^2}{z_2 R i^2} = \delta_5$$

or

$$b = 0.17r_{av} \quad \text{and} \quad m_{11} R_{11} i_{11}^2 = 0.96z_{22} R_{22} i_{22}^2, \quad (33)$$

where $V(\delta)$ – dual function of an optimizing task.

Let us define from the equation, received from the fourth approval of the first theorem of duality, the current proceeding in a core of a short-circuited winding of a rotor:

$$i_{2cm}^2 = \frac{\pi^3}{9} \frac{r^4 k \Delta T}{\rho_c z_{22}^2} \quad (34)$$

Taking into account (34), equations (10) take a form:

$$k_{11} h_{11} = 0.033\pi r_{av}^2 \quad \text{and} \quad d_{cm}^2 = \frac{4}{9} \pi \frac{r_{av}^3}{z_{22}} \quad (35)$$

The expression for resistance of one phase of a short-circuited winding of a rotor has an appearance [13]:

$$R_{22} = \frac{4\rho_{wr} d_{cm}}{\pi d_{cm}^2} + \frac{3\pi\rho_{wr} r_{av}}{2z_{22} k_{11} h_{11} \sin^2\left(\frac{\pi p}{z_{22}}\right)}, \quad (36)$$

where ρ_{wr} – specific conductivity of a short-circuited ring of a winding of a rotor;

p – number of poles.

In equation (36), let us substitute expressions (35) and (3):

$$R_{22} = 0.91 \frac{\rho_{wr} z_{22}}{r_{av}^2} + 45 \frac{\rho_{wr}}{z_{22} r_{av} \sin^2\left(\frac{\pi p}{z_{22}}\right)}, \quad (37)$$

Let us calculate the size of phase current of a short-circuited winding of a rotor from the equation (16):

$$i_{22} = \sqrt{\frac{P_s}{k_{MX, D} z_{22} R_{22}}}, \quad (38)$$

The equation of thermal balance has an appearance [13]:

$$m_{11} i_{11}^2 R_{11} + z_{22} i_{22}^2 R_{22} = 4\pi k_{T, av} \Delta T d_r, \quad (39)$$

In (39), let us substitute (33) and express using it a value of phase current, admissible on a condition of thermal balance:

$$i_{2opt} = 2.53 \sqrt{\frac{k \Delta T r^2}{z R}} \quad (40)$$

The current size received using formula (38) has to be equal in size to current from formula (40). Values (38) and (40) can be several percent.

For calculation of c, b, R_2, i_2, i_{2opt} , it is necessary to know the size of optimum average radius $r_{av.opt}$. From the fourth approval of the first theorem of duality, considering (32), let us calculate $r_{av.opt}$:

$$\frac{8}{9} \pi^2 r_{av.opt}^3 = \delta V(\delta) \quad \text{or} \quad r_{av.opt} = 0.4 \sqrt[3]{V(\delta)} \quad (41)$$

Dual function of this task using (32) has an appearance:

$$V(\delta) = \left(\frac{c_1}{\delta_1}\right)^{\delta_1} \left(\frac{c_2}{\delta_2}\right)^{\delta_2} \left(\frac{c_3}{\delta_3}\right)^{\delta_3} \left(\frac{c_4}{\delta_4}\right)^{\delta_4} \left(\frac{c_5}{\delta_5}\right)^{\delta_5} \left(\frac{c_6}{\delta_6}\right)^{\delta_6} \left(\frac{c_7}{\delta_7}\right)^{\delta_7} \left(\frac{c_8}{\delta_8}\right)^{\delta_8} * \\ * \left(\delta_1 + \delta_2 + \delta_3 + \delta_4\right)^{\delta_1 + \delta_2 + \delta_3 + \delta_4} \left(\delta_6 + \delta_7\right)^{\delta_6 + \delta_7} \\ \text{or} \\ V(\delta) = 6.07 * 10^{-3} \left(k_{11}^y\right)^{0.04} \left(\frac{1}{k \Delta T}\right)^{0.096} \left(\frac{P_2}{n k s}\right)^{0.96} \frac{B^{0.688}}{\left(\frac{mc}{B \delta}\right)^{2.6}} \quad (42)$$

Let us substitute equation (42) in (41):

$$r_{av.opt} = 0.08 \left(k_{11}^y\right)^{0.013} \left(\frac{1}{k \Delta T}\right)^{0.032} \left(\frac{P_2}{n k s}\right)^{0.32} \frac{B^{0.229}}{\left(\frac{mc}{B \delta}\right)^{0.87}} \quad (43)$$

The received solution of optimization has been checked by the example of specific objectives [13].

To simplify a part of work when concluding a new formula for an average radius at the new scheme of a winding, the proportion by which it is possible to determine the new numerical coefficient standing at the beginning of a formula (43) has been made:

$$x = \frac{0.815}{\frac{\pi}{2} \frac{step}{z} + 10} \quad (44)$$

which will finally take the following form:

$$r_{av,opt} = x \left(k_{11} y_{step} \right)^{\frac{\delta}{3}} \left(\frac{1}{k \Delta T} \right)^{\frac{\delta}{3}} \left(\frac{P_2}{n k s} \right)^{\frac{\delta}{3}} \frac{B^{\frac{2}{3}} \delta^7}{mc} \left(\frac{B \delta}{g} \right)^{\frac{2}{3}} \left(\delta_6 + \delta_7 \right) \quad (45)$$

By means of the received expressions, it is possible to optimize parameters of special asynchronous electric drives of a short-circuited rotor, set power P_2 and to design these types of devices. If by optimization of a component of the special asynchronous electric drive, the scheme of the chosen winding is the same, and the number of poles is equal in the reviewed example to four, then it is possible to use formula (43) for calculation r_{cp} without changes. If components with other scheme of a winding are optimized, then formula (43) has to be changed as values of the solution of normalization and orthogonality δ_i will be different.

3. Conclusion

In the article, the problem of parameters optimization of special asynchronous electric drives is considered and solved. The received formulas allow one not only to solve a certain optimizing problem, but also to construct dependences between the optimized parameters of special asynchronous electric drives and components during change of size of power, current in a winding of the stator or a rotor, induction in a gap or steel of magnetic conductors and other parameters. On the constructed dependences, it is possible to choose required optimum values of parameters of special asynchronous electric drives and components without repeated calculations. The solution of an objective will allow one to design special asynchronous electric drives with optimum mass-dimensional and power parameters for various industries.

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