

Investigation of Temperature Change under Influence of Ultrashort Laser Pulses Taking into Account Relaxation Properties of Materials

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Abstract. By using the modified Fourier law's formula considering the relaxation of heat flow and temperature gradient, a mathematical model of the local non-equilibrium process of plate heating with ultrashort laser pulses was developed. The research showed that consideration of non-locality results in the delayed plate heat up irrespective of the laser radiation flow intensity. It was also shown that in consideration of the relaxation phenomena, the boundary conditions may not be fulfilled immediately – they may be set only within a definite range of the initial time.

1. Introduction

Presently, the laser system is available allowing one to generate frequency-variable light pulses with the duration of up to 10^{-15} s. They may be used to create extremely non-equilibrium conditions for fast-relaxing excitations – relaxation time $10^{-11} - 10^{-14}$.

The known heat exchange models are based on parabolic heat conductance equations. As a result, they include an embedded infinite heat propagation velocity. Therefore, any change of the reason will immediately change the consequence. Since no infinite values of any parameters may occur in real processes, the equations derived based on the Fourier law may be adequate to real physical processes only within a particular time range. It is known that parabolic equations are inadequate in describing all the fast processes, as well as temperature change at small and extra small time values, in any heat processes [1 – 11]. In this connection, new, more adequate mathematical models should be developed applicable to these processes. As regards the development of such models, this work elaborates the direction of considering the time-spatial non-locality, which is based on the heat flow and the temperature gradient relaxation in the Fourier law formula.

2. Mathematical task setting

For deriving the differential heat ignition equation, considering local not-equilibrium, let us represent the Fourier law formula in the form of [7]:

$$q = -\lambda \frac{\partial T}{\partial x};$$



$$q = -\lambda \left(\frac{\partial T}{\partial x} + \tau_2 \frac{\partial^2 T}{\partial x \partial t} \right) - \tau_1 \frac{\partial q}{\partial t}, \quad (1)$$

where q – heat flow; T – temperature; x – coordinate; t – time; λ – heat conductance factor; τ_1, τ_2 – relaxation factors.

By substituting (1) in the heat balance equation:

$$c\rho \frac{\partial T}{\partial t} + \frac{\partial q}{\partial x} = 0, \quad (2)$$

let us find:

$$c\rho \frac{\partial T}{\partial t} = \lambda \frac{\partial^2 T}{\partial x^2} + \lambda \tau_2 \frac{\partial^3 T}{\partial x^2 \partial t} + \tau_1 \frac{\partial}{\partial t} \left(\frac{\partial q}{\partial x} \right), \quad (3)$$

where ρ – density; c – heat capacity.

By expressing $\partial q / \partial x$ of (2) and substituting in (3), one obtains:

$$\frac{\partial T}{\partial t} + \tau_1 \frac{\partial^2 T}{\partial t^2} = a \left(\frac{\partial^2 T}{\partial x^2} + \tau_2 \frac{\partial^3 T}{\partial x^2 \partial t} \right), \quad (4)$$

where $a = \lambda / (c\rho)$ – heat conductivity factor.

It is obvious that in case of $\tau_1 = \tau_2 = 0$, equation (4) is reduced to the classic parabolic heat conductance equation.

Let us find the solution to equation (4) under the following boundary conditions:

$$T(x, 0) = T_0 \quad (5)$$

$$\partial T(x, 0) / \partial t = 0 \quad (6)$$

$$-\lambda \frac{\partial T(0, t)}{\partial x} = q(t) \quad (7)$$

$$\frac{\partial T(\delta, t)}{\partial x} = 0, \quad (8)$$

where δ – the plate thickness; T_0 – initial temperature; $q(t) = q_0 \sin(\omega t)$ – time-variable heat flow; q_0 – heat flow oscillation amplitude; $\omega = 2\pi / \eta$ – cyclic frequency; η – full-wave oscillation period.

Let us introduce the following non-dimensional values and parameters:

$$\Theta = \frac{T - T_0}{T_0}; \quad Fo = \frac{at}{\delta^2}; \quad \zeta = \frac{x}{\delta}; \quad Fo_1 = \frac{a\tau_1}{\delta^2}; \quad Fo_2 = \frac{a\tau_2}{\delta^2}; \quad Ki = \frac{q_0 \delta}{\lambda T_0}; \quad Pd = \frac{\omega \delta^2}{a},$$

where Θ , Fo , ζ – non-dimensional temperature, time, coordinate, respectively; Ki – Kirpichev criterion; Pd – Predvoditelev criterion; Fo_1 , Fo_2 – non-dimensional relaxation factors.

Considering the adopted designations, task (4) – (8) will be:

$$\frac{\partial \Theta(\zeta, Fo)}{\partial Fo} + Fo_1 \frac{\partial^2 \Theta(\zeta, Fo)}{\partial Fo^2} = \frac{\partial^2 \Theta(\zeta, Fo)}{\partial \zeta^2} + Fo_2 \frac{\partial^3 \Theta(\zeta, Fo)}{\partial \zeta^2 \partial Fo} \quad (9)$$

$$(Fo > 0; \quad 0 < \zeta < 1)$$

$$\Theta(\zeta, 0) = 1 \quad (10)$$

$$\frac{\partial \Theta(\zeta, 0)}{\partial Fo} = 0 \quad (11)$$

$$-\frac{\partial \Theta(0, Fo)}{\partial \zeta} = Ki |\sin(Pd \cdot Fo)| \quad (12)$$

$$\frac{\partial \Theta(1, Fo)}{\partial \zeta} = 0 \quad (13)$$

3. Numerical Solving Method

To obtain solution of task (9) – (13) in the considered region, based on the finite differences method, let us introduce the temporal-spatial mesh with steps $\Delta \zeta$, ΔFo , respectively, by variables ζ , Fo , so that:

$$\zeta_k = k \Delta \zeta, \quad k = \overline{1, K}; \quad Fo_i = i \Delta Fo, \quad i = \overline{1, I},$$

where K, I is the number of steps by coordinates ζ , Fo .

$$\begin{aligned} \frac{\Theta_k^i - \Theta_k^{i-1}}{\Delta Fo} + Fo_1 \frac{\Theta_k^{i+1} - 2\Theta_k^i + \Theta_k^{i-1}}{\Delta Fo^2} &= \frac{\Theta_{k-1}^i - 2\Theta_k^i + \Theta_{k+1}^i}{\Delta \zeta^2} + \\ &+ Fo_2 \frac{\Theta_{k-1}^i - 2\Theta_k^i + \Theta_{k+1}^i - \Theta_{k-1}^{i-1} + 2\Theta_k^{i-1} - \Theta_{k+1}^{i-1}}{\Delta Fo \Delta \zeta^2} \end{aligned} \quad (14)$$

$$\Theta_k^0 = 0 \quad (15)$$

$$(\Theta_k^1 - \Theta_k^0) / \Delta Fo = 0 \quad (16)$$

$$-\frac{\Theta_1^i - \Theta_0^i}{\Delta \zeta} = Ki |\sin(Pd \cdot Fo_i)| \quad (17)$$

$$\frac{\Theta_K^i - \Theta_{K-1}^i}{\Delta \zeta} = 0. \quad (18)$$

4. Discussion of the results

Fig. 1 shows the temperature calculation results for the cases when relaxation properties of materials are not considered ($Fo_1 = Fo_2 = 0$) and are considered ($Fo_1 = Fo_2 = 0.09$). Their analysis allows concluding that consideration of relaxation properties results in delayed body heat-up. This fact is the evidence that due to the thermoinertial properties of the material, the instant heat-up of the body is impossible under any conditions of the external heat exchange.

Fig. 2 shows the results of calculations allowing one to evaluate the influence of heat flow discontinuity (fig. 3) on the temperature condition of the structure. It follows from their analysis that in the time range, when the heat flow is equal to zero, there is the temperature decrease close to the wall surface ($\zeta = 0$), which turns to be insufficient as the coordinate rises.

Fig. 3 shows the change of heat flow over time for $Ki = 100$.

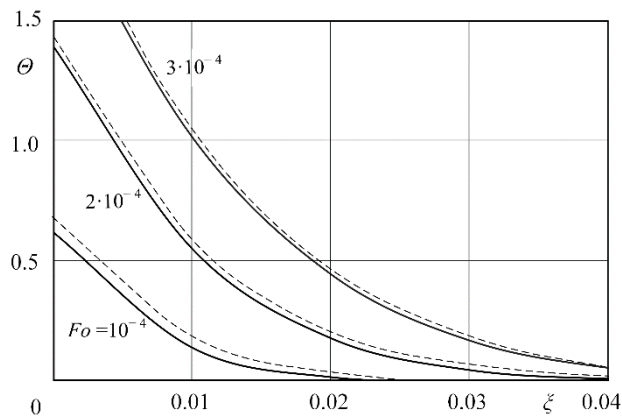


Figure 1. Temperature distribution in plate. — — — — — not considering the relaxation properties of materials ($Fo_1 = Fo_2 = 0$); — — — — — considering the relaxation properties of the material ($Fo_1 = Fo_2 = 0.09$); $Ki = 100$; $Pd = 5000$

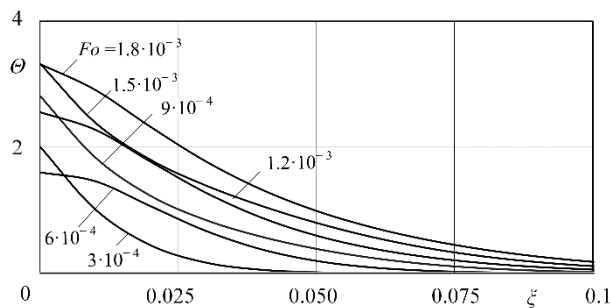


Figure 2. Temperature distribution in plate considering the thermal flow discontinuity (fig. 3) ($Fo_1 = Fo_2 = 0.09$); $Ki = 100$; $Pd = 5000$

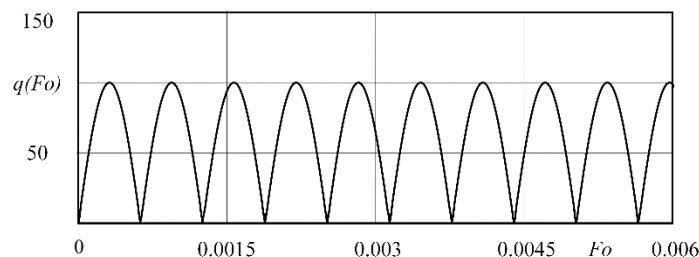


Figure 3. Step change of heat flow over time

5. Acknowledgments

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References

- [1] De Groot S, Mazur P 1964 *Non-Equilibrium Thermodynamics* (Moscow: Mir) 456 p
- [2] Gyarmati I 1974 *Non-Equilibrium Thermodynamics* (Moscow: Mir)
- [3] Sobolev S L 1991 Transfer Process And Progressing Waves In Locally Non-Equilibrium Systems *Success of Physical Science* 3(161) pp 5 – 29
- [4] Sobolev S L 1997 Locally Non-Equilibrium Models Of Transfer Processes *Success of physical science* **10(167)** 1096 – 1106
- [5] Jou D, Casas-Vázquez J, Lebon G 2006 *Extended Irreversible Thermodynamics* (Moscow – Izhevsk: RDE Regular and Chaotic Dynamics: The Computer Research Institute) 528 p.
- [6] Lykov A V 1967 *Heat Conductivity Theory* (Moscow: Vyshaya shkola) 600 p
- [7] Kudinov I V, Kudinov V A 2015 Mathematical Simulation of the Locally Nonequilibrium Heat Transfer in a Body with Account for its Nonlocality in Space and Time *Journal of Engineering*

- Physics and Thermophysics* **2(88)** 406 – 422
- [8] Stefanyuk E V 2009 Control of the laser radiation flow in material processing *Problems of power engineering* **5 – 6** 10 – 17
- [9] Gureyev G D, Gureyev D M 2008 Influence of temporal shape of laser pulse on the nature of surface temperature change at the heating stage *Vestnik Samarskogo Gosudarstvennogo Tekhnicheskogo Universiteta. Seriya Fiziko-Matematicheskie Nauki* **1** 130 – 135
- [10] Rykalin N N 1975 *Laser processing of materials* (Moscow: Machine building)
- [11] Gureyev D M 2001 *Fundamentals of physics of lasers and laser material processing* (Samara: SamSTU)