IOP Conf. Series: Materials Science and Engineering **327** (2018) 052014 doi:10.1088/1757-899X/327/5/052014

Quasi-periodic dynamics in system with multilevel pulse modulated control

Yu A Gol'tsov, A S Kizhuk and V G Rubanov

Belgorod State Technological University named after V.G. Shoukhov,
46 Kostyukov St., Belgorod, 308012, Russia

E-mail: umin@mail.ru

Abstract. In this paper, the authors describe the transitions from the regular periodic mode to quasiperiodicity that can be observed in a multilevel pulse-width modulated control system for a high-power heating unit. The behavior of such system can be described by a set of two coupled non-autonomous differential equations with discontinuous right-hand sides. The authors reduce the investigation of this system to the studying of a two-dimensional piecewise-smooth map. The authors demonstrate how a closed invariant curve associated with quasiperiodic dynamics can arise from a stable periodic motion through a border-collision bifurcation. The paper also considers a variety of interesting nonlinear phenomena, including phase-locking modes, the coexistence of several stable closed invariant curves, embedded one into the other and with their basins of attraction separated by intervening repelling closed curves.

1. Introduction

The required temperature regulation is one of the crucial tasks in many heat technology processes, such as ceramics, glass and glass blocks production, crystal growing and others, as even the slight deviation from the required temperature disrupts the necessary parameters of the technological process and deteriorates the finished product quality.

For example, when growing a synthetic sapphire crystal, it is necessary to provide the temperature change pattern in the crucible from 25°C to 2050°C with a certain degree of its increase and decrease, which implies the application of the automated control system with the function of the software setting of the temperature variation in the crucible with the required accuracy.

Research activities with a commercial heating unit imply certain technological and structural complexities, so an experimental heating unit was designed in order to test and study the control actions; the layout of the unit is presented in Fig. 1 [1, 2].

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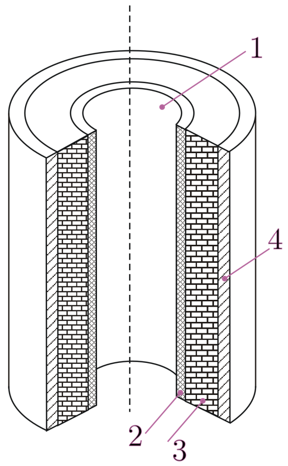

Figure 1. Schematic diagram of heating unit

The heating unit consists of the following areas: internal furnace space 1, filled with air or gas; nichrome electric heater 2, uniformly located in the inner layer of lining 3, made of magnesite bricks, and the outer layer of lining 4, made of the mineral wool in a cylindrical beaker made of galvanized steel (fig. 1). The geometric shape of the furnace is a bounded cylinder with the lining on its top and bottom.

The heat exchange processes in the heating and lined areas take place due to heat conductivity, hardness and opacity of substances, of which they are made. In the internal space of the unit, the heat transfer process is conditioned by the convective and conductive components. The confined internal space of the heating object contains either the heated material or air. In view of this, it can be affirmed that the convection processes are not so intensive in the internal space so the convective component can be excluded. The heat exchange processes in each area are determined with heat conductivity equations.

To solve the synthesis problem of the control law, taking into account the peculiarities of heat exchange processes, the transfer function of a heating unit has been experimentally determined [2]:

$$\mathrm{W(s)} = \frac{K}{(T_1 \cdot s + 1)(T_2 \cdot s + 1)}, \tag{1}$$

where K, T_1, T_2 – the transfer factor and the time constants of an object, respectively.

The currently used temperature regulators with thyristor electric energy converters distort considerably the input current curve shape, which results in the appearance of non–sinusoidal modes in the supply network.

On the other hand, the tendency of heating units' power augmentation for many heat technology processes requires creating high-power controlled power sources. One of the ways to solve this problem is using the multilevel principle of electrical energy conversion [3].

Nowadays the multilevel modulation systems are extensively used in energy-intensive technological processes [3]. At the appropriate switching frequency of semiconductor switches and the required number of areas, the arbitrarily small pulsation factor and the high accuracy of control signal reproduction can be achieved.

At the same time, implementation of multilevel modulation systems' advantages is a complicated problem. It stems from the fact that parameter variations in pulse systems can result in under-frequency oscillations, multiple of the modulation frequency, and quasiperiodic or chaotic modes. The most dangerous of these effects are rigid transitions, when discontinuous changes of dynamics occur against the periodic dynamics as a result of slight parameter alterations or random disturbances. This causes not only reduction of the control quality performance and process flow disruption, but also abrupt failures of technological equipment.

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2. Mathematical model

The motion equation of a heating unit control system, the continuous linear part of which is described with the transfer function (1), is as follows:

$$T_1T_2\frac{d^2T}{dt^2}+\left(T_1+T_2\right)\frac{dT}{dt}+T=\Gamma\cdot\mathrm{K}_F\left(\xi\right), \tag{2}$$

where T – temperature in a heating unit; T_1, T_2 – time constants; $\Gamma=K\cdot U$, where U – supply voltage, K – transfer factor of the continuous linear part; ξ, $\mathrm{K}_F(\xi)$ – the input and output signals of the modulator, respectively. The article considers a system, based on the pulse-width modulation of the first kind (PWM-1).

Let us introduce notations $x_1 = T$, $x_2 = dT/dt$ and rewrite equation (2) in matrix representation:

$$\frac{d\mathrm{X}}{dt}=A\mathrm{X}+b+\mathrm{K}_F\left(\xi\right), \tag{3}$$

$$\mathrm{X}=\begin{bmatrix}x_1\\x_2\end{bmatrix},\ A=\begin{bmatrix}0 & 1\\ -\dfrac{1}{T_1T_2} & -\dfrac{T_1+T_2}{T_1T_2}\end{bmatrix},\ b=\begin{bmatrix}0\\ \dfrac{K\cdot U}{T_1T_2}\end{bmatrix},$$

$$\mathrm{K}_F\left(\xi\right)=\frac{1}{2N}\left[1+sign\left(\xi\right)\right],$$

$$\xi=\alpha\left(V_{ref}-\beta x_1\left(k\cdot a\right)\right)-\frac{V_0}{N}\left(t/a-\lfloor t/a\rfloor\right),\ k=0,\ 1,\ 2,\ ...$$

where a – modulation period, $\lfloor\cdot\rfloor$ — function, which singles out the argument's integral part, V_{ref}– the heating unit's temperature setting signal, β – the transfer factor of a temperature sensor, α – amplification factor; V_0 – reference signal of a modulator; N – number of modulator's areas.

Parameters:

T_1T_2 = 10240 c; $T_1 + T_2$ = 352 c; K = 328,7 °C/B; a = 10 c; 2<U<24 B; β = 0,01 B/°C; V_0 = 5 B; V_{ref} = 5 B; $\alpha > 0$

The mathematical model (3) can be reduced to a simpler form [4]:

$$\frac{dx}{dt}=\lambda_1\left(x-\mathrm{K}_F\left(\overline{\varphi}\right)\right);\ \frac{dy}{dt}=\lambda_2\left(y-\mathrm{K}_F\left(\overline{\varphi}\right)\right),$$

$$\mathrm{K}_F\left(\overline{\varphi}\right)=\frac{1}{2N}\left[1+sign\left(\overline{\varphi}\right)\right],\ \overline{\varphi}=q+x_k-\vartheta y_k-\frac{P}{N\alpha}\left(t/a-\lfloor t/a\rfloor\right), \tag{4}$$

$$\vartheta=\lambda_1/\lambda_2,\ q=\frac{\lambda_1-\lambda_2}{\beta\cdot K\cdot U\cdot\lambda_2}V_{ref},\ P=\frac{\lambda_1-\lambda_2}{\beta\cdot K\cdot U\cdot\lambda_2}V_0=\frac{V_0}{V_{ref}}q,\ x_k=x\left(ka\right),\ y_k=y\left(ka\right).$$

Here $\lambda_1 = -1/T_1$, $\lambda_2 = -1/T_2$ − eigenvalues of matrix A and

$$x_1=\frac{K\cdot U\cdot\lambda_2}{\lambda_2-\lambda_1}\left(x-\vartheta y\right),\ x_2=\frac{K\cdot U\cdot\lambda_1\cdot\lambda_2}{\lambda_2-\lambda_1}\left(x-y\right)..$$

The dynamic system (4) has been reduced to a two-dimension map [4]:

$$\begin{aligned}x_{k+1}&=e^{a\lambda_1}\left(x_k-\frac{s_k}{N}\right)+\frac{1}{N}\left(s_k-1+e^{a\lambda_1\left(1-z_k\right)}\right);\\ y_{k+1}&=e^{a\lambda_2}\left(y_k-\frac{s_k}{N}\right)+\frac{1}{N}\left(s_k-1+e^{a\lambda_2\left(1-z_k\right)}\right),\\ k&=1,\ 2,\ 3,\ ...\end{aligned} \tag{5}$$

where $z_k=t_k/a-k$ − pulse duty factor $0\le z_k\le 1$ and $1\le s_k\le N$.

Variables s_k and z_k are determined:

$$s_k = \begin{cases} 1, & \varphi_k < 0, \\ N, & \varphi_k > \dfrac{P}{\alpha}, \\ \left\lfloor \dfrac{N\alpha}{P} \cdot \varphi_k \right\rfloor + 1, & 1 \le \varphi_k \le \dfrac{P}{\alpha}, \end{cases} \qquad z_k = \begin{cases} 0, & \varphi_k < 0, \\ 1, & \varphi_k > \dfrac{P}{\alpha}, \\ \dfrac{N\alpha}{P} \cdot \varphi_k - s_k + 1, & 1 \le \varphi_k \le \dfrac{P}{\alpha}, \end{cases}$$

$$\varphi_k = q + x_k - \vartheta y_k.$$

3. Bifurcation analysis

In general, case period T of the periodic motion of the dynamic system (3) is multiple of modulation period a: $T = m\,a$, $m = 1, 2, \ldots$. Let us refer to the motion with such period as m-cycle or the cycle of m period.

For bifurcation analysis, supply voltage U and amplification factor α were taken as varying parameters. In Fig. 2 a one-parameter bifurcation diagram, calculated for $\alpha = 33{,}0$ with the alteration of the supply voltage U, is presented.

At low values of U there is only a single stable 1-cycle in the phase field of a dynamic system (3). When increasing the supply voltage, the stable 1-cycle undergoes the so-called «border-collision bifurcation» [4-8] in point U_1. As a result, the 1-cycle disappears and is smoothly replaced with an unstable 1-cycle of another type with a pair of complex conjugate multipliers (unstable focus), surrounded with a stable closed invariant curve.

As it is known, the motion pattern on a closed invariant curve depends on the rotation number; if it is irrational, the invariant curve is densely filled with points of the map and the dynamics becomes quasiperiodic. When the rotation number is rational, the invariant curve has an even number of periodic orbits, half of which are stable and the others are saddle, and the invariant curve itself is composed of the unstable manifolds of saddle cycles.

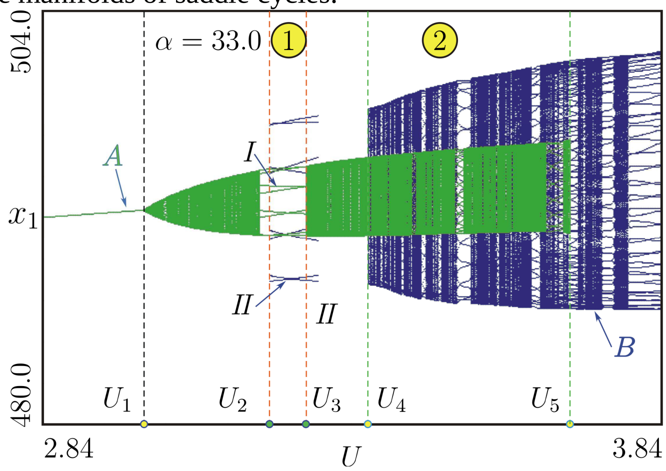

Figure 2. Bifurcation diagram, illustrating the generation of coexisting closed invariant curves

The increase of the supply voltage results in the dynamics complication due to the appearance of multistable behavior areas.

In area 1, there are two coexisting stable 8-cycles (see Fig. 2). The first stable 8-cycle I, belonging to main branch A, appears through the saddle-node bifurcation together with saddle 8-cycle. In the resonant dynamics area the closed invariant curve is composed of unstable manifolds of the saddle 8-cycle.

The second pair of 8-cycles (the stable II and the saddle one) appears hard through the «border-collision», for example, in point U_2. The border of the basins of attraction of the coexisting attractors is represented with stable manifolds of a saddle 8-cycle.

A special type of multistablilty has been identified, when several stable closed invariant curves, embedded one into another and having periodic or quasiperiodic dynamics, coexist in the phase space. It has been demonstrated in the work [9] that it is a typical situation for multilevel modulation systems. In the bifurcation diagram, the area of the coexistance $U_4 < U < U_5$ of two stable closed invariant curves is denoted with 2. The coexisting attractors' basins of attraction are separated with an unstable closed invariant curve. It should be pointed out that it is true only for two-dimensional maps (in multidimensional systems the basins of attraction are separated with stable manifolds of a saddle closed invariant curve).

4. Conclusion

In this paper the authors present the bifurcation analysis findings of a multilevel modulation control system for a heating unit. The behavior of such system can be described by a two-dimension set of two coupled nonautonomous differential equations with discontinuous right-hand sides. The research of this system can be reduced to studying the properties of a two-dimensional piecewise-smooth map.

It has been found out that at low values of supply voltage, the system demonstrates quasiperiodic behaviour, which arises through the border-collision bifurcation. At the high values of amplification factor and supply voltage, the pronounced multistablilty is observed. A special type of multistablilty has been identified, when several stable closed invariant curves, embedded one into another and having periodic or quasiperiodic dynamics, coexist in the phase space. The presented findings are only a first step in understanding complicated nonlinear phenomena in technological processes of controlled heating units. The further development of this research involves the creation of a comprehensive theory of designing a wide variety of heating units, based on up-to-date methods of nonlinear dynamics.

5. Acknowledgments

The authors would like to thank Prof. Zh.T. Zhusubaliyev for his careful reading of the first draft of this manuscript and constructive suggestions.

This work was performed within a development program of the Base University on the basis of BSTU named after V. G. Shukhov

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