

Geometric and computer-aided spline hob modeling

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Abstract. The paper considers acquiring the spline hob geometric model. The objective of the research is the development of a mathematical model of spline hob for spline shaft machining. The structure of the spline hob is described taking into consideration the motion in parameters of the machine tool system of cutting edge positioning and orientation. Computer-aided study is performed with the use of CAD and on the basis of 3D modeling methods. Vector representation of cutting edge geometry is accepted as the principal method of spline hob mathematical model development. The paper defines the correlations described by parametric vector functions representing helical cutting edges designed for spline shaft machining with consideration for helical movement in two dimensions. An application for acquiring the 3D model of spline hob is developed on the basis of AutoLISP for AutoCAD environment. The application presents the opportunity for the use of the acquired model for milling process imitation. An example of evaluation, analytical representation and computer modeling of the proposed geometrical model is reviewed. In the mentioned example, a calculation of key spline hob parameters assuring the capability of hobbing a spline shaft of standard design is performed. The polygonal and solid spline hob 3D models are acquired by the use of imitational computer modeling.

1. Introduction

Modeling of spline hobs, which represent high-precision geometrically complex tools, includes gaining the following objectives subsequently:

objective 1 – profiling, i.e. main helical surface forming in accordance with given work piece surface;

objective 2 – helical surface forming of clearance groove, which serves for chip disposition and cutting face surface forming.

The profiling objective and various approaches to its solution are the subject of the theory of gearing [1] and the theory of cutting tools [2]. This objective is rather geometrical [3, 4] than technological, and its solution is devoted to achieve, in terms of the theory of gearing, the engagement between a worm wheel and a worm in rolling action, which corresponds to the work piece and the spline hob.

The objective of helical surface forming of clearance groove is often disregarded, in comparison with objective 1. Its solution is considered at the stage of design transformation of a worm into a cutting tool, specifically the spline hob.



The matrix-vector method of curve and surface representation is successfully applied in achieving both objectives of geometrical modeling [1]. This method has certain advantages due to the fact that the working piece surface is machined with relative motion of a component billet and a cutting instrument, while vector form of surface representation easily allows for geometric transformations, which describe such motions.

The development of information technology supports the process of mechanical manufacture to attain a very high level. Nowadays information technologies and computer programs are actively being introduced into the design of metal cutting tools. Computer programs have been developed rapidly in the mechanical process for the manufacture of cutters and design of cutters [5, 6, 7]. This is especially the case for worm cutting tools or hobs [8, 9, 10]. Hobs are used for making cylindrical gears by the hobbing method. Their construction and technology are complex, and their accuracy directly influences the accuracy of gears being machined. Obviously, the design of hobs based on geometric modeling with the use of modern information technologies and computer programs is an urgent task.

2. Problem definition

A spline hob is an efficient, high-performance tool, which finds wide application in spline shaft manufacturing. The spline hob represents a multi-edge cutter of complex geometry. Its development constitutes an extensive design-technological task and is based upon geometrical formation of the main helical surface and the clearance groove helical surface.

While the objective of main helical surface forming is well-known, its solution based on planar scheme analysis of the engagement of the initial generating rack of a cutter and a spline shaft, the objective of helical surface forming of clearance groove, comprising clearance for cutting chips and front surface of cutting teeth of spline hob, is a spatial problem. The solution of the former objective is specific to geometric modeling of each given spline hob, designed to machine geared work pieces with certain geometrical form of teeth. Thus, paper [11] considers the solution to the problem of modeling of a spline hob for cylindrical gear wheel machining.

The solution to the above-mentioned spatial problem for the case of spline hob for spline shaft machining has been still inadequately investigated. Thereupon, the development of a geometrical 3D model of the spline hob for spline shaft surface generation, comprising the capability of computer-aided cutter modeling, is a relevant objective.

3. Theory

3.1 Analytical representation of main helical surface profile

Figure 1 depicts the main helical surface profile in the section normal to the teeth of a generating rack. Each of five segments of the profile is represented by a corresponding vector in its own local coordinates. Center coordinates z_1, \dots, z_5 and y_1, \dots, y_5 of local coordinate systems are calculated on the basis of the known scheme of main helical surface profiling, given there is the spline profile and the geometric parameters of engagement in rolling action [2].

According to the accepted coordination, vectors $\bar{r}_1, \bar{r}_3, \bar{r}_5$ in their local coordinate systems are of the following form:

$$\bar{r}_1 = (0, -y_1, -z), \bar{r}_3 = (0, 0, z_3), \bar{r}_5 = (0, -y_5, z_5). \quad (1)$$

Arc segments (vectors \bar{r}_2 and \bar{r}_4) are represented by vector functions of arcs in parametric form:

$$\bar{r}_2(\varphi_2) = (0, R_2 \sin \varphi_2, R_2 \cos \varphi_2), \quad \varphi_{02} \leq \varphi_2 \leq \varphi_{n2}, \quad (2)$$

$$\bar{r}_4(\varphi_4) = (0, R_4 \sin \varphi_4, R_4 \cos \varphi_4), \quad \varphi_{04} \leq \varphi_4 \leq \varphi_{n4}. \quad (3)$$

Initial boundary angles $\varphi_{02}, \varphi_{04}$ of vector functions $\bar{r}_2(\varphi_2), \bar{r}_4(\varphi_4)$ are calculated as follows:

$$\varphi_{02} = \pi - \arcsin \frac{y_2}{|\bar{r}_2|}, \quad \varphi_{04} = \arcsin \frac{(y_4 - y_3)}{|\bar{r}_4|}. \quad (4)$$

Terminal boundary angles $\varphi_{n2}, \varphi_{n4}$ are calculated from terminal points of arc segments, in turn calculated in local coordinate systems $Y_2O_2Z_2$ and $Y_4O_4Z_4$ correspondingly (figure 1):

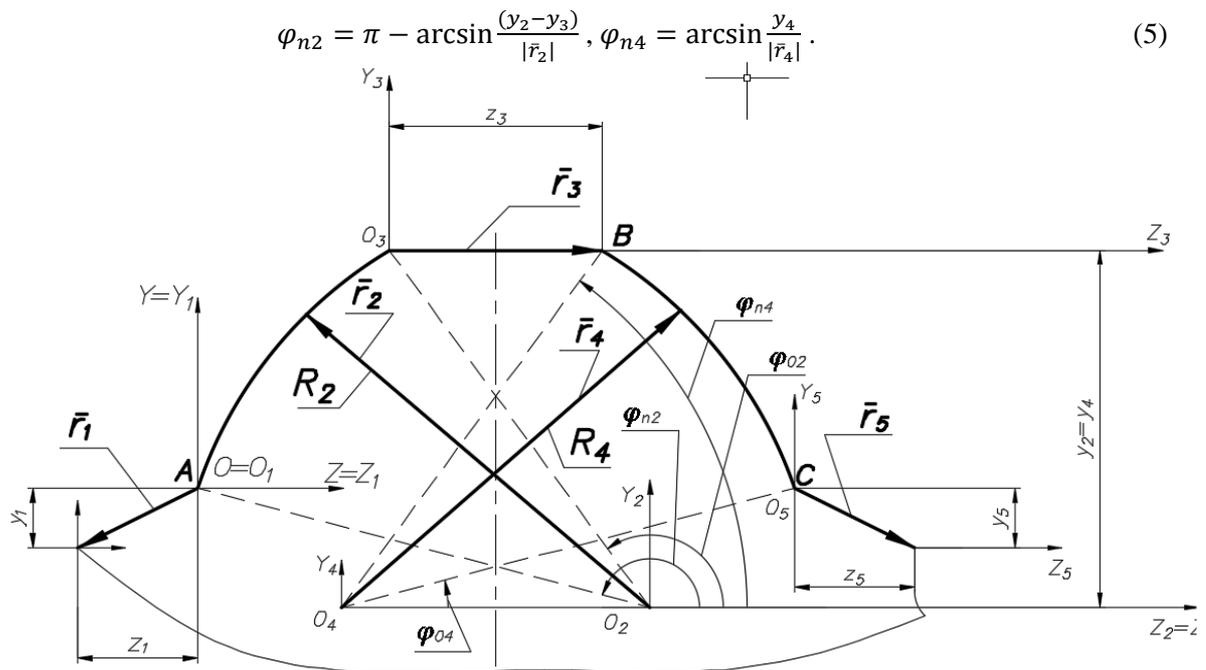


Figure 1. Main helical surface profile coordination.

To describe the whole main helical surface profile, all the vectors representing corresponding profile segments are transformed to a common coordinate system. Let us suppose that coordinate system OYZ refers to a certain spline hob tooth. Then the profile of each spline hob tooth can be represented by a combination of equations of its segments in corresponding local coordinate systems, transformed to coordinate system OYZ of the certain spline hob tooth. The equation of the spline hob tooth profile is of the following standard form:

$$\bar{r}_{i,j} = \bar{r}_i M, \quad (6)$$

where \bar{r}_i is a vector in the local coordinate system, represented by a rowed matrix, j is the number of spline hob tooth, M is a parallel transformation matrix of the following form:

$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ m_i & n_i & 1 \end{bmatrix},$$

where $i=1, \dots, 5$; in case $i=1$ the equation $m_i=n_i=0$ takes place.

Vector $\bar{r}_{0i} = [m_i, n_i]$ describes the location of the i -th local coordinate system of each vector in the coordinate system of a tooth. As a result, the main helical surface profile is represented by a set of vector functions.

3.2 Analytic description of the initial rack

Figure 2 represents an initial rack in the section normal to the coil direction of the main helical surface. This rack is formed by the profile of the main helical surface and its displacement in the plane of the normal section in the direction of axis OZ of the local coordinate system of the profile with pitch t . In this case, in the local coordinate system of the rack for the i -th vector of the j -th tooth, any other given vector in relation to the mentioned coordinate system is found from the expression:

$$\bar{r}_{i,j,r} = \bar{r}_{0k} + \bar{r}_{i,j}. \quad (7)$$

Vector \bar{r}_{0k} of equation (7) is of the following form:

$$\bar{r}_{0k} = n\bar{r}_{0j}. \quad (8)$$

For the first tooth, $j=0$; n represents the amount of teeth on the rack; index j can vary within limits $0 < j < n$.

The description of teeth of a single initial rack is related to the coordinate system of the rack, which coincides with the local coordinate system of the first tooth (figure 2). Each subsequent tooth is described in the coordinate system of the rack by means of parallel displacement on vector \bar{r}_{0j} , the magnitude of which is equal to teeth pitch t .

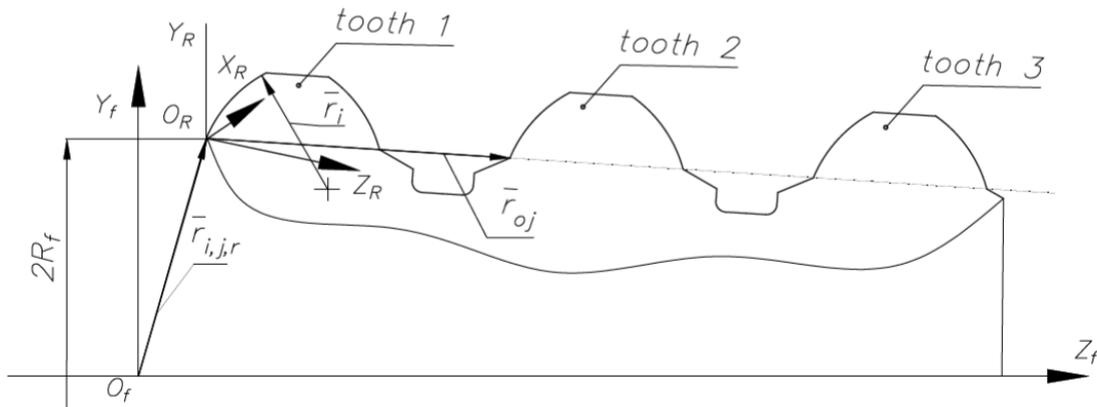


Figure 2. Initial rack profile coordination in normal section.

3.3 Mathematical model of wireframe spline hob formation

The spline hob comprises a number of circumferentially positioned racks, each of them comprising a number of identical teeth. Each rack, as well as its teeth, is found by matrix transformation of turning at a corresponding angle in relation to the global coordinate system of the spline hob (figure 2).

The expression for each vector in the spline hob coordinate system is of the following general form:

$$\bar{r}_{i,j,r,f} = M_2 \bar{r}_{0f} + \bar{r}_{i,j,r}, \quad (9)$$

where $\bar{r}_{i,j,r}$ represents the i -th – vector of the j -th tooth of the r -th rack. The matrix of turn M_2 is of the following form:

$$M_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \psi_1 & -\sin \psi_1 \\ 0 & \sin \psi_1 & \cos \psi_1 \end{bmatrix}.$$

It is accepted in this paper that the coordinate system of the initial rack is situated in plane YOZ. Helical racks formed by cutting edges of the spline hob are positioned on the main helical surface. This means that neighboring points of the same vector or adjacent vectors are positioned on helical lines as well.

Helix parameter a is found from specified setup angle γ_0 between the spline hob axle and the helical clearance groove axle in order to cut clearance grooves. Helix parameter a represents the correlation between pitch of machined groove and circumference at accepted cutter radius R_0 , which refers to the coordinate origin of the first tooth of the first rack. Parameter a is found from the travel of a point of helical line alongside the z axis:

$$z = a\varphi. \quad (10)$$

Helical line pitch t_B with radius R_0 with full circumference $\varphi = 2\pi$ is found as:

$$t_B = \frac{2\pi R_0}{\text{tg } \gamma_0}. \quad (11)$$

The circumferential vector-function makes a full circumference upon travelling the value of the pitch.

Angle φ of equation (10) varies within limits $0 \leq \varphi \leq F_{\max}$. Maximum value F_{\max} is calculated as follows:

$$F_{\max} = \frac{B}{a}, \quad (12)$$

where B represents spline hob length.

It should be noted that involution and variation of value R_0 takes place during travel, which affects the change of coordinates x, y, z of points on all the vectors. Then the vector of the first pitch is of the following form:

$$\bar{r}_{i,j,r} = \begin{pmatrix} R_i \cos \varphi_i \\ R_i \sin \varphi_i \\ a \varphi_i \end{pmatrix}, \quad (13)$$

As a result, taking helical motion into account, coordinates x, y, z of any point of any vector can be calculated and stored in an array. Since the spline hob comprises a number of racks, it has to be considered that upon turning at angle ψ , the coordinate system of a rack is displaced alongside axis OZ due to taking part in helical motion alongside a cylinder with diameter $2R_0$ (figure 2).

The value of displacement $a_1 \psi$ depends on cutting pitch and angle ψ . In case the rack under consideration is the first one, $\psi=0$. Value ψ has a dimension of a radian.

$$\psi = \frac{2\pi}{k}, \quad (14)$$

where k represents the amount of racks on the spline hob. The amount of displacement alongside axis OZ (figure 2) is found relative to the coordination of the initial rack:

$$z_i = a_1 \psi_i. \quad (15)$$

Hereby all the points of the rack are displaced on the value of pitch t_b in the normal section in one turn around the axis of the spline hob. Considering that the displacement takes place alongside the axis OZ, the following should be accepted:

$$z_i = a_1 \psi_i \cos \gamma_0. \quad (16)$$

From this, the value of displacement alongside axis OZ follows:

$$a_1 = \frac{z_1}{2\pi}. \quad (17)$$

The spline hob comprises a number of racks. Displacement of a neighboring rack takes place upon turn around the axis of the spline hob by value ψ_1 , which equals:

$$\psi_1 = \frac{2\pi}{k}, \quad (18)$$

where k represents the amount of racks on a spline hob.

Thus all the points of vectors of the first rack are displaced alongside axis OZ and turned at angle ψ_1 .

As a result, one of the problems of spline hob design automation is resolved, namely, the equations that describe the position of any point of the cutting edge of a spline hob in space are found. Such equations allow us to calculate load characteristics of spline hobbing.

4. Result

The mathematical model that describes all the cutting edges of a spline hob is developed. The coordinates of the points of the cutting edge can be calculated according to the proposed model, which in turn presents an opportunity to perform computation of dynamic load on the segments of the cutting edge.

5. Consideration of the result

The surfaces of the teeth of spline hob are considered as a set of vector functions transformed to the single coordinate system by equation (9). On the basis of the acquired mathematical model, the software working in VisualLISP for AutoCAD environment is developed. This software includes the following modules: the cutting edge wireframe model calculation and formation module and the helical clearance groove calculation module (figure 6). The modules provide the opportunity of forming a polygonal model and a solid model of spline hob based on the points acquired during calculation.

The input data for spline hob profiling constitute the dimensions of a spline shaft in section normal to its axis (figure 3), namely outer diameter D , tooth width b , number of teeth z , inner diameter d ,

inner diameter with consideration for cavities dI , chamfer C , the distance between teeth without consideration for cavities, a .

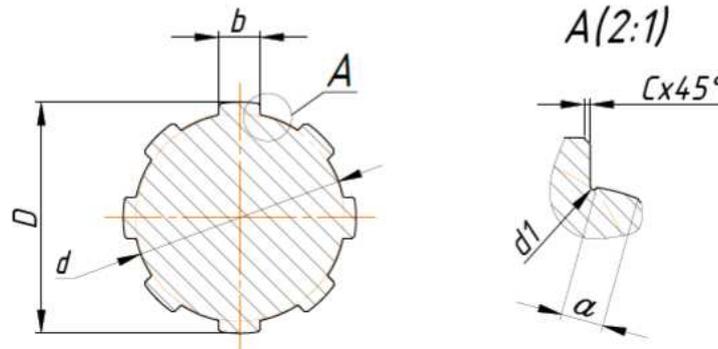


Figure 3. Cross section parameters of a spline shaft.

To demonstrate the work of the software, a calculation and formation of a solid model of a spline hob for spline shaft machining have been performed. The input data are presented in table 1 and constitute a spline shaft D-8x52x60 GOST 1139-80 (SE SEHV 187-75) (figure 3).

Table 1. Dimensions of medium series spline shaft D-8x52x60 GOST 1139-80 (centered by the outer diameter).

$z \times d \times D$	z	d (mm)	D (mm)	b (mm)	dI (mm)	a (mm) no less	C (mm)	
							nominal	tolerance
8x52x60	8	52	60	10	48.7	2.44	0.5	+0.3

For parameters of the specified spline shaft tooth profile and in accordance with the known main helical surface profiling scheme, the coordinates of five local coordinate system centers are acquired (figure 1): $y_1=0, z_1=0; y_2=-1.85, z_2=9.46; y_3=2.94, z_3=1.018; y_4=2.614, z_4=9.46; y_5=0, z_5=12.435$.

For the next step, the key values of spline hob profiling are calculated (table 2). Geometrical representation of normal section of the cutting edge profile is performed (figure 4).

Table 2. Key calculated values of spline hob profiling.

Substitution circle radius (mm)	R_0	19.425714
Substitute circle center abscissa (mm)	y_2	-1.84777
Substitute circle center application (mm)	z_2	9.46172
Spline hob radius (mm)	R_f	45
Specified setup angle in relation to axis $O_f Z_f$ (rad)	γ_0	0.087266463
Clearance groove helical surface pitch (mm)	t_B	3043.114101
Tooth pitch in normal section (mm)	t	22.114
Spline hob length (mm)	B	80
Helix parameter (mm/rad)	a	342.9015691
Circular angle of turn around axis $O_f Z_f$ (rad)	F_{max}	0.291628878

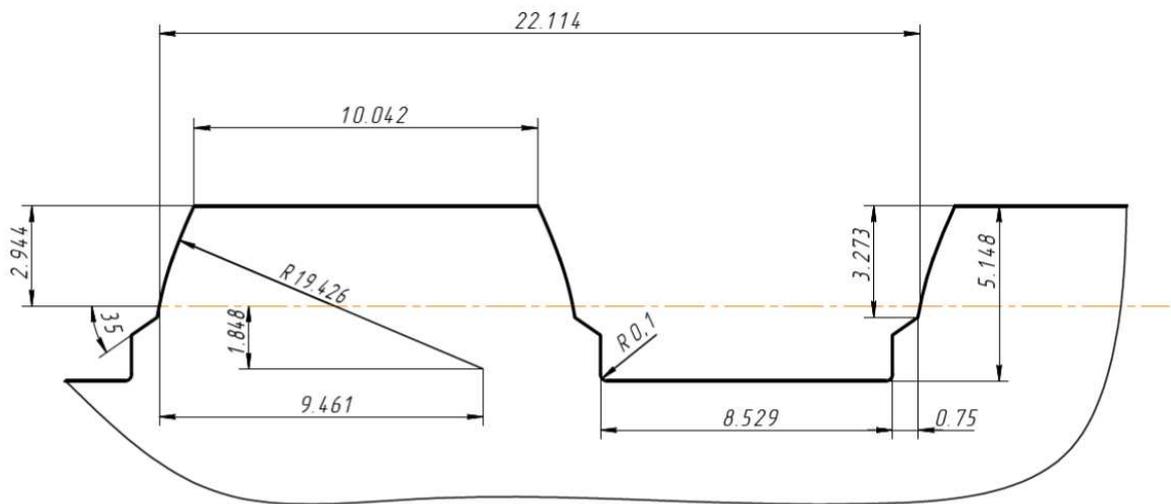


Figure 4. Calculation result of spline hob tooth profile parameters in normal section.

3D models of spline hob structural elements are acquired on the basis of the mathematical model (figure 5, 6).



Figure 5. Solid 3D model of spline hob.

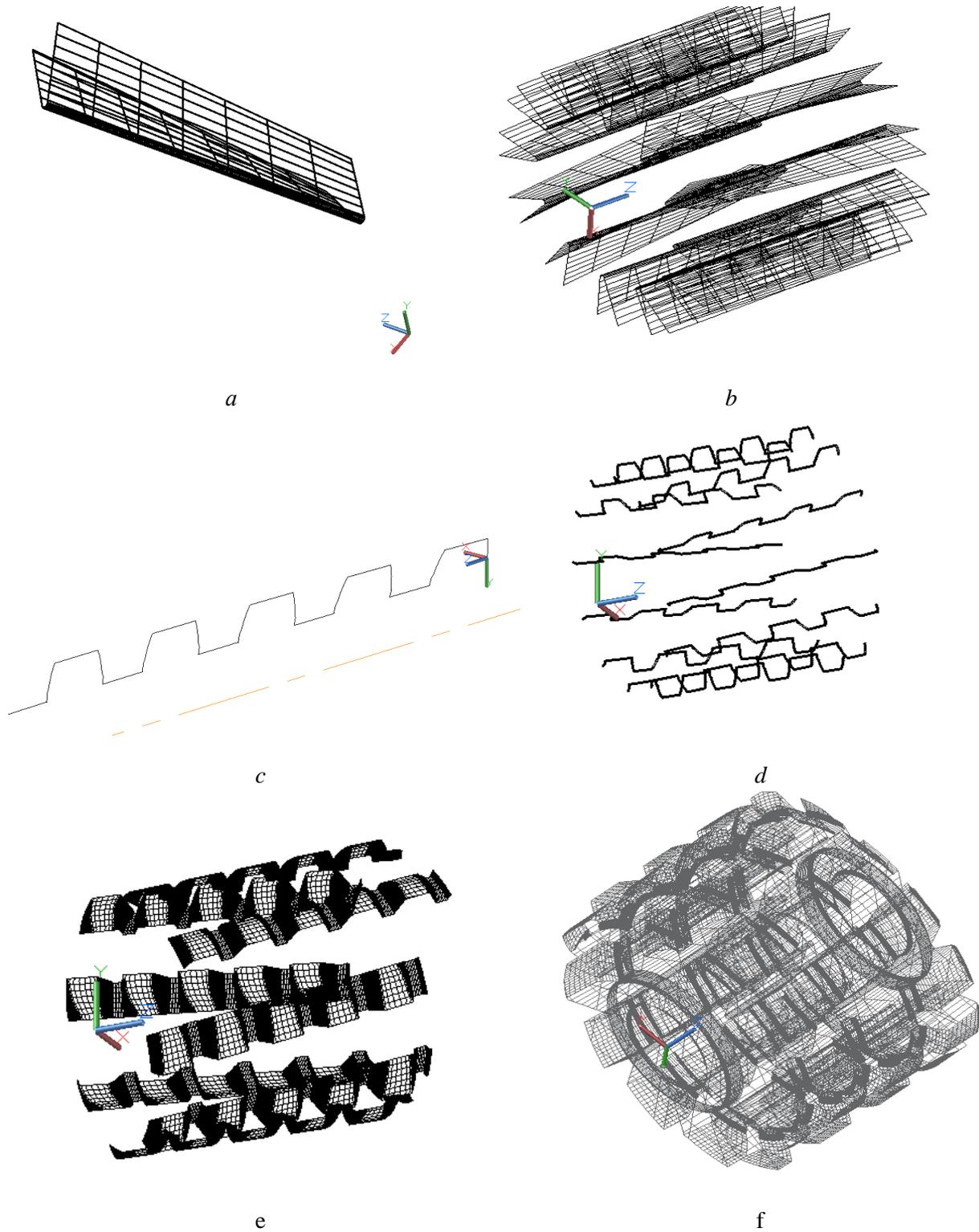


Figure 6. 3D models of spline hob structural elements:
a – helical clearance groove; b – array of helical clearance groves; c – cutting edge profile;
d – array of cutting edge profiles; e – relief sections of the teeth; f – polygonal spline hob model.

6. Conclusion

The proposed geometric model of spline hob cutting edge formation and its mathematical description present an opportunity to calculate coordinates of points of the spline hob surface in a static position as well as with motion imitation. The spline hob is considered as a spatial body; the proposed model allows us to calculate coordinates of points of the spline hob surface that belong to the cutting edge and are either engaged or idle in the process of milling. This, in turn, presents an opportunity to define forces that influence a particular section of the cutting edge. Thus, the proposed geometric model of spline hob formation can be the basis of calculation of dynamic load effecting the spline hob in any given spatial position.

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