Analytical dependences of motion of working part in inertial cone crusher.

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Abstract. The article discusses the analytical dependence of the movement of the inertial cone crusher, which is a hard cylinder that is moving along the inner elliptic trajectory of the stationary cone of the inertial cone crusher.

1. Introduction

Theoretical studies of the steady-state mode of details of the operation of an inertial cone crusher were published in the works A.K. Rundkvist and I.I. Blekhman [1-8]. They investigated the mode of the regular running inner cone on the outside in the approximation of constancy of the angle of nutation. This mode can be realized in case of slow running with the right geometry of the lining cone, as well as in the operating mode with a uniform filling of the crushing cavity material both by the volume and by the grain-size composition.

The lining of the cone, as a rule, has ellipticity of the working surfaces even with their careful mechanical operation. The movable cone, even during slow running, is driven around the outside cone at each point of the surface, describing an elliptical trajectory. With irregular wear of lining, the ellipticity increases and with sufficiently severe wear can experience movement trajectories, close to triangular and polygonal. The presence of irregularity in this character of the movement leads to the variability in time of the crushing force, what causes the deterioration of the technological parameters of the inertial cone crusher and increases stresses in the nodes and details.

Higher stresses, obviously, should occur in the case of the elliptical trajectory of the movable cone.

In this regard and also due to the fact that the movement of the movable cone in an elliptical trajectory in the first approximation reflects the picture modes of the inertial cone crusher, it is advisable to produce a theoretical study of this movement of the movable cone.

2. Statement of the problem

Let us consider the problem of running in the nerinertial cone crusher on the elliptical surface the outside cone in the "flat" setting, namely, let us suppose that the movable cone consists of a rigid cylinder, moving along the inner elliptic trajectory of the stationary cone with axes d and c (d >c) [1, 9].

To solve this problem, let us make the following assumptions:

– The plane of action of centrifugal force of the unbalanced-mass vibration generator coincides with the plane of action of centrifugal force of the rolling cylinder and plane, where the line of contact interactive cones lies;

– in the initial time, the movable cone contact with non-movable in the point "A" (Fig. 1);

– the current position of the contact of the movable cone with non-movable as a result of its motionon the inner surface of themovaablecone is set to distance r from the centre of the elliptic trajectory to the contact and angle θ, which forms the distance with the positive direction of axis «OX» (Fig. 1).

Let us denote Cartesian coordinates of the center O_2 of movable cone through x, y, and through ψ – its rotation.

3. Analytical studies

According to the results of [1, 10], the expression for the motional energy of the system consisting of the movable cone and the unbalanced-mass vibration generator, would be:

$$E = \frac{M+m}{2}(\dot{x}^2 + \dot{y}^2) + \frac{I_{6}\dot{\psi}^2}{2} + \frac{I_c + m\varepsilon^2}{2}\dot{j}^2 + m\varepsilon\dot{j}(\dot{y}\cos\psi - \dot{x}\sin j), \qquad (1)$$

where M, I_{6} – accordingly mass and central mass moment of inertia of the cone; m, I_c, ε – accordingly mass, central mass moment of inertia and eccentricity of the unbalanced mass; x, y, ψ – Cartesian coordinates of the center O_t cone and its pure rotation; φ – rotation of the unbalanced mass.
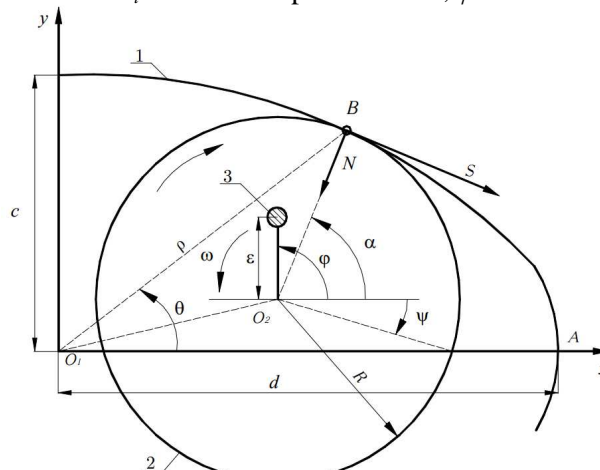

Figure 1. A design scheme of the running inner cone at the surface of the outside. *1* – inner surface of the outside cone; *2* – outside surface of the inner cone; *3* – centre of unbalanced mass.

It is possible to observe that during the running of the movable cone as stationary without slippage, there is the following expression:

$$\dot{x}^2 + \dot{y}^2 = R\dot{f}^2. \qquad (2)$$

Taking into account (2), the expression for motional energy (1) takes the form:

$$T = \frac{1}{2}\left[I_{6} + (M+m)R^2\right]\dot{\psi}^2 + \frac{I_c + m\varepsilon^2}{2}\dot{j}^2 + m\varepsilon(\dot{y}\cos\varphi - \dot{x}\sin\varphi)\dot{\varphi}. \qquad (3)$$

As independent variables for the system with two degrees of clamping, let us choose generalized coordinates θ and ψ (figure 1); then:

$$\dot{x} = \frac{dx}{d\theta}\dot{\theta};$$

$$\dot{y} = \frac{dy}{d\theta}\dot{\theta};$$
(4)

$$\dot{\psi} = \frac{d\psi}{d\theta}\dot{\theta}.$$

Taking into account equation (4), relation (3) can be written in the form:

$$T = \frac{1}{2}\left[I_\text{в} + (M+m)R^2\right]\left(\frac{d\psi}{d\theta}\right)^2\dot{\theta}^2 + \frac{I_c + m\varepsilon^2}{2}\dot{\varphi}^2 + m\varepsilon(\frac{dy}{d\theta}\cos j - \frac{dx}{d\theta}\sin\varphi)\dot{\theta}\dot{\varphi}.$$
(5)

According to the design scheme in figure 1, one can record that:

$$\begin{cases} x = p(\theta)\cos\theta; \\ y = p(\theta)\sin\theta. \end{cases}$$
(6)

In Cartesian coordinates, the equation of the elliptic trajectory has the form:

$$\frac{x^2}{d^2} + \frac{y^2}{c^2} = 1,$$
(7)

Where d, c – accordingly the semimajor and semiminor axes of the elliptic trajectory.
Substitution of equation (6) into equation (7) leads to the ratio:

$$p(\theta) = \frac{c}{\sqrt{1 - e^2\cos\theta}}.$$
(8)

Here,

$$e = \sqrt{1 - c^2\!\big/d^2} \quad (0 < e < 1)$$
(9)

is the eccentricity of the ellipse.
Because of the lack of slippage during the rotation of a movable cone by angle α, in an elliptical trajectory it will describe an angle equal to θ. Because of this, let us get the following ratio:

$$\psi = \frac{1}{R}\int\sqrt{p^2(\theta) + \left(\frac{dp}{d\theta}\right)^2}\,d\theta - a.$$
(10)

Relation (10) establishes a relationship between angles α, θ, and ψ and allows one to compute the value of angle ψ as a function of generalized coordinates θ.
Using ratio (8), let us calculate:

$$\frac{dp}{d\theta} = -\frac{ce^2\cos\theta\sin\theta}{\left(1 - e^2\cos^2\theta\right)^{3\!/\!2}},$$
(11)

$$\sqrt{p^2 + \left(\frac{dp}{d\theta}\right)^2} = \sqrt{\frac{c^2}{1 - e^2\cos^2\theta} + \frac{c^2 e^4\cos^2\theta\sin^2\theta}{\left(1 - e^2\cos^2\theta\right)^3}} =$$
$$= \frac{c\sqrt{1 - 2e^2\cos^2\theta + e^4\cos^4\theta + e^4\cos^2\theta(1 - \cos^2\theta)}}{\left(1 - e^2\cos^2\theta\right)^{3\!/\!2}} = \frac{c\sqrt{1 - e^2\cos^2\theta(2 - e^2)}}{\left(1 - e^2\cos^2\theta\right)^{3\!/\!2}}.$$
(12)

Next, let us write the expression relating angles α and θ, i.e. let us express angle α as a function of generalized coordinates θ.

$$a = \theta - arctg \frac{1}{\rho} \frac{d\rho}{d\theta}. \tag{13}$$

From relation (13), it follows that:

$$\frac{1}{\rho} \frac{dp}{d\theta} = \frac{tg\theta - tga}{1 + tg\theta\,tga}. \tag{14}$$

Taking into account relations (8) and (11), expression (14) takes the following form:

$$-\frac{e^2 \cos\theta \sin\theta}{1 - e^2 \cos^2\theta} = \frac{tg\theta - tga}{1 + tg\theta\,tga}. \tag{15}$$

From relation (13), it follows that:

$$tga = \frac{tg\theta}{1 - e^2}. \tag{16}$$

From relations (16), let us find that:

$$a = arctg \frac{tg\theta}{1 - e^2}. \tag{17}$$

Let us calculate the value of the derivative of angle α on generalized coordinate θ. To do this, let us differentiate with respect to expression (17) for θ and get:

$$\frac{da}{d\theta} = \frac{\dfrac{1}{\left(1 - e^2\right)\cos^2\theta}}{1 + \dfrac{tg^2\theta}{\left(1 - e^2\right)^2}} = \frac{1 - e^2}{\left(1 - e^2\right)^2 \cos^2\theta + \sin^2\theta} = \frac{1 - e^2}{1 - \left(2 - e^2\right)e^2 \cos^2\theta}. \tag{18}$$

Then it is necessary to calculate the derivative of angle ψ on generalized coordinate θ. To do this, let us differentiate with respect to ratio (10) for θ and get:

$$\frac{d\psi}{d\theta} = \frac{1}{R} \sqrt{p^2(\theta) + \left(\frac{d\rho}{d\theta}\right)^2} - \frac{da}{d\theta}. \tag{19}$$

Substituting equations (8), (11) and (18) in relation (19), one can obtain the following expression:

$$\frac{d\psi}{d\theta} = \frac{c}{R} \frac{\left(1 - \left(2 - e^2\right)e^2 \cos^2\theta\right)^{1/2}}{\left(1 - e^2 \cos^2\theta\right)^{3/2}} - \frac{\left(1 - e^2\right)}{1 - \left(2 - e^2\right)e^2 \cos^2\theta}. \tag{20}$$

Let us assume that the eccentricity of the elliptic trajectory is a small amount of the first order of smallness, and then resulting expression (20) will expand in power series of "e" with accuracy up to quantities of the second order of smallness.

$$\frac{d\psi}{d\theta} \cong \frac{c}{R}\left(1-e^2\cos^2\theta\right)\left(1-e^2\cos^2\theta\right)^{-\frac{3}{2}} - (1-e^2)(1-2e^2\cos^2\theta)^{-1} \cong$$

$$\cong \frac{c}{R}\left(1-e^2\cos^2\theta\right)\left(1+\frac{3}{2}e^2\cos^2\theta\right) - (1-e^2)(1+2e^2\cos^2\theta)^{-1} \cong$$

$$\cong \frac{c}{R}\left(1+\frac{e^2}{2}\cos^2\theta\right) - 1 + e^2 - 2e^2\cos^2\theta = \frac{c}{R} - 1 + \left(\frac{c}{2R}-2\right)e^2\cos^2\theta + e^2 \cong$$

$$\cong \frac{c}{R} - 1 + \left[1 + \left(\frac{c}{2R}-2\right)\cos^2\theta\right]e^2. \tag{21}$$

It is obvious that the condition of running the movable cone on a stationary one in the case of the elliptic trajectory is:

$$R\!\big/\!c = \gamma < 1. \tag{22}$$

Taking into account relation (22), let us present equation (21) as:

$$\frac{d\psi}{d\theta} = \frac{1}{\gamma} - 1 + \left(1 + \left(\frac{1}{2\gamma}-2\right)\cos^2\theta\right)e^2 = \frac{1-\gamma}{\gamma}\left[1 + \frac{\gamma + \frac{1-4\gamma}{2}\cos^2\theta}{1-\gamma}e^2\right]. \tag{23}$$

In obtained relation (23), in view of the smallness of parameters γ and e, let us assume that the values are proportional, γe^2 are the amount of the third order of smallness.

Owing to the made assumption, relation (23) takes the form:

$$\frac{d\psi}{d\theta} \cong \frac{1-\gamma}{\gamma}\left[1 + \frac{e^2}{1-\gamma}\cos^2\theta + ...\right]. \tag{24}$$

Let us write the relations between Cartesian coordinates x and y of generalized coordinate θ. According to figure 1, it is possible to write expressions:

$$x = \rho\cos\theta - R\cos a, \tag{25}$$

$$y = \rho\sin\theta - R\sin a. \tag{26}$$

Let us compute the derivative of relation (25) as generalized coordinate θ:

$$\frac{dx}{d\theta} = \frac{d\rho}{d\theta}\cos\theta - \rho\sin\theta + R\sin a\frac{da}{d\theta}, \tag{27}$$

because of ratio (17):

$$\sin a = \frac{\sin\theta}{\left(1-e^2\left(2-e^2\right)\cos^2\theta\right)^{\frac{1}{2}}} \cong \sin\theta\left(1 + e^2\cos^2\theta\right). \tag{28}$$

Taking into account relations (11), (18) and (28), equation (27) is reduced to the form:

$$\frac{dx}{d\theta} = -\sin\theta\left[\frac{c}{\left(1-e^2\cos^2\theta\right)^{\frac{3}{2}}} - \frac{R\left(1-e^2\right)}{\left(1-e^2\left(2-e^2\right)\cos^2\theta\right)^{\frac{3}{2}}}\right]. \tag{29}$$

Obtained expression (29) can be decomposed in a series of small parameters γ and e with the accuracy equal to the values of the second order of smallness, thus it is possible to obtain:

$$\frac{dx}{d\theta} \cong -c(1-\gamma)\sin\theta\left[1+\frac{3e^2\cos^2\theta}{2(1-\gamma)}+...\right]. \tag{30}$$

Second, let us calculate the derivative of ratio (26) for generalized coordinate θ:

$$\frac{dy}{d\theta} = \frac{d\rho}{d\theta}\sin\theta + \rho\cos\theta - R\cos a\frac{da}{d\theta}, \tag{31}$$

Here,

$$\cos a = \frac{\left(1-e^2\right)\cos\theta}{\left(1-e^2\left(2-e^2\right)\cos^2\theta\right)^{1/2}} \cong \cos\theta\left(1-e^2\sin^2\theta\right). \tag{32}$$

Taking into account relations (11), (18) and (32), one obtains the following expression:

$$\frac{dy}{d\theta} = \frac{c\left(1-e^2\right)\cos\theta}{\left(1-e^2\cos^2\theta\right)^{3/2}} - \frac{R\left(1-e^2\right)\cos\theta}{\left(1-e^2(2-e^2)\cos^2\theta\right)^{3/2}}. \tag{33}$$

Decomposition of obtained expression (33) in a series into small parameters γ and e leads to a result:

$$\frac{dy}{d\theta} \cong c(1-\gamma)\cos\theta\left[1-\frac{1-\frac{3}{2}\cos^2\theta}{1-\gamma}e^2+...\right]. \tag{34}$$

4. Conclusions
Thus, the authors of the paper described the process of movement of movable cones on the surface of a stationary cone in an elliptical path under certain assumptions.

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