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On damping of screw dislocation bending vibrations in dissipative crystal: limiting cases

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Abstract. The expression for the generalized susceptibility of the dislocation obtained earlier was used. The electronic drag mechanism of dislocations is considered. The study of small dislocation oscillations was limited. The contribution of the attenuation of low-frequency bending screw dislocation vibrations to the overall coefficient of dynamic dislocation drag in the long-wave and short-wave limits is calculated. The damping of short-wave bending screw dislocation vibrations caused by an external action of an arbitrary frequency has been investigated. The contribution of long-wave bending screw dislocation vibrations damping in the total drag coefficient at an arbitrary frequency is found.

1. Introduction

In the process of using mechanical engineering products, vibration dynamic loads are generated on the corresponding structures. These loads affect the material crystal structure of constructions and act on dislocations (linear crystal defects), causing their oscillations, which are damped due to various mechanisms. One of the first theoretical works on the drag of a moving dislocation was carried out by Eshelby [1]. He considered in it the damping of a rectilinear dislocation oscillations due to thermoelastic energy dissipation and radiation of elastic waves. Subsequently, a large number of specific mechanisms calculations for the dislocations drag were carried out. Reviews of these mechanisms in [2, 3] are given. Along with this, any dissipative processes in the most general form can be taken into account in the equations of motion by including friction forces in the right-hand side of the Lagrange equation. Such approach, based on dissipative processes accounting through the dispersion of elastic modules, was developed in [4]. In [6], on the basis of [5] results generalization, the equations for the small oscillations of a crystal with dislocation taking into account the dissipative processes were obtained, and the generalized susceptibility of dislocation in a dissipative crystal is found. Also in work [6], cases $\omega = 0$ and $k_z = 0$ are considered, where ω is frequency, k_z is wave vector component along dislocation line. In papers [7, 8], the dynamic inhibition of dislocation by defects in a crystal is considered. The closest to the present paper is paper [7], in which the dynamic retardation of the motion of screw dislocations at point defects is investigated taking into account the excitation of transverse vibrations of dislocation elements. In recent paper [9], fundamentals of dislocation dynamics simulations are given. Unfortunately, in this work, dislocations were represented in the form of straight lines; bending vibrations were not taken into account. In this paper, on the basis of the expression for the dislocation generalized susceptibility in a dissipative crystal [6], various limiting cases of screw dislocation bending vibrations damping are investigated.

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2. Damping of low-frequency bending dislocation vibrations

Damped bending vibrations of screw dislocation lying along axis Oz were investigated. The case of small oscillations of a dislocation near the equilibrium position was limited. In the low-frequency limit, from [6] for the imaginary part of the screw dislocation inverse generalized susceptibility, taking into account only the terms corresponding to the dislocation bending vibrations, let us obtain:

$$\operatorname{Im} g_s^{-1}(k_z,\omega) = -\frac{\rho b_s^2}{2\pi}\omega\int_{|k_z l|}^{k_m l} dx\left[(3-8\xi)\frac{(k_z l)^2}{x^3}\gamma_t + 4\xi^2\frac{(k_z l)^2}{x^3}\gamma_l\right.$$
$$\left. -4(1-2\xi)\frac{(k_z l)^4}{x^5}\gamma_t - 4\xi^2\frac{(k_z l)^4}{x^5}\gamma_l\right]. \tag{1}$$

Here ρ is density of the crystal, b_s is screw component of the Burgers vector, k_m is maximum wave number, l is electron mean free path, $x = kl$ is dimensionless variable, $\xi = c_t^2 / c_l^2$, c_t and c_l are the transverse and longitudinal sound velocities in a nondissipative crystal, $\gamma_t(x)$ and $\gamma_l(x)$ are the damping coefficients of transverse and longitudinal sound waves in a dissipative medium. As a concrete damping mechanism for oscillations, electronic drag was considered, for which the damping coefficients of the transverse and longitudinal sound are expressed in the form [10]:

$$\gamma_t(x) = \gamma^0\left(\frac{2x^3}{3[(1+x^2)\operatorname{arctg}x - x]} - 1\right) \quad \text{and} \quad \gamma_l(x) = \gamma^0\left(\frac{x^2\operatorname{arctg}x}{3(x-\operatorname{arctg}x)} - 1\right), \tag{2}$$

where γ^0 is a constant depending on the material. For the purpose of analytical integration in formula (1), damping coefficients (2) were approximated as follows:

$$\gamma_t(x) \approx \frac{4}{3\pi}\gamma^0\frac{x^2}{x+20/3\pi} \quad \text{and} \quad \gamma_l(x) \approx \frac{\pi}{6}\gamma^0\frac{x^2}{x+5\pi/8}. \tag{3}$$

Substituting expressions (3) into formula (1) and performing the integration, one can obtain:

$$\operatorname{Im} g_s^{-1}(k_z,\omega) = -\frac{\rho b_s^2}{2\pi}\gamma^0\omega\left\{\frac{1}{5}(3-8\xi)(k_z l)^2\ln\frac{|k_z l|+20/3\pi}{|k_z l|} + \frac{16}{15}\xi^2(k_z l)^2\ln\frac{|k_z l|+5\pi/8}{|k_z l|}\right.$$
$$-\frac{16}{3\pi}(1-2\xi)(k_z l)^4\left[\left(\frac{3\pi}{20}\right)^3\ln\frac{|k_z l|+20/3\pi}{|k_z l|} - \left(\frac{3\pi}{20}\right)^2\frac{1}{|k_z l|} + \frac{3\pi}{40}\frac{1}{(k_z l)^2}\right]$$
$$\left.-\frac{2\pi}{3}\xi^2(k_z l)^4\left[\left(\frac{8}{5\pi}\right)^3\ln\frac{|k_z l|+5\pi/8}{|k_z l|} - \left(\frac{8}{5\pi}\right)^2\frac{1}{|k_z l|} + \frac{4}{5\pi}\frac{1}{(k_z l)^2}\right]\right\}. \tag{4}$$

Let us now consider two limiting cases: long-wave and short-wave, using expression (4). In the long-wave limit ($|k_z l| << 1$), there is:

$$\operatorname{Im} g_s^{-1}(k_z,\ \omega) = -\frac{\rho b_s^2}{10\pi}\gamma^0\omega\left[\left(3-8\xi+\frac{16}{3}\xi^2\right)(k_z l)^2\ln\frac{1}{|k_z l|}\right.$$
$$\left.-\left(1-\ln\frac{20}{3\pi}\right)\left(2-4\xi+\frac{8}{3}\xi^2\right)(k_z l)^2\right]. \tag{5}$$

In the short-wave limit ($1 << |k_z l| << k_m l$), one obtains:

$$\operatorname{Im} g_s^{-1}(k_z,\ \omega) = -\frac{\rho b_s^2}{2\pi}\gamma^0 \omega \frac{4}{9}\left[\frac{5}{\pi} - \frac{16}{\pi}\xi + \pi\xi^2\right]|k_z l|. \tag{6}$$

It is known that the imaginary part of the generalized susceptibility is proportional to the average energy dissipated per unit time in the system [11]. Consequently, the imaginary part of the inverse generalized susceptibility of screw dislocation, found in the present section, determines an additional dissipation of energy due to bending dislocation vibrations.

3. Damping of short-wave bending dislocation vibrations

In the short-wave case ($1 << |k_z l| << k_m l$), the sound waves damping coefficients (2) can be approximately written as $\gamma_t(x) \approx (4/3\pi)\gamma^0 x$ and $\gamma_l(x) \approx (\pi/6)\gamma^0 x$. The expression for the imaginary part of the screw dislocation inverse generalized susceptibility in a dissipative crystal [6]; taking into account the major terms with respect to k terms, let us write:

$$\operatorname{Im} g_s^{-1}(k_z,\ \omega) = -\frac{\mu b_s^2}{2\pi}\omega \int_{|k_z|}^{k_m} k dk \left[\frac{3k_z^2 c_t^2 \gamma_t}{(c_t^2 k^2 - \omega^2)^2 + 4\omega^2\gamma_t^2} + \frac{4k_z^2 c_t^2 \gamma_l - 8k_z^2 c_l^2 \gamma_t}{(c_l^2 k^2 - \omega^2)^2 + 4\omega^2 \gamma_l^2}\right]. \tag{7}$$

Here taking into account only the terms corresponding to the dislocation bending vibrations, μ is shear modulus of the crystal. Using dimensionless variable $x = kl$ in (7), one can obtain:

$$\operatorname{Im} g_s^{-1}(k_z,\omega) = -\rho b_s^2 \gamma^0 \omega (k_z l)^2 \left[\frac{2}{\pi^2}\int_{|k_z l|}^{k_m l} \frac{x^2 dx}{\left(x^2 - \omega^2/\omega_t^2\right)^2 + \left(\frac{8}{3\pi}\gamma^0 \omega/\omega_t^2\right)^2 x^2} - \xi\left(\frac{16}{3\pi^2} - \frac{1}{3}\xi\right)\int_{|k_z l|}^{k_m l} \frac{x^2 dx}{\left(x^2 - \omega^2/\omega_l^2\right)^2 + \left(\frac{\pi}{3}\gamma^0 \omega/\omega_l^2\right)^2 x^2}\right],$$

where $\omega_t = c_t/l$, $\omega_l = c_l/l$. After integrating, let us obtain, taking into account the main summands:

$$\operatorname{Im} g_s^{-1}(k_z,\omega) = -\frac{3}{4\pi}\rho b_s^2 \omega_t^2 (k_z l)^2 \operatorname{arctg}\frac{8\gamma_0 \omega}{3\pi\omega_t^2 |k_z l|} + \left(\frac{16}{\pi^3} - \frac{1}{\pi}\xi\right)\rho b_s^2 \omega_t^2 (k_z l)^2 \operatorname{arctg}\frac{\pi\gamma_0\omega}{3\omega_l^2 |k_z l|}. \tag{8}$$

At low frequency values, expression (8) is written as follows:

$$\operatorname{Im} g_s^{-1}(k_z,\omega) = -\left(\frac{2}{\pi^2} - \frac{16}{3\pi^2}\xi + \frac{1}{3}\xi^2\right)\rho b_s^2 \gamma_0 \omega |k_z l|. \tag{9}$$

Result (9) practically coincides with result (6) obtained in section 2.

4. Damping of long-wave bending dislocation vibrations

Using the result of [6], let us write the expression for the imaginary part of the screw dislocation inverse generalized susceptibility in a dissipative crystal:

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$$\operatorname{Im} g_s^{-1}(k_z,\ \omega) = -\omega B_s - \frac{\mu b_s^2}{2\pi}\omega \int_{|k_z|}^{k_m} kdk \left[\frac{1}{(c_t^2k^2-\omega^2)^2+4\omega^2\gamma_t^2} \left(\frac{\omega^4\gamma_t}{c_t^2k^2} \right.\right.$$

$$\left. + 3k_z^2c_t^2\gamma_t - 4\frac{k_z^4c_t^2\gamma_t}{k^2} - 6\frac{k_z^2\omega^2\gamma_t}{k^2} + 8\frac{k_z^4\omega^2\gamma_t}{k^4} \right) + \frac{1}{(c_l^2k^2-\omega^2)^2+4\omega^2\gamma_l^2}$$

$$\left.\times\left(4k_z^2c_t^2\gamma_l - 4\frac{k_z^4c_t^2\gamma_l}{k^2} - 8k_z^2c_l^2\gamma_t + 8\frac{k_z^2\omega^2\gamma_t}{k^2} + 8\frac{k_z^4c_l^2\gamma_t}{k^2} - 8\frac{k_z^4\omega^2\gamma_t}{k^4} \right)\right]. \tag{10}$$

Here B_s is the drag coefficient of rectilinear screw dislocation moving with a constant velocity [2-4], the linear approximation with respect to attenuation coefficients γ_t and γ_l was used. In the integral (10), let us pass to dimensionless variable $x = kl$ and note the rapid decrease of the integrand as one approaches the upper limit. Then the integral can be represented in the form:

$$\int_{|k_zl|}^{k_ml} xdx[\ldots] = \int_0^{k_ml} xdx[\ldots] - \int_0^{|k_zl|} xdx[\ldots] \approx \int_0^{\infty} xdx[\ldots] - \int_0^{|k_zl|} xdx[\ldots]. \tag{11}$$

Let us also note that in expression (10), one can neglect the terms containing k_z^4/k^4. The first integral on the right-hand side of (11) can be calculated using the theory of residues, similar to the calculation in [6]. In calculating the second integral, the authors take into account that in the long-wave approximation ($|k_zl| << 1$) from the formulas for the attenuation coefficients of sound waves (2), it follows that $\gamma_t \approx (1/5)\gamma^0 x^2$, $\gamma_l \approx (4/15)\gamma^0 x^2$. As a result, for the imaginary part of the screw dislocation inverse generalized susceptibility in a dissipative crystal, taking into account only the main terms in powers of k_zl, one may obtain:

$$\operatorname{Im} g_s^{-1}(k_z,\ \omega) = -\omega B_s - \frac{\rho b_s^2}{8}\omega|\omega| + \frac{\rho b_s^2}{20\pi}\gamma^0\frac{\omega^3}{\omega_t^2}$$

$$+ \rho b_s^2\omega\left(C_1\frac{\omega_t^2}{2|\omega|} + C_2\frac{\gamma^0}{10\pi} - C_3\frac{\gamma^0}{10\pi}\ln\frac{\omega_t}{|\omega|} \right)(k_zl)^2, \tag{12}$$

where C_1, C_2, C_3 are positive constants of the order of unity that depend weakly on the Poisson's ratio. In expression (12), the first term with B_s corresponds to the drag of a rectilinear screw dislocation moving with a constant velocity. The second term with $\omega|\omega|$ corresponds to the radiation friction of a rectilinear dislocation. The third term with $\gamma^0\omega^3$ corresponds to the damping of the oscillations of a rectilinear dislocation in a dissipative medium. The fourth term with $(k_zl)^2$ corresponds to the damping of the long-wave bending vibrations of a screw dislocation, the terms in parentheses correspond to radiation loss, to drag in a dissipative medium, and to their interference contribution. Let us note that the first three terms of formula (12) coincide with the result of [6], in which bending vibrations of the dislocation were not taken into account. At the same time, the result (12) can not be compared with the results of an analysis of screw dislocation low-frequency bending vibrations since in this paragraph it was assumed that $\omega >> 1$ Hz.

Let us consider the attenuation of long-wave ($|k_zl| << 1$) bending vibrations of a screw dislocation at low and medium frequencies ($\omega << c_t|k_z|$ or $\omega/\omega_t << |k_zl|$), where $\omega_t = c_t/l \sim 10^{10}\ s^{-1}$. Then the expression for the imaginary part of the edge dislocation inverse generalized susceptibility in a dissipative crystal [6] takes the form:

$$\begin{aligned}
\operatorname{Im} g_s^{-1}(k_z,\omega) = -\omega B_s - \frac{\mu b_s^2}{2\pi}\omega \int_{|k_z|}^{k_m} kdk \Bigg[\left(\frac{1}{\omega_t^4 (kl)^4} + \frac{2\omega^2}{\omega_t^6 (kl)^6} \right) \Bigg(3k_z^2 c_t^2 \gamma_t - 4\frac{k_z^4 c_t^2 \gamma_t}{k^2} \\
+ 12\frac{k_z^2 \omega^2 \gamma_t^3}{c_t^2 k^4} - 16\frac{k_z^4 \omega^2 \gamma_t^3}{c_t^2 k^6} - 6\frac{k_z^2 \omega^2 \gamma_t}{k^2} + 8\frac{k_z^4 \omega^2 \gamma_t}{k^4} \Bigg) + \left(\frac{1}{\omega_l^4 (kl)^4} + \frac{2\omega^2}{\omega_l^6 (kl)^6} \right) \\
\times \Bigg(4k_z^2 c_t^2 \gamma_l - 4\frac{k_z^4 c_t^2 \gamma_l}{k^2} - 16\frac{k_z^2 \omega^2 \gamma_t^2 \gamma_l}{c_t^2 k^4} + 16\frac{k_z^4 \omega^2 \gamma_t^2 \gamma_l}{c_t^2 k^6} - 8k_z^2 c_l^2 \gamma_t \\
+ 8\frac{k_z^2 \omega^2 \gamma_t}{k^2} + 8\frac{k_z^4 c_l^2 \gamma_t}{k^2} - 8\frac{k_z^4 \omega^2 \gamma_t}{k^4} \Bigg) \Bigg]. \qquad (13)
\end{aligned}$$

Substituting expressions (3) into formula (13) and integrating under condition $|k_z l| << 1$, let us obtain the following, taking into account the major terms:

$$\begin{aligned}
\operatorname{Im} g_s^{-1}(k_z,\,\omega) = -b_s^2 \omega B_s - \frac{\rho b_s^2}{10\pi}\gamma^0 \omega \Bigg[\left(3 - 8\xi + \frac{16}{3}\xi^2 \right)(k_z l)^2 \ln\frac{1}{|k_z l|} \\
- \left(2 - 4\xi + \frac{8}{3}\xi^2 \right)(k_z l)^2 - \left(2\xi - \frac{8}{3}\xi^2 \right)\frac{\omega^2}{\omega_l^2} + \left(\frac{4\pi}{5}\xi - \frac{515}{45\pi}\xi^2 \right)\frac{\omega^2}{\omega_l^2}|k_z l| \Bigg]. \qquad (14)
\end{aligned}$$

Result (14) in the low-frequency limit $\omega \to 0$ coincides with result (5) obtained in section 2.

5. Conclusion

The results obtained in this paper are most applicable to metals in the low temperature range. Numerical estimates show that the contribution of screw dislocation bending vibrations drag reaches 10% in the total coefficient of screw dislocation drag. The investigations carried out in this work can be used to study the influence of external effects on the crystal plasticity. This study is also important in the design and manufacture of machines and mechanisms for operation at low temperatures.

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