

# Analytical approaches to optimizing system "Semiconductor converter-electric drive complex"

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**Abstract.** In the electric drives of the machine-building industry, the problem of optimizing the drive in terms of mass-size indicators is acute. The article offers analytical methods that ensure the minimization of the mass of a multiphase semiconductor converter. In multiphase electric drives, the form of the phase current at which the best possible use of the "semiconductor converter-electric drive complex" for active materials is different from the sinusoidal form. It is shown that under certain restrictions on the phase current form, it is possible to obtain an analytical solution. In particular, if one assumes the shape of the phase current to be rectangular, the optimal shape of the control actions will depend on the width of the interpolar gap. In the general case, the proposed algorithm can be used to solve the problem under consideration by numerical methods.

## 1. Introduction

The level of development of the modern element base, for example, a power converter [1] technology and microprocessor control systems remove the need for the selection ("standard" or fixed) of voltages and currents at the inputs and outputs of the power elements, which opens up additional previously ignored possibilities for improving mass-size characteristic due to the variation of "nominal" and other parameters (the number of phases, the form of the linear density of the skinned current) [2]. This is mostly relevant for a number of executive mechanisms of the machine-building industry. Further, the weight and size characteristics of the electric machine's components, which are optimal for the design of a single machine, may not be the best when operating it, for example, from a regulated power source [3].

A lot of work has been devoted to the optimization of the electric drive's power part [4], but the problem does not appear to have been resolved[5]. The analysis of the most important works is made for its objective evaluation, devoted to the optimization of the semiconductor converter-electric drive complex.

## 2. Statement of the optimization problem

The traditional methods for selecting the power of electric drives for general industrial installations usually come from known load diagrams of the electric drive and are limited only by the permissible provision for using the electric machine for heating, overload capability (at the maximum permissible torque), maximum speed, etc. [6].

In servo electric drives, the task looks more complicated due to the significant influence of the dynamic parameters of the electromechanical converter on the quality of the processes in the electric drive and in most cases its solution is not unique [7]. Here the most common criteria is positioning



time [8,9], using electric drive with the best generalized indicators of the pickup type is:

$$P = \frac{T_n^2}{J}. \quad (2.1)$$

Quality (nominal angular acceleration) is:

$$D = \frac{T_n}{J}. \quad (2.2)$$

Other optimization criteria are also not excluded, in particular, taking into account the presence of compliance in the mechanical transmission - precision with the use of phased optimization [10].

The requirements for the electric drive make it necessary to formulate the optimization criteria in another way at the development stage of the electric drive. Thus, it is proposed in [11] to take the maximum of the electromagnetic torque in specified dimensions, as optimization criterion, by improving the geometry of the design of electric motors, and the optimization parameters are the radius of the rotor and the number of pole pairs. Here, the valve motor with ferrite and rare-earth magnets is shown. There is a valve motor with ferrite magnets with a length of the magnetic circuit of  $l = 120$  mm, an external diameter -  $D = 120$  mm, a coefficient characterizing the relationship between iron and magnetic material -  $v = 2\pi r l_m / \pi R^2 = 0,12$  and induction in the gap -  $B_r = 0,33$  T. The value of the electromagnetic torque was  $6.6$  Nm, and the optimum number of pole pairs and the rotor diameter are equal to five and  $D_p = 84$  mm, respectively. A similar calculation was carried out for a valve motor with excitation from rare-earth magnets, with the induction assumed to be  $B_r = 1$  T, and coefficient  $v = 0,05$ . The calculation showed that with the optimum number of poles 4 and with diameter  $D_p = 66$  mm, the torque reaches a value of  $11.4$  Nm.

The joint work of the converter and the synchronous jet machine were taken into account in [12]. The maximum of the electromagnetic torque was used as the criterion, by optimizing the geometric dimensions of the machine. By varying the ratio of the diameter of rotor  $D_p$  to the value of outer diameter  $D$ , it was possible to achieve a maximum with  $D_p/D = 0,6$  with the ratio of inductances  $L_d/L_q = 10$  [13].

The minimum cost for the active materials of the converter - power drive system [14] was taken into account by the introduction of optimization criterion  $q = Q/T_n$ , where  $Q$  is the mass of active materials in the elements of the electric drive,  $T_n$  is the nominal electromagnetic torque of the electric machine. At the same time, the shape of the triangle formed by MDS vectors in a generalized electric machine was optimized taking into account the unit costs for each term. The optimization results for a number of specific electric drives with different power supplies [15,16] have shown that the need form of the torque triangle formed by the MDS vectors depends on the unit costs for the active materials both in the electrical machines themselves and in the power supplies of the stator (armature) circuit and the excitation. The efficiency of matching the power circuits of the electric drive and the power source [17], which are optimal in terms of weight and size, largely depends on the value of the unit costs for the power source: it is the highest for electric drives with small value of unit costs for the power source and, conversely, effect is insignificant and often even absent in the case of power sources with high unit costs [18].

These variants of optimization of the electric drive's power part assume a sinusoidal form of the linear density of the skinned current. In practice, the form of the skinned current is determined by the design of the machine in traditional electric motors: in AC motors (synchronous and asynchronous), it is a sinusoid, in a DC motor - a rectangle. In non-traditional electric drives, due to the multiphase nature of the machine and the presence of electric energy converter between the network and the electric drive, this form can be arbitrary [19] when the number of phases of the stator winding is increased and each of these windings can be fed independently of the others. There is an additional possibility, redistributing the values of the losses in phases at a particular instant of time, and increasing the electromagnetic torque of the electric drive.

The statement of the optimization problem of the electromechanical system can be considered correct, if the following are indicated and justified: optimization criteria, optimization parameters, limit and functional connect.

### 3. Calculation and experiment

The optimization criteria will be chosen so as to provide the best mass-dimensional parameters of the electric drive.

As an optimization criterion let us take [20]:

$$q = \frac{S}{\int_{-\tau}^{+\tau} x^2 \cdot dt}. \quad (3.1)$$

Criterion  $q$  is proportional to the ratio of the magnitude of the electromagnetic torque to the value of losses in the stator winding at single axial length of the rotor, the circumference of the stator boring, the active resistance of the stator winding. It is convenient in that it has zero dimension with respect to the value of the stator current since both the numerator and the denominator are equally dependent (in the second) on this current.

### 4. Results of optimization

In the general case, it is more convenient to look for MDS curve ( $y(t)$ ) in the gap rather than the shape of the linear density of skinned current ( $x(t)$ ). So the functional will have the form:

$$T = \frac{1}{2} \cdot \int_{-\tau}^{+\tau} (y(t) \cdot y'(t) + y(t) \cdot y'(t)) \cdot dt. \quad (4.1)$$

Under the limiting, it is:

$$y \leq y_{max};$$

$$\frac{1}{2\tau} \cdot \int_{-\tau}^{+\tau} (y')^2 dt \leq A. \quad (4.2)$$

Such problem belongs to the isoperimetric class, in which it is required to find the extremum of the functional in the presence of so-called isoperimetric conditions. As is known, these problems can be reduced to problems on the conditional extremum, by introducing new unknowns. To obtain the basic necessary condition, it is necessary to find an auxiliary functional (the Lagrange functional):

$$T = \int_{-\tau}^{+\tau} \left( \frac{1}{2} (y(t) \cdot y'(t) + y(t) \cdot y'(t)) + \lambda_1 \cdot (y'(t))^2 + \lambda_2 \cdot y'(t) \right) dt, \quad (4.3)$$

where  $\lambda_1, \lambda_2$  are Lagrangian constants, and write the Euler equation for it. Arbitrary constants  $\lambda_1, \lambda_2$  in the general solution of the system of Euler equations, constants  $C_1, C_2$  are found from the isoperimetric and limit conditions:

$$y(-\tau) = F_{max}, \text{ where } F_{max} - \text{the maximum value of the MDS.}$$

In the particular case of a rotor with a circular cross section, when there are no negative sections of the curve of specific forces  $z(t)$ , there is no sign of the modulus in the functional, so it is:

$$T = \int_{-\tau}^{+\tau} ((y(t) \cdot y'(t) + \lambda_1 \cdot (y'(t))^2 + \lambda_2 \cdot y'(t))) dt. \quad (4.4)$$

The functional reduces to form  $T = \int F(y, y') dt$ , then the Euler equation has first integral  $F - y' \cdot F_{y'} = A$ .

The extremal of the functional is according to an arbitrary function satisfying a condition and this in complete agreement with the electromagnetic torque is equal to zero for the case of a rotor of circular section.

In the general case of finding the extremal of the functional, it is necessary to consider its subdifferential and solve the problem using the theory of convex analysis, while the solution becomes rather cumbersome, which makes it difficult to use at a later stage of designing the electric drive.

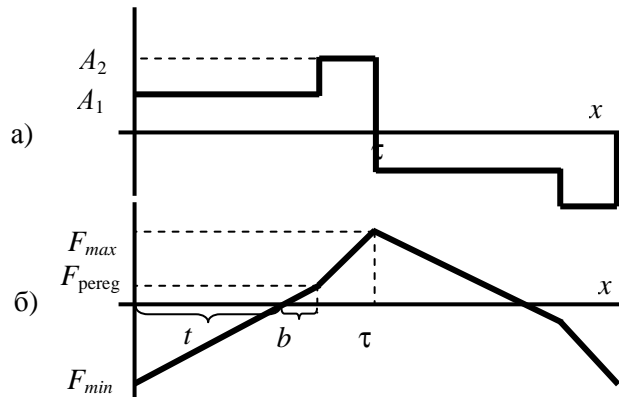
Let us show the simplest form of linear load  $A(x)$  in an electric machine when value  $A = \text{Const}$  on each segment of the pole span of the electric drive (Figure 1, a) [21].

In this case, it is necessary to find the optimal relationship between the values of the linear load in the pole zone and the zone of the interpolar gap, which in the general case may be unequal to each other.

Let us do preliminary mathematical calculations. Figure 1 shows the accepted law of the change the linear density of the skinned current along the stator bore, where  $A_1$  is the current density of phase

windings which located above the pole of electric drive, and  $A_2$  is the current density of the phase windings which located above the interpolar gap. Then the amplitude value of the MDS  $F_{max}$  can be determined from expression [22]:

$$F = \int A \cdot dx + C, \quad (4.5)$$



**Figure 1.** Distribution of linear load  $A$ , magnetomotive force  $F$  along stator boring  $x$ .

where  $C$  is selected from condition  $|F_{max}| = |F_{min}|$ . In this case:

$$F_{max} = (A_1 \cdot \alpha_\delta + A_2(1 - \alpha_\delta)) \cdot \frac{1}{2} \tau. \quad (4.6)$$

For definiteness, let us assume that  $A_1 \cdot \alpha_\delta \cdot \tau > A_2 \cdot (1 - \alpha_\delta) \cdot \tau$ .  $t$  is the distance (in fractions of the polar pitch) along the stator bore from  $F_{min}$  to  $F = 0$  (Figure 1, b), then:

$$t = \frac{1}{2 \cdot A_1} \cdot [A_1 \cdot \alpha_\delta + A_2(1 - \alpha_\delta)]. \quad (4.7)$$

So  $F_{pereg}$  is the MDS value at which the slope of the MDS curve changes:

$$F_{pereg} = F_{max} - A_2 \cdot (1 - \alpha_\delta) \cdot \tau = \frac{1}{2} \cdot [A_1 \cdot \alpha_\delta - A_2(1 - \alpha_\delta)] \cdot \tau. \quad (4.8)$$

The length of section  $b$  at which the tangential forces are directed opposite to the forces creating the resultant electromagnetic torque (Fig. 1b) is:

$$b = (\alpha_\delta - t) = \frac{1}{2} \cdot [\alpha_\delta - n \cdot (1 - \alpha_\delta)], \quad (4.9)$$

where  $n = A_2/A_1$ .

In a system linear in the magnetic field, the electromagnetic torque is proportional to the MDS [23]:

$$T = 2 \left( \frac{1}{2} \cdot F_{max} \cdot t \cdot A_1 - \frac{1}{2} \cdot b \cdot F_{pereg} \cdot A_1 \right) \cdot \tau \cdot l_\delta \cdot \frac{D}{2} \cdot \frac{\mu_0}{L_\delta} \quad (4.10)$$

$$T = \frac{1}{2} \cdot A_1 \cdot A_2 \cdot \alpha_\delta \cdot (1 - \alpha_\delta) \cdot \tau^2 \cdot l_\delta \cdot \frac{D \cdot \mu_0}{L_\delta}, \quad (4.11)$$

where  $l_\delta$  is the length of the magnetic core in the axial direction;  $D$  is the diameter of the rotor;  $L_\delta$  is the air gap.

The square of the effective value of the current is:

$$I_{quad.m..}^2 = (I_a^2 \cdot T \cdot \alpha_\delta + I_b^2 \cdot T \cdot (1 - \alpha_\delta)) \cdot \frac{1}{T} = \frac{\tau^2}{m^2 \cdot w_\phi^2} \cdot [A_1^2 \cdot \alpha_\delta + A_2^2 \cdot (1 - \alpha_\delta)], \quad (4.12)$$

where  $m$  is the number of phases;  $w_\phi$  is the number of turns per phase;  $f = 1/T$  – rotor speed.

To indirectly take into account the limitations on heating, it is more convenient to divide the electromagnetic torque by the square of effective value of current.

In this case, the minimum size of the electric drive can be taken into account by the criterion [24]:

$$q = \frac{T}{I^2} = \frac{1}{2} \cdot \frac{n \cdot \alpha_\delta \cdot (1 - \alpha_\delta)}{\alpha_\delta + n^2 \cdot (1 - \alpha_\delta)} \cdot l_\delta \cdot \frac{D}{L_\delta} \mu_0. \quad (4.13)$$

The problem of finding an extremal is reduced to finding the extremum of a function of two variables. The necessary conditions for the extremum can be written in the form:

$$\frac{\partial q}{\partial n} = 0; \quad \frac{\partial q}{\partial \alpha_\delta} = 0. \quad (4.14)$$

Let us find the partial differential coefficient of the function with respect to  $n$  and equate it to zero:

$$\frac{\partial q}{\partial n} = \frac{1}{2} \cdot \alpha_\delta \cdot (1 - \alpha_\delta) \cdot \frac{\alpha_\delta - n^2 \cdot (1 - \alpha_\delta)}{(\alpha_\delta + n^2 \cdot (1 - \alpha_\delta))} \cdot l_\delta \cdot \frac{D}{L_\delta} \cdot \mu_0 = 0. \quad (4.15)$$

The stationary point (the possible extremum point) is:

$$n = \sqrt{\frac{\alpha_\delta}{1 - \alpha_\delta}}. \quad (4.16)$$

If one substitutes the obtained value of  $n$  into the function, then one obtains:

$$q = \frac{1}{4} \cdot \sqrt{\alpha_\delta \cdot (1 - \alpha_\delta)} \cdot l_\delta \cdot \frac{D}{L_\delta} \cdot \mu_0. \quad (4.17)$$

Criterion  $q$  depends only on one variable  $\alpha_\delta$ ; therefore, when finding the second stationary point, it is sufficient to find the derivative and equate it to zero:

$$\frac{dq}{d\alpha_\delta} = \frac{1}{8} \cdot \frac{(1 - \alpha_\delta) - \alpha_\delta}{\sqrt{\alpha_\delta \cdot (1 - \alpha_\delta)}} \cdot l_\delta \cdot \frac{D}{L_\delta} \cdot \mu_0. \quad (4.18)$$

Solving equations, let us find stationary points:

$$\alpha_\delta = 0,5; n = 1.$$

A sufficient condition for the existence of an extremum of a function of two variables is:

$$\left(\frac{\partial^2 q}{\partial n \partial \alpha_\delta}\right)^2 - \frac{\partial^2 q}{\partial n^2} \cdot \frac{\partial^2 q}{\partial \alpha_\delta^2} < 0. \quad (4.19)$$

In this case:

$$\begin{aligned} \frac{\partial^2 q}{\partial n \partial \alpha_\delta} &= \frac{1}{8} \\ \frac{\partial^2 q}{\partial n^2} &= -\frac{1}{16} \\ \frac{\partial^2 q}{\partial \alpha_\delta^2} &= -\frac{1}{2}. \end{aligned}$$

On the basis of this, one can conclude that the stationary points are extreme, and since:

$$\begin{aligned} \frac{\partial^2 q}{\partial n^2} &< 0 \\ \frac{\partial^2 q}{\partial \alpha_\delta^2} &< 0. \end{aligned}$$

So with such parameters of the pole span and the ratio of the excitation current to the armature current, the maximum of the functional is satisfied.

The physically obtained results can be explained as follows:

- for an arbitrary value of pole span  $\alpha_\delta$  and  $n$  corresponding to the MDS created by the stator windings will not change its sign, so the specific forces along the pole span have one sign, as a result, the electromagnetic torque takes a maximum value;

- similarly, for a given value of the ratio of the excitation current to armature current  $n$ , the extreme value of the functional is satisfied at value  $\alpha_\delta$  for which the MDS does not change its sign [25].

## 5. Conclusion

An analytical method of searching the optimum shape of the phase current by the criterion of the best use of the "semiconductor converter-electric drive complex" is proposed. It is shown that under certain restrictions on the phase current form, it is possible to obtain an analytical solution. In particular, if one assumes the shape of the phase current to be rectangular, the optimal shape of the control actions will depend on the width of the interpolar gap. In the general case, the proposed algorithm can be used to solve the problem under consideration by numerical methods.

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