

# Analysis of the effectiveness of various cross-sections in large-span post-tensioned ceilings

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**Abstract.** The correct construction of large span, slim post-tensioned concrete slabs is conditioned by an appropriate cross-section selection. It is generally accepted that the thinnest slab can be constructed using the full cross-section as the largest compression stress storage. However, completely different cross-sections may help to overcome large spans. The paper presents the results of the computational analysis of several types of cross-sections (full, with internal relieving inserts and ribbed) in the application to a post-tensioned slab with a span of 15.0m. Based on the results presented, appropriate conclusions were drawn.

## 1. Introduction

Post-tensioned concrete slabs prestressed with unbounded tendons have been used in the world for several decades. They were initially applied in the USA, Hongkong, Singapore, Australia, and later also in European countries. The architectural trend that has been going on for many years is aimed at creating large, support-free spaces in buildings. On the other hand, obtaining the highest possible height of rooms makes it necessary to use the thinnest floors possible. All of this forces designers to thorough analysis and refinement of constructional solutions that have been used for years and to look for new varieties.

Improvements in the efficiency (increase in span) of ceilings are also sought in ceilings made from pre-tensioned hollow core slabs. A larger span can be obtained using an additional concrete topping [1], or using additional unbounded tendons inside the steel beams supporting the pre-tensioned concrete hollow core slabs [2]. An interesting solution of a semi-prefabricated ceiling on prestressed concrete boards, increasing the spread of traditional reinforced concrete floors at the same thickness, was published in paper [3].

Undoubtedly, much better results in the construction of large span ceilings can be obtained in post-tensioned concrete technology, using unbounded post-tensioning. These tendons with small diameters, a low friction coefficient and small curvature radiuses allow to shape long, high-curved routes. In addition to applications in ceilings, their advantages allow shaping slender rod structures in buildings, e.g. [4]. In Poland, in recent years, several very thin prestressed ceiling slabs with unbounded tendons have been designed and successfully constructed [5].

There are many types of cross-sections as well as design variations of floor slabs. Each of them has its advantages and disadvantages. It is difficult to say unequivocally that one is better, and another is worse. Its usefulness and effectiveness will depend on the conditions of use. In this paper, the results of the analysis of three types of cross-sections will be presented: full, with internal relieving inserts, and ribbed, in application to a one-way simply supported slab with a span of 15.0m. Post-tensioned one-way concrete slabs are more often used in buildings due to their simpler execution in relation to two-way slabs.



## 2. Design assumptions

Computational analysis has been applied to the one-way flat roof slab shown in Figure 1. The slab is freely supported on four walls with a thickness of 250mm. The axial spacing of the walls in the shorter direction is 15.0m while in the longer 50.0m. By omitting the end zones, the slab therefore works unidirectionally. The slab thickness was originally set to 400mm (1/37.5 of the span).

The following slab loads were assumed: slab self-weight -  $25\text{kN/m}^3$ , layers on the roof -  $1.5\text{kN/m}^2$ , load of devices on the roof and under the slab -  $2.5\text{kN/m}^2$  (live load), snow -  $1.0\text{kN/m}^2$  (live load).

Figure 2 shows the prestressing profiles adopted from the principle of using the maximum tendon sag. For this purpose, a minimal structural distance of the tendons from the lower edge in the middle of the span (50mm) and prestressing in the middle of the cross-section's center of gravity over the supports have been assumed. For cross-sections with two axes of symmetry (table 1 - cross-sections 1, 2 and 3) a 150mm sag was obtained, for ribbed cross-sections (with a raised center of gravity) 199mm were obtained for cross-sections no. 4 and 5 and 229mm for cross-section no. 6.

It was assumed that the slab will be made of concrete class C30/37 according to [6]. For prestressing, commonly used for unbounded tendons 0.6" strands with a cross-sectional area of  $150\text{mm}^2$  and steel strength of 1860MPa were assumed. The value of initial force in tendon is 220kN.

Two design situations were considered:

- the initial design situation (just after prestressing) - the dead load of the slab was taken into account and the prestressing was reduced by immediate losses of 10% and the increasing factor equal to 1.05 [6],
- final design situation - all operating loads, prestressing after time-dependent losses assumed as 20% of the initial force and a decreasing factor equal to 0.95 were taken into account.

As a criterion for correctness of the cross-section's work, apart from limiting deflection, boundary stresses in concrete were assumed. Maximum stresses were limited to:

- for compression in the initial design situation -  $0.6f_{ck}(t_0)$ , where  $f_{ck}(t_0)$  means the characteristic compressive strength of concrete during prestressing.  $f_{ck}(t_0)$  was assumed as  $0.8f_{ck}$ . Ultimately, the limit compressive stresses amount to  $\sigma_{cc,lim} = 0.6 \times 0.8 \times 30 = 14.4\text{MPa}$ .
- for tension in final design situation -  $f_{ctm}$  that is the average concrete tensile strength of 2.9MPa. In the initial situation, this value was reduced by 20%. This limitation results from the necessity of ensuring lack of cracking. The cross-sections cracked at the bottom, resulting in a significant reduction in stiffness, cause a radical increase in deflections. However, cracking the top in the initial stage is not dangerous for the work of the structure, but it requires the use of an appropriate upper ordinary reinforcement in the slab.

The total long-time deflection of the ceiling was limited to 1/300 of the span, i.e., to 50mm. The long-time deflection was determined in accordance with [7], using the increasing factor to the deflection

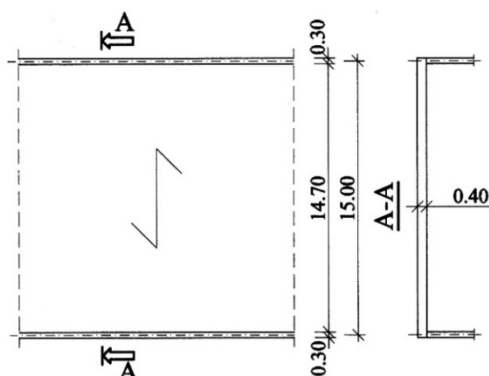


Figure 1. Slab geometry.

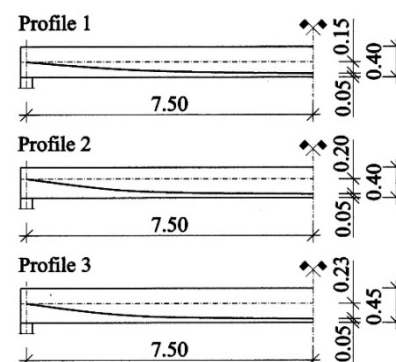


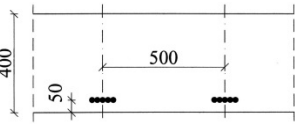
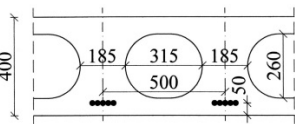
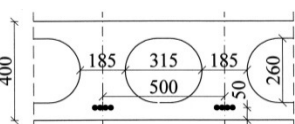
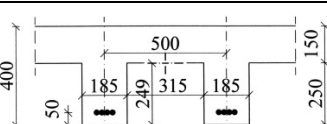
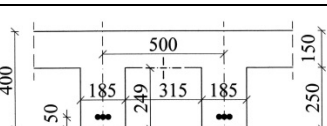
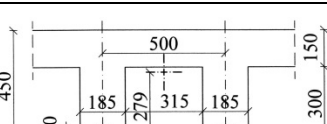
Figure 2. Tendons profiles.

calculated from the elastic analysis. The value of this coefficient is 3.0 for dead loads and prestressing and 1.5 for live loads.

Four cross-sections were considered in the analysis:

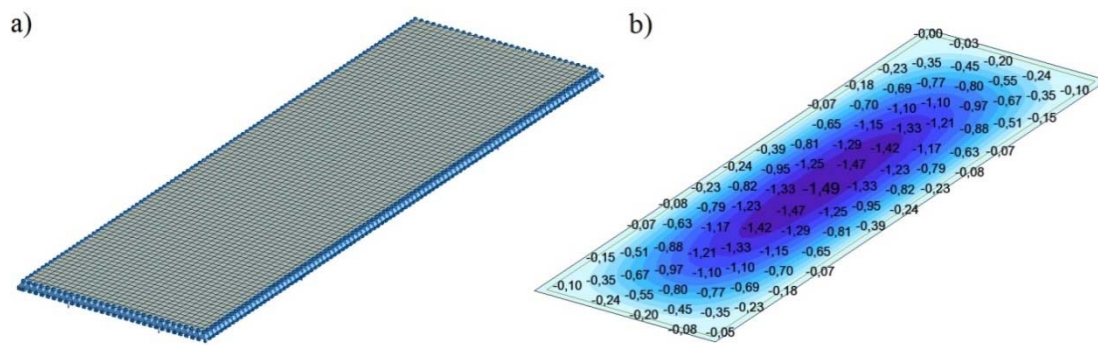
- solid cross-section with a height of 400mm (no. 1 in table 1),
- cross-section with a height of 400 mm with relieving inserts *Slim Line* of the *Cobiax* system (no. 2), with a height of 260mm and a diameter of 315mm. The spacing of the inserts was assumed to be 500mm in the transverse direction and 350mm in the load-bearing direction. The reduction of the slab's weight due to the use of inserts amounted to 226kg/m<sup>2</sup>, i.e., 22.6% while the reduction of the moments of inertia 11.4%,
- ribbed cross-section with a height of 400mm (no. 4 and 5) with a 150mm-thick slab and ribs with a width of 185mm in a 500mm spacing,
- ribbed cross-section with a height of 450mm (no. 6) with a 150mm-thick slab and ribs with a width of 185mm in a 500mm spacing.

Table 1 Analyzed cross-sections as well as their properties.

Cross-section		Tendon profile (see fig.2)	Cross-section area m <sup>2</sup>	Moment of inertia m <sup>4</sup>	Slab weight reduction		Moment of inertia reduction %
No.	Shape				kg/m <sup>2</sup>	%	
1		1	0.400	0.00533	-	-	-
2		1	0.2652	0.00472	226	22.6	11.4
3		1	0.2652	0.00472	226	22.6	11.4
4		2	0.242	0.00305	394	39.4	42.8
5		2	0.242	0.00305	394	39.4	42.8
6		3	0.261	0.00434	473	42.0	42.8

### 3. Calculation

In order to perform the calculations, the slab calculation model was performed in the Finite Element Method (Figure 3a). Linear hinged support was assumed on the perimeter, with only one longest edge deprived of the possibility to move horizontally. Surface 4-node, square finite elements with 500mm



**Figure 3.** FEM model of analyzed slab (a), elastic slab deflection for cross-section no. 1 (b).

sides were used. The prestressing load was applied as a substitute horizontal load on the edges and a vertical surface load determined according to the formula:

$$w = \frac{8P \cdot \delta}{L^2} \quad (1)$$

where  $P$  is the force in the tendon,  $\delta$  is the tendon sag and  $L$  is the tendon length. Computational analysis was carried out in the range of linear-elastic concrete work.

#### 4. Calculation results

The results of the computational analysis carried out for all 6 considered cross-sections are summarized in table 2. In section no. 1 (solid slab) correct results were obtained by using 5 prestressing strands every 500mm. The use of relieving inserts in cross-section no. 2, with identical prestressing, allowed to reduce the deflection (in relation to the solid cross-section no. 1) from 44.7 to 23.4 mm. However, the correct deflection value (44.7mm) can be obtained by reducing the number of tendons in the bundle from 5 to 4 (cross-section no. 3). The positive effect of internal inserts relieving both the deflection of the ceiling and the amount of prestressing. This is due to a much smaller reduction in cross-section stiffness (11.4%) in relation to the slab's weight reduction (22.6%).

In the case of a ribbed cross-section (no. 4) with a similar height (400mm) and with 4 tendons in the rib (as in the case of a properly working cross-section with relieving inserts), a considerable exceeding of the compressive stresses was obtained in the initial design situation (19.9 in relation to 14.4MPa). Due to the large drop in the moment of inertia (42.8%) and surface area (39.4%), this cross-section proved to be too weak to transfer such a prestressing. Subsequently, the number of strands in this cross-section was reduced in the rib to 3 (section 5), obtaining the appropriate stresses in the initial design situation. Such prestressing, however, turned out to be too small in a situation of use where tensile stresses of 4.25 MPa were obtained. This means cracking of the cross-section. In the case of a cracked cross-section, one should expect long-time deflections significantly greater than 3 times the

Table 2 Values of concrete stresses and slab deflection.

Cross-section No.	Initial design situation		Final design situation		Elastic deflection	Long-time deflection
	Upper stress	Bottom stress	Upper stress	Bottom stress		
	MPa		MPa			
1	3.83	6.39	10.2	-1.93	14.9	44.7
2	3.68	11.8	10.4	1.99	7.8	23.4
3	4.69	7.67	11.2	-1.42	14.9	44.7
4	-1.22	19.9	-	-	-	-
5	1.06	11.7	9.14	-4.25	1.26	63.0
6	0.85	10.5	7.13	-1.72	6.1	18.3

elastic deflection. Assuming an analogy to cracked reinforced concrete cross-sections, this deflection can be up to 5 times greater than elastic deflection, i.e., it can be 63mm. This is already a value much greater than the 50mm limit. This cross-section proved to be too weak to the assumed geometrical conditions and ceiling loads. Its height has been increased to 450mm (cross-section 6). For such a cross-section, correct values were obtained with 3 tendons in the rib. The use of ribbed cross-section and increasing its height from 400 to 450mm allowed to reduce the number of strands in the bundle with 5 for solid cross-section and 4 for cross-section with internal relief inserts up to 3.

## 5. Conclusions

Based on the performed computational analysis, it was found that:

- Although the solid cross-section constitutes the largest compressive stress storage (important in case of prestressed structures), the use of internal relieving inserts with the same prestressing allows to reduce the slab's deflection. The same effect (as to deflections) can be achieved by reducing the amount of prestressing (in the analyzed case by 20%). Although it is commonly recognized that the full cross-section, due to its highest compressive stress storage capacity, allows the smallest thickness of the slab, the internal relieving inserts seem to be a good solution when overcoming a large span with slender slabs. Their advantage lies in a much smaller reduction in the stiffness of the cross-section in relation to the slab's weight reduction.
- Ribbed cross-sections, due to a significant reduction in stiffness and surface area, are much weaker compared to solid ones and those with internal relief inserts. Although they allow a significant reduction in the amount of prestressing (in the analyzed case - by 20 and 40% respectively), this occurs at the expense of increasing the height.

## 6. References

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