Output Error Analysis of Planar 2-DOF Five-bar Mechanism

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Abstract. Aiming at the mechanism error caused by clearance of planar 2-DOF Five-bar motion pair, the method of equivalent joint clearance of kinematic pair to virtual link is applied. The structural error model of revolute joint clearance is established based on the N-bar rotation laws and the concept of joint rotation space, The influence of the clearance of the moving pair is studied on the output error of the mechanis. and the calculation method and basis of the maximum error are given. The error rotation space of the mechanism under the influence of joint clearance is obtained. The results show that this method can accurately calculate the joint space error rotation space, which provides a new way to analyze the planar parallel mechanism error caused by joint space.

1. Introduction

For the planar 2-DOF Five bar parallel mechanism, its members can be regarded as rigid components, and the end errors of the output rod are derived from the rod length tolerance and joint clearance, while the joint clearance leads to the uncertainty of the output attitude of the end effector of the mechanism. However, the proper movement of the deputy clearance ensures the normal operation of institutions necessary conditions. The clearance is too small so that the mechanism may stuck and unable to run. And a large amount of space not only can not guarantee the accuracy of movement, but also causes the movement of side impact mechanism[1].Therefore, it is necessary to study the problem of kinematic pairs. Aiming at the precision analysis of planar parallel mechanism, many scholars have made a great deal of researches, Yu et al.[2] proposed a method of compensating clearance of the kinematic pair by optimizing the 3-RRR parallel mechanism. Zhang et al.[3] combined the principle of virtual work and optimization techniques and put forward to analyze the pose error of planar mechanism by numerical calculation method; Ting [4-5] studied the influence of the clearance error of the kinematic pair of the crank-slider mechanism and the four-bar mechanism with the end effector on the output uncertainty by establishing the kinematics model. Compared with the tedious and complicated numerical calculation and analysis method, it is simpler and More effective to establish the error model.

2. Model of joint clearance for planar 2-DOF Five-bar linkages

The planar 2-DOF Five-bar parallel mechanism includes two active input joints. The clearance error model is shown in Figure 1.One of the obvious characteristics of the rotating joint is that the diameter of the pin is different from the diameter of the hole in the rod. It is assumed that the pin and the hole

are always in contact with each other in the research, There is no deformation or rupture between the pin and the hole in each joint, so the center distance between the pin and the hole is kept at a constant value. In the paper, the tiny distance caused by the rotating joint's clearance can be equivalent to a virtual link in the error model.The length of this virtual link is different from the radius between the hole in the rotating joint and the pin, which is also 1/2 of the distance of the rotating joint clearance. Each joint clearance adds a degree of freedom to the mechanism which results in an uncertainty of the mechanism's output orientation.
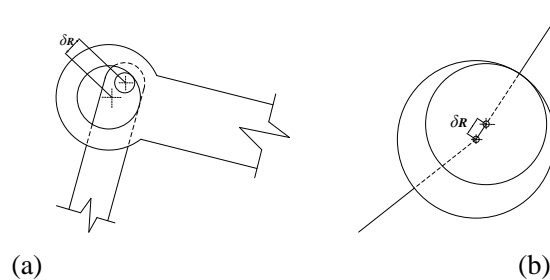

(a)   (b)

Figure 1. Model of joint clearance for the revolute pair

3. N-bar rotatability laws and joint rotation space

a. N-bar rotatability laws

Based on the N-bar rotatability laws proposed by the Ting[6-7], Since a clearance link is essentially a short link ,a clearance link can have full revolution with respect to any link in the chain. The full rotatability of any clearance links is the key issue of the clearance link model.

Let lmax be the longest rod in the Five rods. Let lmin1 be the shortest rod. Let lmin2 be the second shortest rod, And the remaining two rod lengths are l_m and l_n respectively, and the following classification is made:

ClassI: $l_{max}+l_{min1}+l_{min2} \leq l_m+l_n$
ClassII: $l_{max}+l_{min1}+l_{min2} \geq l_m+l_n$
ClassIII: $l_{max}+l_{min1}+l_{min2} = l_m+l_n$

An N-bar linkages has full rotatability if and only if
1) The linkage belongs to classIchain, and
2) There is one and only one non-input joint between any pair of long links.

b. Joint rotation space

Joint rotation space is the input variable curve range. The movement of each joint is a single reciprocating cycle. For the whole mechanism, the motion of joint affects each other, and the motion state of the mechanism can be clearly expressed by the movement of all joints. A point in the rotation space of any joint of the N-bar linkage corresponds to a configuration of the mechanism.Therefore, the joint rotation space is a synthesis of the possible configurations of all mechanisms of an institution. The characteristics of the movements of a mechanism can be expressed in the form of a map of the characteristics of a mechanism so as to make the complex movement clearly and concisely presented.

c. Invariant link rotatability[8]

Invariant link rotatability suggests that in any planar single-loop chain formed by a set of links and revolve joints, the rotatability between any two links, whether they are adjoined or not, is independent of the order of connection. Because of the invatiant link rotatability, all clearance links can be grouped together. This reduces the complexity of the linkage.The independent rotation of each clearance link may be used to model the position uncertainty under worst as well as non-worst situations.

4. Influence of kinematic joint clearance on output error

The joint clearance is equivalent to a short rod, so that the Five-bar parallel mechanism is equivalent to a Ten-bar parallel mechanism, as shown in Figure 2.The number of degrees of freedom also increase from 2 to 7. However, with the design of the actual mechanism, the total length of all the equivalent

clearance rod is much less than the sum of the rod length of the original ideal rod. Therefore, the equivalent parallel mechanism does not change the type of the mechanism.If both inputs are applied to the ideal connecting rod, the equivalent parallel mechanism has seven degrees of freedom, and the additional five degrees of freedom are due to the uncertainty of the joint space motion.It can be considered that because these five arbitrary degrees of freedom are applied to the five clearance links (short links), the parallel mechanism still has complete rotatability, which in turn leads to the existence of the entire mechanism error rotation space.

The ideal rod length of the planar Five-bar parallel mechanism is a, b, c, d, e, respectively, and the rod length of the equivalent clearance rod is δ,namely,

$$a + e < b + c + d \tag{1}$$

The equivalent joint clearance connecting rod,

$$a + (e + 5\delta) < b + c + d \tag{2}$$

Therefore, according to the mechanism classification criterion mentioned above, the type of the planar parallel mechanism doesn't change after the equivalent clearance connecting rod is added, which ensures that the complete rotation between any original ideal connecting rod is stable and unchangeable.According to the Invariant link rotatability, given the input θ_2, θ_3, the lengths of l_1 and l_2 can be obtained from the input rod and the rod length of the frame,

$$l_1^2 = a^2 + e^2 - 2ae \cdot \cos\theta_2 \tag{3}$$

$$l_2^2 = b^2 + l_1^2 - 2bl_1 \cdot \cos(\theta_3 + \alpha) \tag{4}$$

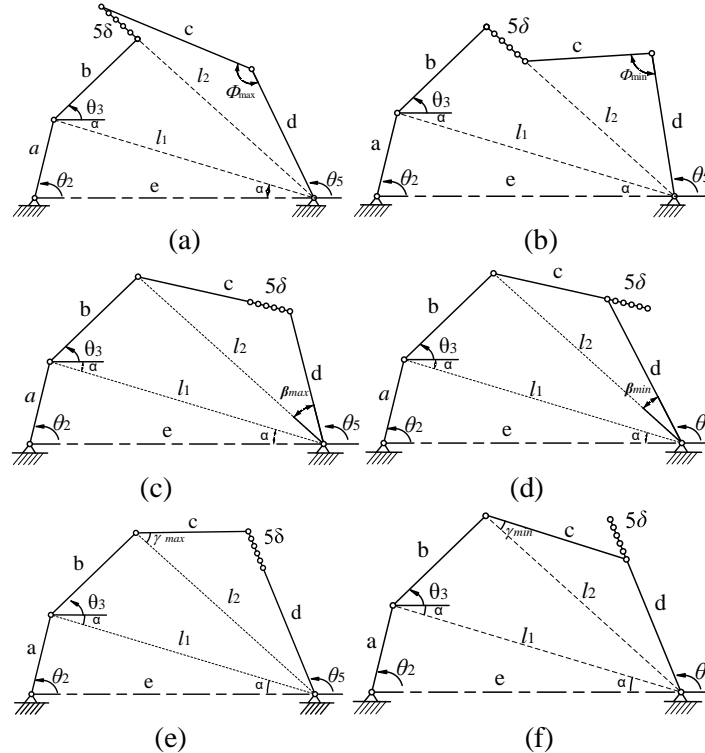The output range of the error between the two links of the mechanism can be obtained,

Figure 2. Output error of planar 2-DOF Five-bar

According to the joint rotatability ,The possible rotation range between links c and d is Φ_{max} and Φ_{min},

$$\phi_{\max,\min} = \arccos\left[\frac{c^2 + d^2 - (l_2 \pm 5\delta)^2}{2cd}\right] \tag{5}$$

According to the joint rotatability ,The possible rotation range between links d and l_2 is β_{max} and β_{min},

$$\beta_{\max,\min} = \arccos\left[\frac{d^2 + l_2^2 - (c \pm 5\delta)^2}{2d \cdot l_2}\right] \tag{6}$$

According to the joint rotatability ,The possible rotation range between links c and l_2 is γ_{max} and γ_{min},

$$\gamma_{max,min} = \arccos\left[\frac{c^2 + l_2^2 - (d \pm 5\delta)^2}{2c \cdot l_2}\right] \qquad (7)$$

Through the above calculation, according to any joint clearance on the direction of the output rod error is equivalent,Planar Five-bar parallel mechanism in the rotary joint space under the influence of the angular error between the rod is easy to get.

5. Solution of working space error caused by joint space

Based on the established error geometry model, according to the rotation mechanism of the link mechanism used in this paper, the mechanism shown in Figure 3.(a) can finally evolve into Figure 3.(b). Based on the given organization parameters, the error space of the equivalent planar parallel mechanism as shown in Figure4. can be precisely calculated. Table 1 shows the dimension parameters.
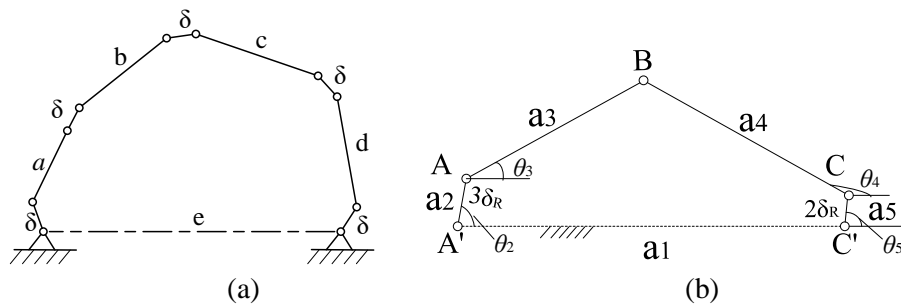

Figure 3. Joint clearance equivalent to Five parallel rod linkage

Table 1

δ_R(mm)	a1	a2	a3	a4	a5
0.76	304.80	2.29	152.40	274.32	1.52

The equivalent rod length of the rotating joint in the designed mechanism is estimated as $\delta_R = 0.76$mm. Based on the theory of N-bar rotation mentioned above and the classification of Five-bar mechanism, as shown in Figure 3, the two shortest rods are all connected rods. The four rotating joints that are involved in the rotation are all rotating pairs. According to whether the input joint is applied, the following two situations exist:

a.   The two input joints are not adjacent, as shown in Figure 4, θ_2 and θ_5 are input joints. When A'A and AB are collinear, the Five-bar linkages degenerate into a four-bar linkages. When A'A and AB coincidence, it also degenerates into a four-bar ring, the ring equation can be:

$$a_1 + a_5 \cdot e^{i\theta_5} + a_4 \cdot e^{i\theta_4} = (a_2 + a_3) \cdot e^{i\theta_2} \qquad (8)$$

$$a_1 + a_5 \cdot e^{i\theta_5} + a_4 \cdot e^{i\theta_4} = |a_2 - a_3| \cdot e^{i\theta_2} \qquad (9)$$

The above equation (8) can be obtained by Euler's formula:

$$\begin{cases} a_1 \sin 0 + a_5 \sin\theta_5 + a_4 \sin\theta_4 = (a_2 + a_3)\sin\theta_2 \\ a_1 \cos 0 + a_5 \cos\theta_5 + a_4 \cos\theta_4 = (a_2 + a_3)\cos\theta_2 \end{cases} \qquad (10)$$

The above equation (9) can be obtained by Euler's formula:

$$\begin{cases} a_1 \sin 0 + a_5 \sin\theta_5 + a_4 \sin\theta_4 = |a_2 - a_3|\sin\theta_2 \\ a_1 \cos 0 + a_5 \cos\theta_5 + a_4 \cos\theta_4 = |a_2 - a_3|\cos\theta_2 \end{cases} \qquad (11)$$

Eliminate θ_4, the above equation can be arranged as:

$$[a_1^2 + a_2^2 + a_3^2 + 2a_2 a_3 - a_2 a_5 \cos(\theta_2 - \theta_5) - 2a_3 a_5 \cos(\theta_2 - \theta_5)$$
$$- 2a_1 a_2 \cos\theta_2 - 2a_1 a_3 \cos\theta_2 + 2a_1 a_5 \cos\theta_5 - a_4^2 + a_5^2] = 0 \qquad (12)$$

$$[a_1^2 - a_4^2 + a_5^2 + 2a_1 a_5 \cos\theta_5 + |a_2 - a_3|^2 - 2a_5 \cdot |a_2 - a_3| \cdot \cos(\theta_2 - \theta_5) - 2a_1 \cdot |a_2 - a_3| \cdot \cos\theta_2] = 0 \qquad (13)$$

It can be seen from the above formula that the range of all input variables of the equivalent Five-bar mechanism A'ABCC' kinematic chain is determined by Eqs.(12)and(13). The range of all the

input variables is called joint rotation space. The calculated error rotation space is shown in Figure 4(a).

b.   When the two input joints are adjacent,the θ_4 and θ_5 are input joints. When A'A and AB are collinear, the Five-bar ring degenerates into a Four-bar ring. When A'A and AB coincide, it also degenerates into a Four-bar ring, the ring equation (8) (9) finishing as follows:

$$[a_1^2 - a_2^2 - 2a_2a_3 - a_3^2 + 2a_4a_5\cos(\theta_4 - \theta_5) + 2a_1a_5\cos\theta_5 + 2a_1a_4\cos\theta_4 + a_4^2 + a_5^2] = 0 \qquad (14)$$

$$[a_1^2 - 2a_2^2 + 2a_2a_3 + a_5^2 + 2a_4a_5\cos(\theta_4 - \theta_5) + 2a_1a_5\cos\theta_5 + 2a_1a_4\cos\theta_4 - a_3^2 + a_4^2] = 0 \qquad (15)$$

From equations (14) and (15), the error rotation space of two input joints can be calculated as shown in Figure 4(b).
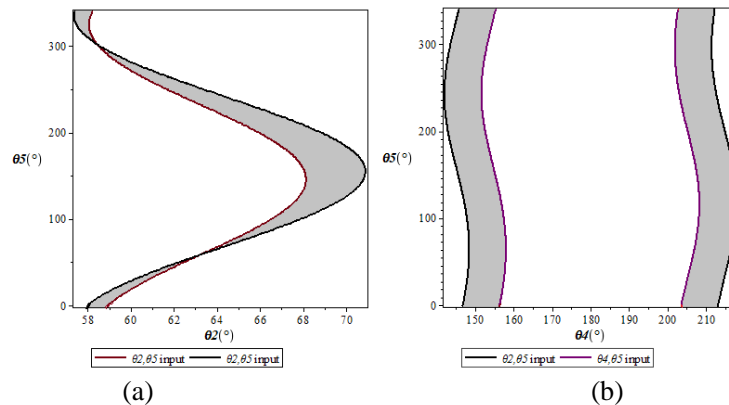

(a)   (b)

Figure 4. Planar 2-DOF Five-bar error rotation space

6. Conclusion

In this paper, the output error of 2-DOF Five-bar planar parallel mechanism due to the uncertainty caused by the rotation of the joint was analyzed. The kinematic accessory joint space is equivalent to a virtual short rod. Establish 2-DOF Five-pole kinematic joint clearance error model. The influence of joint clearance on the output error of planar Five-bar mechanism is analyzed separately, and the calculation result of maximum error is obtained.An example of the length parameter of the mechanism is given. The accurate output rotation space due to the joint clearance is obtained. Due to the existence of joint space, the output error of planar parallel mechanism is inevitable. The method proposed in this paper selects reasonable joint clearance error value when designing planar parallel mechanism, and makes the error region within the controllable safety range has guiding significance.

7.   Reference

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