

Alternative Constraint Handling Technique for Four-Bar Linkage Path Generation

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Abstract. This paper proposes an extension of a new concept for path generation from our previous work by adding a new constraint handling technique. The propose technique was initially designed for problems without prescribed timing by avoiding the timing constraint, while remain constraints are solving with a new constraint handling technique. The technique is one kind of penalty technique. The comparative study is optimisation of path generation problems are solved using self-adaptive population size teaching-learning based optimization (SAP-TLBO) and original TLBO. In this study, two traditional path generation test problem are used to test the proposed technique. The results show that the new technique can be applied with the path generation problem without prescribed timing and gives better results than the previous technique. Furthermore, SAP-TLBO outperforms the original one.

1. Introduction

In daily life and manufacturing, it is common to see many applications of four-bar linkage mechanisms. Steering wheels, oil wells, double wishbone suspensions, windshield wipers, automatic doors, and rock crushers are applications of such a mechanism. One task for designing a four-bar linkage is a path generation problem. This synthesis problem is aimed at finding significant link lengths of the mechanism to achieve a point on a coupler link moving along the desired path [1-16]. The problem is usually converted to be an optimization problem, which is posed to find the dimensions of a mechanism and some other parameters. The objective function of this problem is to minimize the errors between predefined target points and positions of one particular point on the connecting link, while the constraints are imposed for preventing an unusable mechanism solution. This design problem is the kinematic design of a mechanism involving position analysis. The path generation problem is of interest by many researchers due to the advantage of using such a mechanism such as work reported by Slesongsom and Bureerat[16]. Almost all of the previous work tried to improve the design performance of the path synthesis problem in both the constraint handling [10-11, 16] and the performance of an optimizer [1-9, 16]. The path synthesis optimization problem normally composes of two types of constraints. The first constraint is the sequence of shortest link (crank) being able to rotate with a complete revolution in either direction, which agrees with the Grashof criterion. The second constraint is set to obtain unknown timings of the crank in a sequential order. From the



literature, it is found that a simple exterior penalty function technique is the most popular technique used to deal with constraints [1-7, 9-11], but it still lacks in efficiency as reported in [8, 11, 13, 16]. Very recent work in [16] proposed a new technique to neglect the second constraint from the optimization problem and a first constraint is still tackle with the traditional penalty technique, which is found to give the considerably better results than the traditional exterior penalty technique. The new technique can improve the design results but, in turn, increases computational time. The traditional path generation test problems used in this work are the problems without prescribed timing constraints. Researchers rarely studied and developed a new technique for constraint handling technique, therefore, the present work is conducted to show that a new technique proposed in [16] can be effectively used with a new constraint handling technique. The method is one kind of traditional penalty technique, which has been improved for optimal truss sizing [17]. It is an efficient technique for constraint handling. At the present study the efficiency of this technique for handling constraint of the path generation problem is needed to study. The optimizers used in this study are meta-heuristics (MHs) namely: Self-Adaptive Population Size Teaching-Learning Based Optimization (SAP-TLBO) [16], and original teaching - learning based optimization (TLBO) [18]. Performance testing of both optimizers for solving the mechanism synthesis is described with descriptive statistics, which focus on minimum (best), maximum (worse), mean value, and standard deviation of the obtained solutions. The rest of this paper is organized as follows. Position analysis of a four-bar linkage, an optimization problem, and a constraint handling technique are briefly presented in Section 2. The numerical experiments are shown in Section 3 while the design results are given in Section 4. The conclusions of this study are drawn in Section 5.

2. Methodology

2.1. Position analysis

A model of a four-bar linkage, shown in Fig. 1, for position analysis is an important computation required for path synthesis. It is composed of four simple links, which are paired by 4 revolute joints and one link is set as a frame. It is a constrained chain mechanism fully operated by one actuator. The design variables of four-bar linkage lengths are r_1 , r_2 , r_3 , and r_4 and other parameters, which shall give minimum error between the desire path (x_d, y_d) and actual trace points (x_p, y_p) on the coupler link. The coupler curve can be formed when a crank moves over a complete cycle. The coupler point coordinates in the global coordinate are shown in Fig.1 and can be expressed as

$$x_p = x_{O2} + r_2 \cos(\theta_2 + \theta_1) + L_1 \cos(\phi_0 + \theta_3 + \theta_1) \quad (1)$$

$$y_p = y_{O2} + r_2 \sin(\theta_2 + \theta_1) + L_1 \sin(\phi_0 + \theta_3 + \theta_1)$$

where x_{O2} and y_{O2} are the coordinates of the O_2 pin joint in the global coordinates.

ϕ_0 can be obtained by considering the couple link BCP using the cosine law, which is expressed as

$$\phi_0 = \cos^{-1} \left[\frac{L_1^2 + r_3^2 - L_2^2}{2L_1 r_3} \right] \quad (2)$$

The values of angles θ_3 , θ_4 , and γ for the known link lengths r_1 , r_2 , r_3 , and r_4 at any crank angle (θ_2) can be obtained by common kinematic analysis given in [16].

2.2. Optimization problem and constraint handling

The path generation problem without prescribed timing is transformed to an optimization problem with an objective function being assigned to minimize the error between targeted and obtained coupler curves. The position error is the sum of squares of distances between each P_d and the corresponding P_o , where P_d is a set of predefined points, and P_a is a set of obtained points from an input design vector of linkage dimensions and other parameters. The members of a design solution \mathbf{x} include r_1 , r_2 , r_3 , r_4 ,

L_1 , and L_2 , the global position of O_2 (x_{O2} , y_{O2}), and the angle of frame 1 (θ_1). For the without prescribed timing problem, θ_2^i is undefined and included in the design variables. The optimization problem without prescribed timing is then written as

$$\min f(\mathbf{x}) = \sum_{i=1}^N [(x_{d,i} - x_{p,i})^2 + (y_{d,i} - y_{p,i})^2] \quad (3)$$

subject to

$$\theta_2^1 < \theta_2^2 < \dots < \theta_2^i < \dots < \theta_2^N \quad (4)$$

$$\min(r_1, r_2, r_3, r_4) = \text{crank}(r_2) \quad (5)$$

$$2\min(r_1, r_2, r_3, r_4) + 2\max(r_1, r_2, r_3, r_4) < (r_1 + r_2 + r_3 + r_4) \quad (6)$$

$$\mathbf{x}_l \leq \mathbf{x} \leq \mathbf{x}_u \quad (7)$$

where $\mathbf{x} = \{x_{O2}, y_{O2}, r_1, r_2, r_3, r_4, L_1, L_2, \theta_1, \theta_2^i\}^T$, N is the number of predefined points (x_d, y_d) on the prescribed or target curve, and \mathbf{x}_l and \mathbf{x}_u are lower and upper bounds of a design vector \mathbf{x} respectively. The constraints (4) are set to obtain the values of θ_2^i being in ascending or descending order, which is considered if the problem is without prescribed timing. The constraints (5) and (6) correspond to the Grashof's criterion. Normally, an exterior penalty function is used to deal with constraints (5-7). It was however found to be inefficient for solving the above design problem [16], as a result, the unknowns θ_2^i are to be pre-determined and not treated as design variables for optimization. If the mechanism is a crank-rocker, which means constraint (5) and (6) are met. To tackle both constraints a new technique from [17] is used instead the traditional penalty constraint. Equation (5-6) is come from a common form as $g_i(\mathbf{x}) \leq g_{0,i}$. Then, a set of M input angle values (θ_2) equally spaced from 0 to 2π radian $T_2 = \{0, \dots, \theta_2^i, \dots, 2\pi\}$ is generated. With more number of T_2 and more efficient of the new constraint handling, better design results can be expected. Once the positions of point P corresponding to all values in T_2 are calculated, the objective function (3) is obtained as

$$f(\mathbf{x}) = \sum_{i=1}^N \min d_{ij}^2 \quad (8)$$

where $d_{ij}^2 = (x_{d,i} - x_{p,j})^2 + (y_{d,i} - y_{p,j})^2$ for $j = 1, \dots, M$. The detail of this technique can see in [16].

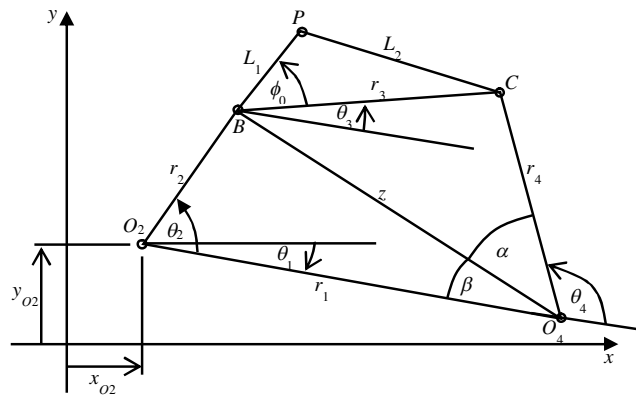


Figure 1. Four-bar linkage in the global coordinate system [16]

For detail of the constraint handling technique from [17] is showed here in brief. The constraint handling is similar technique with usual penalty technique, which is composed of two part is an

objective function combine with constraint functions multiply with switching parameter. The design variable is invalid the constraint a large real switching parameter is added to modify the objective function value (ON state). Otherwise, it is an "OFF state" state, which means switching parameter is 0. With this technique the constraints can tackle, the solution is obtain is a feasible solution. If the switching parameter is too small or too large, it can effect to finding solution of an optimizer. The inefficient of the traditional technique is found by [8, 11, 13, 16]. Alternative penalty technique has been proposed by [18], it is found is an efficient technique. This idea is created the new equivalent objective function by using four constraint handling rules as follow.

- 1) Any feasible solution has a better equivalent objective function value than any infeasible one;
- 2) Between two feasible solutions, one with the better objective function value has a better equivalent objective function value;
- 3) Between two infeasible solutions, one with a similar constraint violation has better equivalent objective function value; and
- 4) As infeasible solution with small constraint violation ($c_i < c_0 > 0$) is considered feasible in the optimisation process where c_0 is a small positive constraint tolerance.

The fourth rule is using to exploring infeasible solution nearby constraint boundary if the feasible solution cannot find. In construction of a new objective unconstraint function from rules 1-3 can be defined as

$$f_p(\mathbf{x}, c_0) = \begin{cases} f' = \frac{f(\mathbf{x})}{f(\mathbf{x}) + k} & \text{if } \max |c_i(\mathbf{x})| \leq c_0 \\ 1 + \max |c_i(\mathbf{x})| & \text{otherwise} \end{cases} \quad (9)$$

where $k > 0$ is a constant to be defined (for this paper $k = 10$), and c_0 is iteratively reducing to 0. Before using previous equation the constraint should rearrange firstly as follow.

$$c_i(\mathbf{x}) = g_i(\mathbf{x}) / g_{0,i} - 1 \leq 0 \quad (10)$$

The equation (10) is in form parameterless.

3. Numerical Experiment

Two without prescribed timing optimisation problem of four-bar linkage path generation is used to study the performance of the proposed design technique. The first one is path generation with prescribed timing with straight line path in vertical direction, while the second is a problem with an elliptic path [6].

Case-1 : Path generation without prescribed timing

Design variables are

$$\mathbf{x} = \{r_1, r_2, r_3, r_4, r_{px}, r_{py}, \theta_0, x_0, y_0, \theta_2^1, \theta_2^2, \theta_2^3, \theta_2^4, \theta_2^5, \theta_2^6\}^T.$$

Target points are

$$r_d^i = \{(20,20), (20,25), (20,30), (20,35), (20,40), (20,45)\}$$

Limitation of design variables:

$$5 \leq r_1, r_2, r_3, r_4 \leq 60$$

$$-60 \leq L_1, L_2 \leq 60$$

$$-60 \leq x_0, y_0 \leq 60$$

$$0 \leq \theta_0, x_0, y_0, \theta_2^1, \theta_2^2, \theta_2^3, \theta_2^4, \theta_2^5, \theta_2^6 \leq 2\pi$$

Case-2 : Path generation without prescribed timing

Design variables are

$$\mathbf{x} = \{r_1, r_2, r_3, r_4, L_1, L_2, \theta_0, x_0, y_0, \theta_2^1, \theta_2^2, \theta_2^3, \theta_2^4, \theta_2^5, \theta_2^6, \theta_2^7, \theta_2^8, \theta_2^9, \theta_2^{10}\}^T.$$

Target points are:

$$r_d^i = \{(20,10), (17.66, 15.142), (11.736, 17.878), (5, 16.928), (0.60307, 12.736), (0.60307, 7.2638), (5, 3.0718), (11.736, 2.1215), (17.66, 4.8577), (20, 10)\}.$$

Limitation of design variables are:

$$5 \leq r_1, r_2, r_3, r_4 \leq 80$$

$$-80 \leq L_1, L_2 \leq 80$$

$$-80 \leq x_0, y_0 \leq 80$$

$$0 \leq \theta_0, \theta_2^1, \theta_2^2, \theta_2^3, \theta_2^4, \theta_2^5, \theta_2^6, \theta_2^7, \theta_2^8, \theta_2^9, \theta_2^{10} \leq 2\pi$$

In order to measure the performance of the new design technique and the SAP-TLBO and original TLBO are used to tackle the design problem. The parameter settings of those methods are detailed as follows:

1. Self-Adaptive Population Size Teaching-Learning Based Optimization (SAP-TLBO) [16].
IRreset = 20, IRange = 5, $\delta = 1$.
2. Teaching-learning-based optimization (TLBO) [18]: Parameter settings are not required.

The definitions of terms and variables can be found in their original sources. For solving four-bar linkage path synthesis, the population size $n_p = 100$ is used for all algorithms, while the maximum number of function evaluations which is a termination criterion is set to be 50,000. Each algorithm is run for 30 times for performance comparison of search consistency.

4. Design Results

The design results from the various optimizers with the new synthesis technique are shown in Table 1. The table shows the number of successful runs (a run which an optimiser can find a feasible solution), the mean objective function values from 30 optimization runs, the worse result (max), the best result (min), and the standard deviation (std). Fig. 2-3 show the best path traced by the coupler point and illustrates the kinematic diagram of the best linkage. In Case-1, there are 6 target points. It was found that SAP-TLBO gives the best result (error = 0.0000598) and the best mean objective value (error = 0.031747), which is better than TLBO. The result of Case-2 showed SAP-TLBO gives the better result than TLBO in both mean (error = 0.059718) and min (error = 0.002465). Furthermore, the best results are better than the best results by [16] (Case-1: error = 0.04760 (mean), 0.00088 (min) respectively, Case-2: error = 1.644548 (mean), 0.027145 (best) respectively). The number of successful runs of all optimizers (present technique) is approximate equal to the previous work, but it higher than the traditional penalty technique [16]. The results concluded that SAP-TLBO with the new technique is better than those from using the traditional technique in both cases.

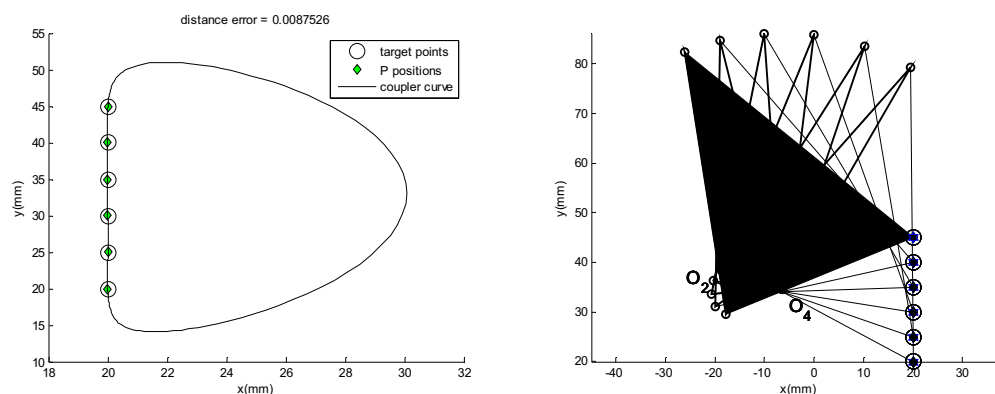


Figure 2. The best coupler curves and the best mechanism of Case-1.

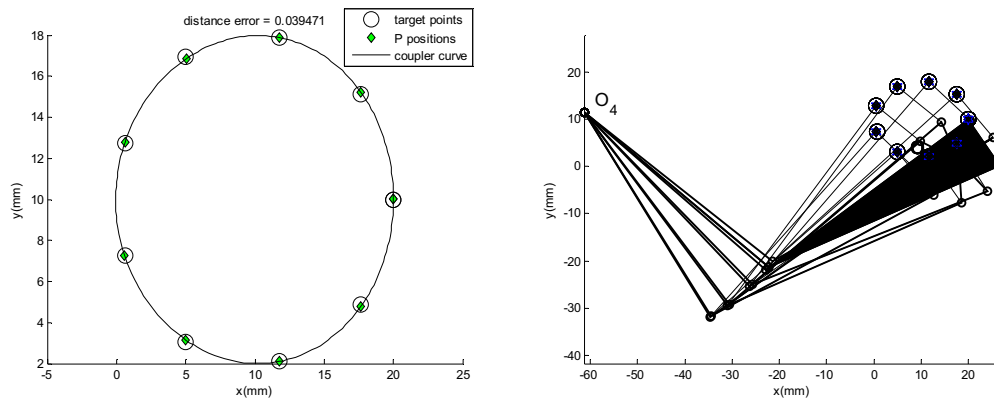


Figure 3. The best coupler curves and the best mechanism of Case-2.

Table 1. Comparative design results for Case-1 and Case-2 with a new technique.

Parameters	Case-1		Case-2	
	TLBO	SAP-TLBO	TLBO	SAP-TLBO
r_1	12.3163	9.3329	80.0000	79.8358
r_2	5.0000	5.0546	8.6507	8.9499
r_3	37.6311	53.5740	66.3060	53.5336
r_4	32.7912	51.7039	47.0738	50.8890
L_1	-59.5259	-40.8228	14.9526	-11.9790
L_2	-52.4788	-59.3263	71.9939	52.6266
x_0	-34.5185	-15.7705	18.0731	17.8703
y_0	40.8644	34.1240	22.4629	1.1295
θ_0	0.0907	2.6586	94.9288	172.5849
mean	0.063749	0.031747	0.209332	0.059718
max	0.718959	0.206311	0.839434	0.352928
min	0.000433	0.0000598	0.003037	0.002465
std	0.160546	0.060032	0.302877	0.081523
Success.*	23	20	27	27

* Success. = no. of successful runs

5. Conclusions

This paper proposes a new technique to find the optimum parameters of a four-bar linkage for path synthesis. This technique is combining a technique to remove timing constraints from the optimization and a new penalty technique for handling Grashof's criterion. It is found that the new technique with SAP-TLBO is outperformed the original TLBO and the new penalty technique can upgrade the results

from the previous study. However, it is recommended that the technique is an alternative technique for solving path generation problem without prescribed timing. For future work, a proper technique for handling Grashof's criterion should be investigated.

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References

- [1] Smaili A and Diab N 2007 *Mechanism and Machine Theory* **42**(1) 115–130
- [2] Cabrera J A, Nadal F, Muñoz J P and Simon A 2007 *Mechanism and Machine Theory* **42**(7) 791-806
- [3] Bulatović R R and Dordević S R 2009 *Mechanism and Machine Theory* **44**(1) 235–246
- [4] Nariman-Zadeh N, Felezi M, Jamali A and Ganji M 2009 **44**(1) 180–191
- [5] Acharyya S K and Mandal M 2009 *Mechanism and Machine Theory* **44**(9) 1784-1794
- [6] Lin W Y 2010 *Mechanism and Machine Theory* **45**(8) 1096–1107
- [7] Cabrera J A, Ortiz A, Nadal F and Castillo J J 2011 *Mechanism and Machine Theory* **46**(2) 127-141
- [8] Peñuñuri F, Peón-Escalante R, Villanueva C and Pech-Oy D 2011 *Mechanism and Machine Theory* **46**(10) 1335–1349
- [9] Khorshidi M, Soheilypour M, Peyro M, Atai A and ShariatPanahi M 2011 *Mechanism and Machine Theory* **46**(10) 1453–1465
- [10] Mateka S B and Gogate G R 2012 *Mechanism and Machine Theory* **52** 158–179
- [11] Ebrahimi S and Payvandy P 2015 *Mechanism and Machine Theory* **85** 189–204
- [12] Slesongsom S and Bureerat S 2015 *Research and Applications in Mechanical Engineering* **3**(2) 46-53
- [13] Phukaokaew W, Slesongsom S and Bureerat S 2016 *Proceeding of 7th TSME – ICOME conference*
- [14] Heo J C and Yoon G H 2013 *Mechanism and Machine Theory* **64** 18-38
- [15] Buskiewicz, J 2015 *Inverse Problem in Science and Engineering* **23** 818 -850
- [16] Slesongsom S and Bureerat S 2017 *Knowledge-Based Systems* **135** 180–191
- [17] Bureerat S and Pholdee N 2016 Optimal truss sizing using an adaptive differential evolution algorithm *Journal of Computing in Civil Engineering* **30**(2) doi:10.1061/(ASCE)CP.1943-5487.0000487
- [18] Rao R V, Savsani V J and Vakharia D P 2011 *Computer-Aided Design* **43**(3) 303-315