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Thermal resistance approach to analyse temperature distribution in thick hollow FGM sphere: under dirichlet boundary condition

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Abstract. A thick hollow FGM sphere with radially varying material property is considered under thermal loading. Thermal conductivity is assumed to vary radially following two different power functions separately and results for both have been analysed. Temperature distribution is obtained using Thermal Resistance Approach (TRA) where spatially varying thermal conductivity is assumed to be a discontinuous piece-wise function of radial distance rather than the continuous power functions. Whole body is considered to be constructed of layers having different material property but each layer is assumed to be homogeneous. Then temperature values are obtained considering each layer's thermal resistance and the total thermal resistance network. Results were compared with other established studies and results obtained by FEM.

1. Introduction

Functionally graded materials are a class of superior composite which are different from the traditional composite materials where variation of material properties along any direction is discontinuous but in FGMs this variation is continuous and smooth which allows researchers to tailor the gradient of material property variation with more flexibility. FGMs are used in various engineering fields because of their advantages over various metal alloys and composite materials to meet extreme material requirements. Many researchers have worked on FGMs since they were first introduced in Japan in 1984 [1]. Yahya et al [1] analysed the behaviour of thick FGM spheres under combined thermal and pressure loading. Jabbari et al [2] presented a general solution for the one dimensional steady state thermo-mechanical stresses in hollow thick porous FGM sphere. Jabbari et al [3] also developed an analytical method to obtain the solution for 2D transient thermal and mechanical stresses in a hollow FGM sphere and piezoelectric layers. S Faruqui et al [4] developed thermal resistance method for solving the temperature distribution problem in hollow FGM cylinders. S Karampour [6] presented solution for 2D thermal and mechanical stresses in poro-FGM spherical vessel. X L Peng and X F Li [7] analysed the thermo-elastic behaviour of cylindrical FGM vessels. Zimmerman R W and Lutz M P worked on thermal stresses and thermal expansion in uniformly heated FGM cylinders. In this paper temperature distribution in FGM sphere is analysed using thermal resistance approach. Results have been obtained for two different power functions of material gradient and also were compared with existing results in the literature. The advantage of this method over others is that results can be obtained for any type of material property variation where obtaining analytical solutions are much more complex due to nonlinearity.

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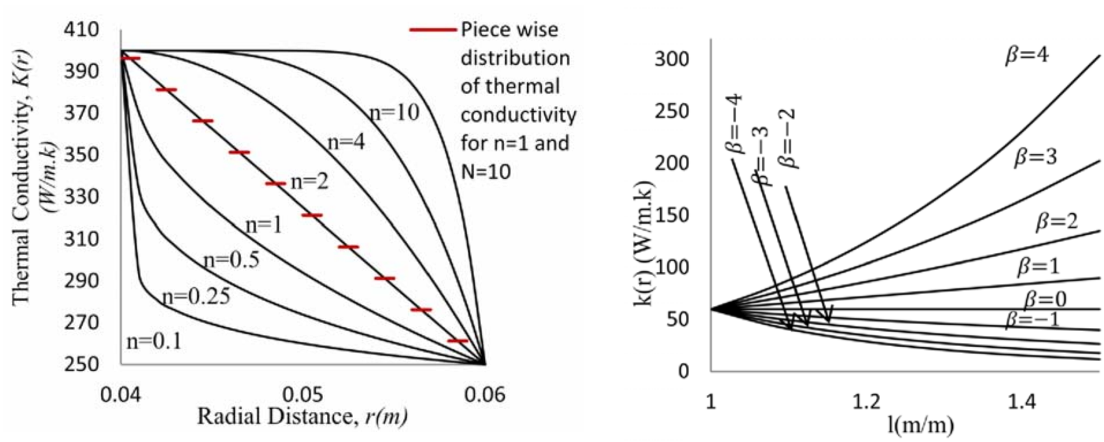


(a) (b)

Figure 1. (a)Radial distribution of $K_{fgm}(r)$ following power function equation (1) and an assumed piecewise distribution for Thermal Resistance Approach. (b) Radial distribution of $K_{fgm}(r)$ following equation (2).

2. Formulation of problem

A hollow spherical body is considered with inner radius r_i and outer radius r_o, thermal conductivity $K_{fgm}(r)$ is assumed to vary radially from k_i to $k_o$following any of the power functions of equation (1) and (2) where k_i and k_o are thermal conductivities of innermost and outermost surfaces respectively.

$$K_{fgm}(r) = k_i + (k_o - k_i)\left(\frac{r-r_i}{r_o-r_i}\right)^n \tag{1}$$

$$K_{fgm}(r) = k_c e^{\beta l} \tag{2}$$

In equation (1) "n" is the power law index and in equation (2) k_c is a material constant, "e" is the Euler constant, "β" is the inhomogeneity parameter and $l = \frac{r}{r_i}$.

2.1 Thermal problem and boundary condition

Taking spherical coordinate system into account and assuming the temperature of the spherical body varies only in radial direction and no internal heat generation the obtained governing equation is

$$\frac{d}{dr}\left(k_{fgm}(r) r^2 \frac{dt}{dr}\right) = 0 \tag{3}$$

Solutions were obtained under dirichlet boundary condition when temperature values at innermost and outermost surfaces of the body are known. These values were taken as $T(r_i) = T_i$ and $T(r_o) = T_o$ respectively.

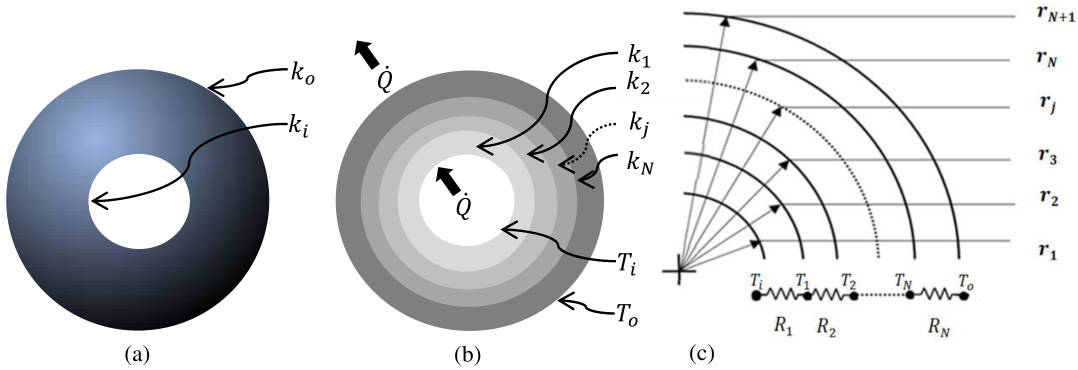


Figure 2. (a) Continuous distribution of k_{fgm}, (b) Layered distribution of k_{fgm} , (c) Thermal resistance network across the layers.

3. Proposed solution procedure (Thermal Resistance Approach)

For dirichlet boundary conditions a solution can be obtained by dividing the whole spherical body into layers as shown in Figure 2(b) and considering each layer a constituent element having homogenous material property throughout. Each element's thermal conductivity can be assigned to it from equation (1) or (2) by taking average of the values at innermost and outermost boundaries of that particular layer. The radial variation of thermal conductivity of the whole body is then assumed to be a piece-wise function of radial distance, r which is shown in Figure 1(a). If the spherical body is divided into 'N' number of layers or elements, each layer will have a thermal conductivity according to this piece-wise variation. After getting 'N' number of different thermal conductivities for 'N' different layers, thermal resistance can be computed for each of them. Thermal resistance R_j across the $\mathrm{j^{th}}$ layer can be determined using the following equation

$$R_j = \frac{r_{j+1} - r_j}{4\pi r_{j+1} r_j K_j} \tag{4}$$

Where $\mathrm{r_j}$ and $\mathrm{r_{j+1}}$ are inner and outer radii of the j^{th} layer respectively. The total thermal resistance across the inner and outer surface of the whole body can be calculated from equation (5) where $\mathrm{R_1, R_2, R_3}$, etc. are the thermal resistances of different layers.

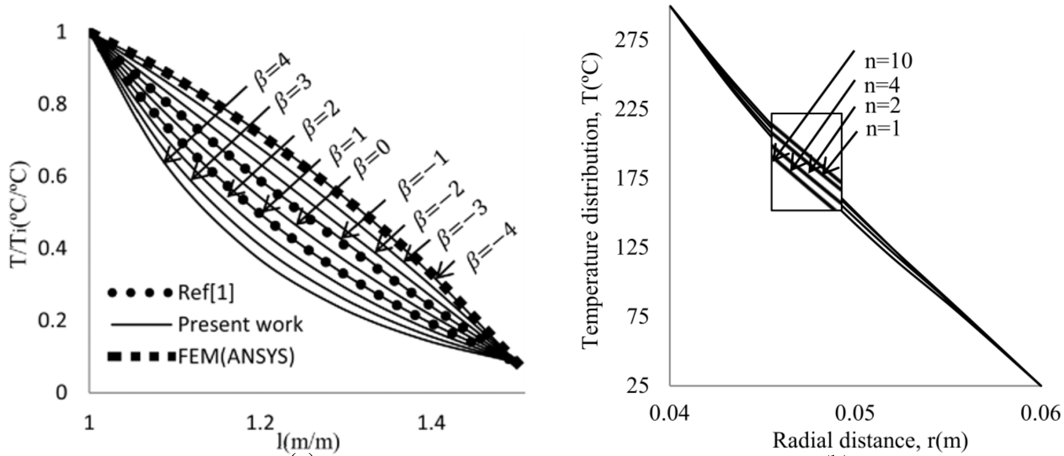
$$R_{total} = R_1 + R_2 + \cdots + R_j + \cdots + R_N \tag{5}$$

However, in this approach it is assumed that there is no heat loss or additional thermal contact resistance at the contact surface of two adjoin layers. Temperature values are obtained at each of the contact surfaces of constituent layers. Temperature at contact surface of the $\mathrm{j^{th}}$ and $\mathrm{(j+1)^{th}}$ element at $\mathrm{r = r_{j+1}}$ is computed from (6). When $\mathrm{T(r_i) = T_i}$ and $\mathrm{T(r_o) = T_o}$,

$$T_{j+1} = \frac{(R_1+R_2+R_3+\cdots+R_{j-1}+R_j)T_o + (R_{j+1}+R_{j+2}+R_{j+3}+\cdots+R_{N-1}+R_N)T_i}{R_{total}} = \frac{\left(\sum_1^j R\right)T_o + \left(\sum_{j+1}^N R\right)T_i}{R_{total}} \tag{6}$$

3.1 Finite element analysis and validate FEM

A geometry specimen was modeled using commercial FEM solver ANSYS for a comparative study of the proposed method. Due to symmetry only one eight of a sphere was considered. The FEM model consisted of tota 1238 elements. Variation in material property was obtained by dividing the specimen into 5 homogeneous layers each layer having same but distinct material property. The meshing region as well as the obtained temperature distribution is shown in Figure 4(a).



(a) (b)

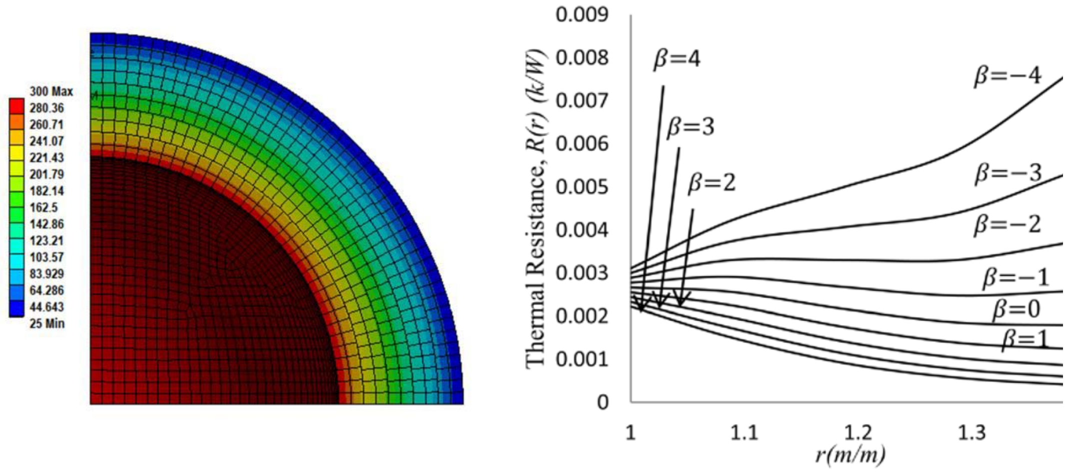
Figure 3. (a) Radial distribution of temperature for power function equation (2), (b) Radial distribution of temperature for power function equation (1).

4. Result, validation and discussion

Results were obtained considering radial distribution of thermal conductivity to follow equation (1) and (2) separately. Obtaining results for power function equation (2) the values of material and geometry parameters were taken as reference [1]. The material property was taken as $k_c = 60\frac{W}{m℃}$ and geometric properties, inner and outer radii of the sphere were taken as $r_i = 40mm$ and $r_o = 60mm$ respectively Boundary condition was $T(r_i) = 300°C \,\&\, T(r_o) = 25°C$. Results were obtained for inhomogeneity parameter $\beta = -4, -3, -2, -1, 0, 1, 2, 3$ and 4 as shown in figure 3(a). From the figure 3(a) it can also be observed that results obtained by thermal resistance approach are in well agreement with results obtained by Yahya et al [1] and FEM of this study using ANSYS solver. For power function equation (1) boundary condition and geometric parameters were kept same as previous and reference [1]. Material properties were taken as $k_i = 400\ \frac{W}{m℃}$ and $k_o = 250\ \frac{W}{m℃}$. Figure 1(a) shows the radial distribution of k_{fgm} for power law index $n = 0.1, 0.25, 0.5, 1, 2, 4$ and 10. Figure 3(b) shows obtained results for $n = 1, 2, 4, 10$ by thermal resistance approach. Table 1 contains a comparison among obtained results from thermal resistance approach and Yahya et al [1] for $\beta = 1 \,\&\, -1$ and we can conclude that the results agree with each other. Figure 4(a) shows mesh and the radial temperature distribution obtained by FEM using ANSYS solver. Figure 4(b) is the radial distribution of thermal resistance for different values of β.

Table 1. Radial temperature distribution from this work and Yahya et al [1]

	$T/_{T(r_i)}$					
$^{r}/_{r_i}$	$\beta = 1$			$\beta = -1$		
	This Work	Yahya[1] (Analytical)	Yahya[1] (FEM)	This Work	Yahya[1] (Analytical)	Yahya[1] (FEM)
1	1	1	1	1	1	1
1.0875	0.745182	0.74516	0.74574	0.810369	0.81036	0.81052
1.1875	0.520155	0.52008	0.52050	0.611509	0.61148	0.61162
1.2875	0.345443	0.34538	0.34568	0.428719	0.42870	0.42881
1.3875	0.207095	0.20707	0.20729	0.259595	0.25959	0.25968
1.5	0.083333	0.083333	0.08333	0.083333	0.083333	0.083333



(a) (b)

Figure 4. (a) Temperature distribution obtained by FEM (ANSYS) for $\beta = -4$, (b) Radial distribution of thermal resistance for different

5. Conclusion

A solution to the temperature distribution problem of FGM hollow sphere is presented in this study for two different power law functions separately using thermal resistance approach. Solutions were obtained for different values of inhomogeneity parameter β for distribution of equation (2) and different values of power law indexes n for equation (1). Obtained results are also compared with some well-established data present in the literature and with results obtained from FEM using ANSYS solver. It is observed that results obtained in this study using thermal resistance approach comply very well with them.

6.Referance

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