

Channel Estimation and Pilot Design for Massive MIMO Systems with Block-Structured Compressive Sensing

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Abstract. Through utilizing the technology of compressive sensing (CS), the channel estimation methods can achieve the purpose of reducing pilots and improving spectrum efficiency. The channel estimation and pilot design scheme are explored during the correspondence under the help of block-structured CS in massive MIMO systems. The block coherence property of the aggregate system matrix can be minimized so that the pilot design scheme based on stochastic search is proposed. Moreover, the block sparsity adaptive matching pursuit (BSAMP) algorithm under the common sparsity model is proposed so that the channel estimation can be caught precisely. Simulation results are to be proved the proposed design algorithm with superimposed pilots design and the BSAMP algorithm can provide better channel estimation than existing methods.

1. Introduction

As the key technology of 5G wireless communication in the coming decades, the massive multiple-input multiple-output (MIMO) draws the tremendous attention from researchers. Owing to the quantities of antennas in base station (BS), the energy efficiency and the data rate in massive MIMO systems can be promoted dramatically [1]. For massive MIMO systems, the ideal channel state information (CSI) plays a genuinely important part to lead a well-performed system. However, it is a challenge because the large channels between the users and the BS have to be estimated accurately [2].

The channel estimation can result in the vast pilot overhead within massive MIMO systems under the large scale BS antennas. Inspiring by the reality that actual physical channels usually hold sparse structure [3], numerous investigators have utilized the CS [4] technique to evaluate the channel impulse response (CIR) for single-user and MIMO-OFDM systems [5, 6, 7]. Unlike the linear estimation methods - the least squares (LS) and minimum mean square error (MMSE) - the performance of channel estimation can be improved under the methods of CS to reduce the pilot overhead burden. Recently, researchers found that the different channels exhibit a common sparsity pattern due to the similar time of arrival (TOA) [8]. In [9] and [10], the common sparsity is exploited to improve the estimation performance with the same pilot subcarriers among all transmit antennas in massive MIMO systems, the pilots are distributed equidistantly in [9] while [10] utilizes the randomly spaced pilots. Meanwhile, the path delays obtained from dealing with uplink transmission are required as aided information in [10]. Although the block-structured CS methods have been investigated in some researches with the prior information of the sparsity which is already known, the block coherence of measurement matrix is required to be optimized efficiently.



In this paper, the pilot design method based on stochastic search is proposed by means of minimizing total interblock coherence and total subblock coherence. Meanwhile, the block sparsity adaptive matching pursuit (BSAMP) algorithm without prior information of the sparsity is proposed to obtain the CIRs correspondently to the block sparse model. The performance of channel estimation can be significantly improved through pilot design method and BSAMP algorithm which have been put forward in contrast with the existing methods.

The rest parts of the research are arranged ahead. In Section 2, the massive MIMO system based on block sparse structure is described. Then the pilot design method and the BSAMP algorithm are proposed in Section 3 and Section 4, respectively. Section 5 reveals the simulation results. Eventually, the conclusion of this research is conducted in Section 6.

Notations: $(\cdot)^T$, $(\cdot)^H$, $\langle \cdot \rangle$, $\|\cdot\|_p$, \cup , \emptyset , $(\cdot)^\dagger$ and $supp_K(x)$ represent respectively the matrix transpose, the Hermitian transpose of a matrix, the inner product, ℓ_p -norm, the set union, the empty set, the Moore-Penrose matrix inversion, and the support of the K largest elements of x .

2. Massive MIMO System Model

Concern a large scale MIMO-OFDM system which has a BS and several users having single-antenna, set BS antennas amount as M , and denote the CIR vector between the k th BS antenna and a known terminal as $\mathbf{h}_k = [h_k(0), h_k(1), \dots, h_k(L-1)]^T$ with L as the spread of maximum channel delay. Because of the sparsity of wireless channels, only K nonzero entries are contained in \mathbf{h}_k and $K \ll L$.

Assuming adopting N OFDM subcarriers to transmit data, the k th transmitting antenna in BS employs N_p ($0 < N_p < N$) subcarriers through matching the indices $\mathbf{p} = [p_1, p_2, \dots, p_{N_p}]$ to transmit pilot symbols $\mathbf{x}^k = [x_1^k, x_2^k, \dots, x_{N_p}^k]$. Unlike the conventional orthogonal pilots, we concern all transmitting antennas use the same subcarriers to transmit pilots for reducing pilot overhead, but each pilot sequence is unique for distinguishing channels from different transmitting antennas. In this paper, we set pilot sequences according to the Hadamard matrix which is an orthogonal matrix consisting of +1 and -1.

After eliminating the cyclic prefix (CP) and taking DFT, the obtained pilot symbols can be illustrated as

$$\mathbf{y} = \sum_{m=1}^M \text{diag}(\mathbf{x}^m) \mathbf{F}_p \mathbf{h}_m + \mathbf{n} = \sum_{m=1}^M \mathbf{X}_m \mathbf{F}_p \mathbf{h}_m + \mathbf{n} \quad (1)$$

Where, $\mathbf{X}_m = \text{diag}(\mathbf{x}^m)$ indicates a diagonal matrix with the diagonal elements which are constituting \mathbf{x}^m , \mathbf{F}_p is the submatrix corresponding the N_p rows and the first L columns of $N \times N$ DFT matrix. \mathbf{n} indicates the additive white Gaussian noise (AWGN). Then, we can rewrite (1) as

$$\mathbf{y} = \mathbf{A} \mathbf{h} + \mathbf{n} \quad (2)$$

Where, $\mathbf{A} = [\mathbf{X}_1 \mathbf{F}_p, \mathbf{X}_2 \mathbf{F}_p, \dots, \mathbf{X}_M \mathbf{F}_p]$ denotes the $N_p \times LM$ matrix and $\mathbf{h} = [\mathbf{h}_1^T, \mathbf{h}_2^T, \dots, \mathbf{h}_M^T]^T$ denotes an aggregate channel vector.

Since the CIRs of different transmitting antennas have the same support due to the similar TOA, we can use the inherent block structure in the large scale MIMO channels to estimate channels. Specifically, the nonzero positions in \mathbf{h}_m are the same while the nonzero coefficients are different. By means of rewriting the entries of \mathbf{h} as $\mathbf{c} = [\mathbf{c}_0^T, \dots, \mathbf{c}_l^T, \dots, \mathbf{c}_{L-1}^T]^T$ where $\mathbf{c}_l = [h_1(l), \dots, h_M(l)]^T$, (2) is expressed below

$$\mathbf{y} = \sum_{l=0}^{L-1} \Phi_l \mathbf{c}_l + \mathbf{n} = \Phi \mathbf{c} + \mathbf{n} \quad (3)$$

Where, $\Phi = [\Phi_0, \dots, \Phi_l, \dots, \Phi_{L-1}]$, and $\Phi_l = [A_l, \dots, A_{(M-1)L+l}]$ is a $N_p \times M$ matrix.

3. Pilot Design Scheme

In CS theory, restricted isometry property (RIP) is the well-known way to obtain accurate recovery of sparse vectors. However, it is computationally infeasible to check this property [11]. Fortunately, the

coherence measure of measurement matrix \mathbf{A} is alternative to replace RIP. The coherence of \mathbf{A} is represented as

$$\mu(\mathbf{A}) = \max_{1 \leq l, k \leq L, l \neq k} \frac{|\langle \mathbf{A}_l, \mathbf{A}_k \rangle|}{\|\mathbf{A}_l\|_2 \cdot \|\mathbf{A}_k\|_2} \quad (4)$$

In block-structured CS case, according to [12] the definition of block coherence is formulated as

$$\mu_B(\Phi) = \max_{1 \leq l, k \leq L, l \neq k} \frac{\rho(\Phi_l^H \Phi_k)}{\|\Phi_l\|_2 \cdot \|\Phi_k\|_2} \quad (5)$$

Where, $\rho(\mathbf{R}) \square \lambda_{\max}^{1/2}(\mathbf{R}^H \mathbf{R})$ indicates the spectrum norm of matrix \mathbf{R} , $\lambda_{\max}(\mathbf{R}^H \mathbf{R})$ signifies the largest eigenvalue of the matrix $\mathbf{R}^H \mathbf{R}$.

To obtain the stable sparse channel estimation performance, pilot design is usually to minimize the block coherence in (5). According to [13], we can design pilot by minimizing total interblock coherence and subblock coherence rather than minimizing the block coherence.

Therefore, the optimization problem can be formed as

$$\mathbf{p}_{opt} = \arg \min_p \varepsilon(\mathbf{p}) = \arg \min_p \mu_B'(\mathbf{p}) + v'(\mathbf{p}) + \|\boldsymbol{\eta}(\mathbf{p}) - \mathbf{1}\|_2^2 \quad (6)$$

Where, $\mu_B'(\mathbf{p}) = \sum_{l=1}^L \sum_{l \neq k} \|\Phi_l^H \Phi_k\|_F^2$ denotes the total interblock coherence under a given pilot pattern \mathbf{p} , $v'(\mathbf{p}) = \sum_{l=1}^L \|\Phi_l^H \Phi_l\|_F^2 - \|\boldsymbol{\eta}(\mathbf{p})\|_2^2$ is the total subblock coherence, $\boldsymbol{\eta}(\mathbf{p})$ is a vector consisting of the diagonal entries of $\Phi^H \Phi$, the third term is normalization penalty.

Intuitively, we can get the best pilot pattern by exhaustively searching. However, it is computationally infeasible to search all possible pilot patterns. So the low-complexity practical method is proposed to achieve near-optimal pilot pattern through stochastic search. The details are provided below.

Algorithm 1- Pilot Design Algorithm

Initializations: Set T_1 . $\mathbf{C} \leftarrow \mathbf{0}^{T_1 \times N_p}$. $\mathbf{v} \leftarrow \mathbf{0}^{1 \times T_1}$.

for $i = 1, 2, \dots, T_1$

 randomly generate $\mathbf{p} \subset \Omega = \{1, 2, \dots, N\}$.

 Obtain $\varepsilon(\mathbf{p})$ according to (6).

$\mathbf{C}[i] = \mathbf{p}$, $v(i) \leftarrow \varepsilon(\mathbf{p})$.

end for (i)

$r = \arg \min_{k=1, 2, \dots, T_1} v(k)$.

Output $\mathbf{p}_{opt} = \mathbf{C}[r]$.

4. Block Sparsity Adaptive Matching Pursuit (BSAMP) Algorithm

Given the common sparsity of wireless communication channels, the BSAMP algorithm is put forward to evaluate channels based on block sparse model in (3). Unlike the existing block-structured CS methods, the method proposed in this work can estimate CIRs without prior information of the sparsity. Moreover, the proposed BSAMP algorithm can update multiple vectors simultaneously, because the CIRs of different BS and terminal antenna pairs have the same locations of nonzero taps. On the contrary, the classical SAMP algorithm [14] only updates one vector in each iteration. Since the comparative change of residual between two successive stages is less than a definite threshold, the BSAMP algorithm stops working. The BSAMP algorithm is detailed in Algorithm 2.

Algorithm 2- Proposed BSAMP Algorithm**Input:** obtained pilot symbols \mathbf{y} , sensing matrix Φ , the step size is s .**Initialization:**The stage index $j=1$, the initial residual $\mathbf{r}_0 = \mathbf{y}$, the iteration index $k=1$, the set $F_0 = \emptyset$ and $\Delta = s$ **Repeat**

$$\mathbf{z} \leftarrow \Phi^H \mathbf{r}_{k-1}$$

$$\omega(l) \leftarrow \sum_{v=0}^{M-1} |z(v + lM)|^2, 0 \leq l \leq L-1$$

$$\Gamma \leftarrow F_{k-1} \cup \text{supp}_\Delta(\omega)$$

$$\mathbf{u} \leftarrow \Phi_\Gamma^\dagger \mathbf{y}$$

$$e(l) \leftarrow \sum_{v=0}^{M-1} |u(v + lM)|^2, 0 \leq l \leq \Gamma$$

$$F \leftarrow \text{supp}_\Delta(\mathbf{e})$$

$$\hat{\mathbf{c}} \leftarrow \Phi_F^\dagger \mathbf{y}$$

$$\mathbf{r} \leftarrow \mathbf{y} - \Phi_F \hat{\mathbf{c}}$$

if meet stop condition **then**

break;

else if $\|\mathbf{r}\|_2 \geq \|\mathbf{r}_{k-1}\|_2$ **then**

$$j = j + 1$$

$$\Delta = j \times s$$

else

$$F_k = F$$

$$\mathbf{r}_k = \mathbf{r}$$

$$k = k + 1$$

end if**Until** meet stop condition;**Output:** estimated CIR matrix $\hat{\mathbf{c}}$ **5. Simulation Results**

Through carrying out the simulations, the performance of the proposed BSAMP algorithm and pilot design scheme are investigated during this part. Concern a large scale MIMO system including $M=8$ transmitting antennas, the total amount of subcarriers is $N=1024$ for transmission. The maximum channel delay spread is set to be $L=200$, there are only $K=6$ nonzero taps in channel vector. We use $N_p=128$ subcarriers to transmit pilot symbols. The normalized mean square error (MSE) is as reference to assess performance. We define the MSE as

$$\text{MSE} = \frac{1}{N_{MC}} \sum_{n=1}^{N_{MC}} \frac{1}{M} \left\| \hat{\mathbf{h}}_m^n - \mathbf{h}_m^n \right\|_2^2 / \left\| \mathbf{h}_m^n \right\|_2^2 \quad (7)$$

Where, $\hat{\mathbf{h}}_m^n$ and \mathbf{h}_m^n are the estimated and true channel vectors in the n th iteration, respectively. N_{MC} is the number of iterations.

Firstly, the performances of the proposed BASMP and the structured subspace pursuit (SSP) [9] are compared in terms of MSE. Meanwhile, because of the ideal information of sparse channel support, the result of the exact LS algorithm is also provided for comparison as performance bound. As shown in the Fig. 1, the classical SP algorithm cannot accurately recover MK -sparse signal with $N_p \ll ML$. In contrast, the SSP algorithm outperforms classical SP algorithm specially at high SNR regimes. And

the proposed BSAMP algorithm has similar performance with SSP algorithm without the prior information of sparsity and approaches the exact LS algorithm. From [15], we know that the traditional CS algorithms need $N_p = 8 \times 6 \times \log(200 / 6) \approx 74$ pilots to estimate CIRs accurately while the block-structured CS algorithms only needs $N_p = 8 \times 6 + 6 \times \log(200 / 6) \approx 58$ pilots. This implies that the block-structured CS method can realize channel estimation precisely by low pilot overhead. Next, it reveals clearly from Fig. 2 that the proposed pilot design algorithm and random pilot scheme has different MSE performance through doing the comparison. The iteration value is to be set as 500000. It can be arrived at the proposed pilot design algorithm exerts the better performance than the random pilot scheme.

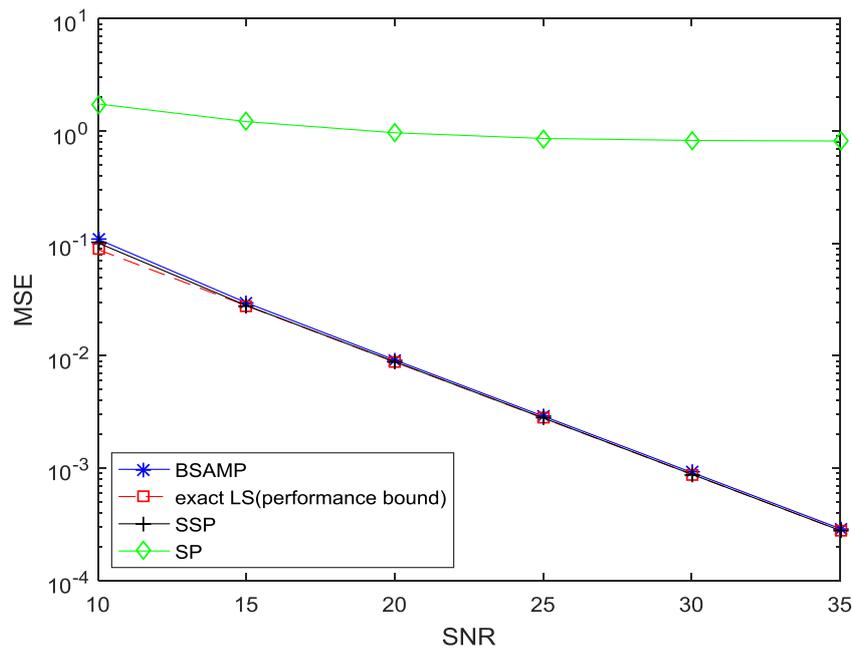


Figure 1. MSE performance comparison for different algorithms.

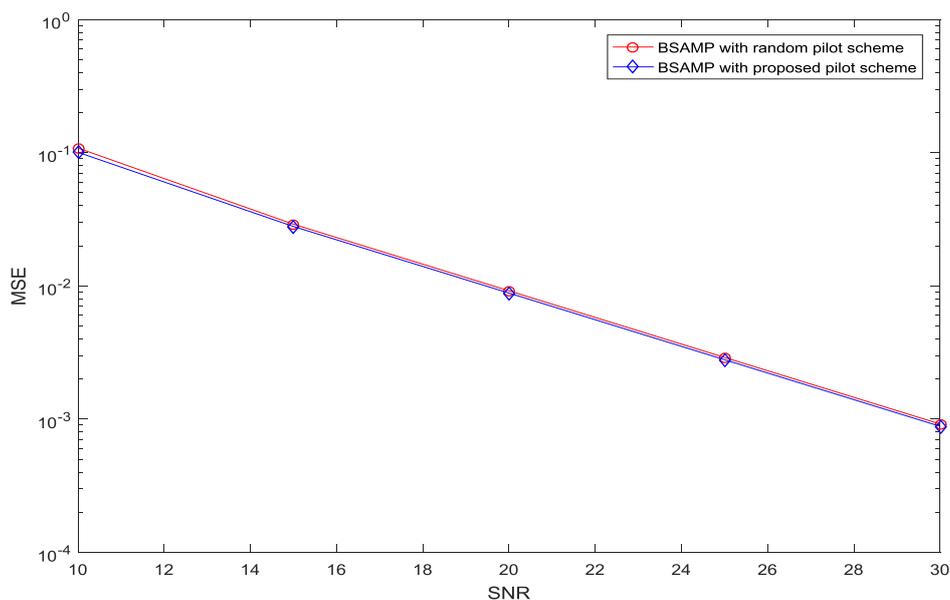


Figure 2. MSE performance comparison for different pilot design scheme.

6. Conclusion

Under the model of the block-structured CS within massive MIMO systems, the pilot design scheme and channel estimation are investigated. By minimizing the block coherence, the pilot design scheme is proposed through the stochastic search manner. Moreover, owing to the common sparsity of wireless communication channels, the BSAMP algorithm is proposed without prior information of the sparsity to obtain the channel estimation in superimposed pilot design. The results of simulation demonstrate the BSAMP algorithm is superior to classical CS based methods with low pilot overhead. And the proposed pilot design method through stochastic search has better MSE performance of channel estimation than random pilot scheme.

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References

- [1] T. L. Marzetta, "Noncooperative cellular wireless with unlimited numbers of base station antennas," *IEEE Trans. Wireless Commun.*, vol. 9, no. 11, pp. 3590-3600, Nov. 2010.
- [2] F. Rusek, D. Persson, B. K. Lau, E. G. Larsson, T. L. Marzetta, O. Edfors, and F. Tufvesson, "Scaling up MIMO: opportunities and challenges with very large arrays," *IEEE Signal Process. Mag.*, vol. 30, no. 1, pp. 40-60, Jan. 2013.
- [3] W. Li, J. C. Preisig, "Estimation of rapidly time-varying sparse channels," *IEEE Journal of Oceanic Engineering*, vol. 32, no. 4, pp. 927-939, Oct. 2007.
- [4] E. J. Candes, J. Romberg, and T. Tao, "Stable signal recovery from incomplete and inaccurate measurements," *Comm. Pure Appl. Math.*, vol. 59, no. 8, pp. 1207-1223, Aug. 2006.
- [5] G. Taubock and F. Hlawatsch, "A compressed sensing technique for OFDM channel estimation in mobile environments: Exploiting channel sparsity for reducing pilots," in *Proc. IEEE Int. Conf. Acoust. Speech Signal Process.*, pp. 2885-2888, Mar. 2008.
- [6] G. Taubock, F. Hlawatsch, D. Eiwien, and H. Rauhut, "Compressive estimation of doubly selective channels in multicarrier systems: Leakage effects and sparsity-enhancing processing," *IEEE J. Sel. Top. Signal Process.*, vol. 4, no. 2, pp. 255-271, Apr. 2010.
- [7] W. U. Bajwa, J. Haupt, A. M. Sayeed, and R. Nowak, "Compressed channel sensing: A new approach to estimating sparse multipath channels," *Proceedings of the IEEE*, vol. 98, no. 6, pp. 1058-1076, June 2010.
- [8] Y. Barbotin, A. Hormati, S. Rangan, and M. Vetterli, "Estimation of sparse MIMO channels with common support," *IEEE Trans. Commun.*, vol. 60, no. 12, pp. 93-100, Dec. 2012.
- [9] Z. Gao, L. Dai, and Z. Wang, "Structured compressive sensing based superimposed pilot design in downlink large-scale MIMO systems," *Electronics Letters*, vol. 50, no. 12, pp. 896-898, June 2014.
- [10] Y. Nan, L. Zhang, and X. Sun, "Efficient downlink channel estimation scheme based on block-structured compressive sensing for TDD massive MU-MIMO systems," *IEEE Wireless Comm. Lett.*, vol. 4, no. 4, pp. 345-348, Aug. 2015.
- [11] A. S. Bandeira, E. Dobriban, D. Mixon, and W. F. Sawin, "Certifying the restricted isometry property is hard," *IEEE Trans. Inform. Theory*, vol. 59, no. 6, pp. 3448-3450, June 2013.
- [12] R. Calderbank, A. Thompson, and Y. Xie, "On block coherence of frames," *Applied and Computational Harmonic Analysis*, vol. 38, no. 1, pp. 50-71, 2015.
- [13] L. Zelnik-Manor, K. Rosenblum, and Y. C. Eldar, "Sensing matrix optimization for block-sparse decoding," *IEEE Trans. Signal Process.*, vol. 59, no. 9, pp. 4300-4312, Sept. 2011.
- [14] Thong T. Do, Lu Gan, Nam Nguyen and Trac D. Tran, "Sparsity adaptive matching pursuit algorithm for practical compressed sensing," *Conference on Signals, Systems and Computers*, pp. 581-587. Oct. 2008.
- [15] R. G. Baraniuk, V. Cevher, M. F. Duarte, and C. Hegde, "Model-based compressive sensing," *IEEE Transactions on Information Theory*, vol. 56, no. 4, pp. 1982-2001, Apr. 2010.