

Terminal Sliding Mode Tracking Controller Design for Automatic Guided Vehicle

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Abstract. Based on sliding mode variable structure control theory, the path tracking problem of automatic guided vehicle is studied, proposed a controller design method based on the terminal sliding mode. First of all, through analyzing the characteristics of the automatic guided vehicle movement, the kinematics model is presented. Then to improve the traditional expression of terminal sliding mode, design a nonlinear sliding mode which the convergence speed is faster than the former, verified by theoretical analysis, the design of sliding mode is steady and fast convergence in the limited time. Finally combining Lyapunov method to design the tracking control law of automatic guided vehicle, the controller can make the automatic guided vehicle track the desired trajectory in the global sense as well as in finite time. The simulation results verify the correctness and effectiveness of the control law.

1. Introduction

AGV belongs to a type of wheeled mobile robot. As a typical nonholonomic system, the issue of AGV path tracking control has received widespread attention.

Sliding mode control is an effective method for nonlinear system control. Its sliding mode can be designed, and has nothing to do with the parameters and external disturbances of the controlled object, which makes the sliding mode control method has the advantages such as fast response, insensitive to disturbance, no need for the system online identification and so on. In recent years, based on sliding mode control, combined with other control methods, the path tracking of mobile robots has been deeply studied and a series of achievements have been made. Reference [1] designed the equivalent control law by using PI sliding mode surface, and achieved the effective tracking on the robot path by using the velocity change function instead of the sign function. Reference [2] designed the system state variables by using nonsingular terminal sliding mode technique combined with the idea of inversion control, which simplified the design of control law. References [3] and [4] designed the AGV control law by using the inversion control method and the Lyapunov method as well as the idea of fast terminal sliding mode control respectively. Reference [5] proposed a bi-power approach law, which improved the convergence velocity of the system. Reference [6] designed an adaptive fuzzy terminal sliding mode controller, achieving the motion control of an omni-directional mobile robot. Reference [7] proposed a quasi-sliding mode control method, which uses the saturation function instead of the sign function to reduce buffeting, achieving good effects.

In this paper, the path tracking is studied using the AGV kinematics model. Based on the fast terminal sliding mode control, a new nonlinear sliding mode surface is proposed, which can make the system converge quickly to the equilibrium point. Then the AGV path tracking control law is designed by



combining with the Lyapunov function. Finally, the validation is carried out.

2. AGV Kinematics Model

The three-wheeled AGV is taken as the study object, and its motion process is as shown in Fig.1.

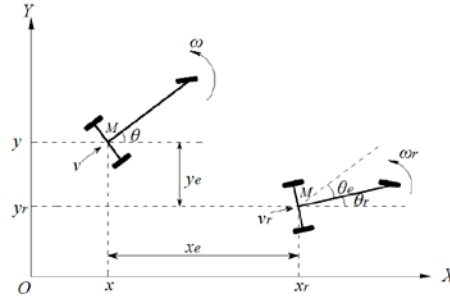


Fig.1 Motion Sketch Map of AGV

The midpoint M of the axial connecting line between the two rear wheels of the AGV is taken as a reference point, and the current pose status of the AGV can be expressed by the position (x, y) of the M point in the coordinate system and the body heading angle θ . Let $\mathbf{P} = [x, y, \theta]^T$, where, $\mathbf{P}$ means the current pose. Suppose that the AGV is a rigid body, and its wheels purely roll on the horizontal plane without sliding, the kinematics equation of the AGV can be expressed as^[8]:

$$\dot{\mathbf{P}} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos \theta & 0 \\ \sin \theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \quad (1)$$

Where, v and ω are the instantaneous linear velocity and the instantaneous angular velocity of the AGV, respectively, which are the control inputs in the kinematics model. The reference pose of the AGV is expressed as $\mathbf{P}_r = [x_r, y_r, \theta_r]^T$, and the control input command as $[v_r, \omega_r]^T$. According to the position relationship shown in Fig.1, the pose error equation of the AGV can be obtained via the coordinate transformation:

$$\mathbf{P}_e = \begin{bmatrix} x_e \\ y_e \\ \theta_e \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x \\ y_r - y \\ \theta_r - \theta \end{bmatrix} \quad (2)$$

The difference equation of pose error is^[8]:

$$\dot{\mathbf{P}}_e = \begin{bmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{\theta}_e \end{bmatrix} = \begin{bmatrix} y_e \omega - v + v_r \cos \theta_e \\ -x_e \omega + v_r \sin \theta_e \\ \omega_r - \omega \end{bmatrix} \quad (3)$$

The AGV path tracking based on the kinematic model is essentially the process in which the AGV converges any of its initial pose to the reference pose under the AGV control input $[v, \omega]^T$ within a limited time, namely $\lim_{t \rightarrow \infty} \|\mathbf{P}_e\| = 0$.

3. Design of AGV Control Law

3.1. Terminal Sliding Mode Design

The terminal sliding mode control can make the system state converged to the equilibrium point within a limited time, which is different from the asymptotic convergence of the ordinary sliding mode control.

The dynamic performance of the system is superior to the ordinary sliding mode control. Besides, compared to the linear sliding mode control, there is no switching term in the expression of the terminal sliding mode control, which can effectively eliminate the buffeting of the system and make the system run more smoothly. A common terminal sliding mode is [9]:

$$\mathbf{s} = \dot{\mathbf{x}} + \beta \mathbf{x}^{q/p} = \mathbf{0} \quad (4)$$

Where, $\mathbf{x} \in \mathbb{R}$ is the state variable, $\beta > 0$, p and q are positive odd numbers and $2q > p > q$.

In Eq. (4), the nonlinear part is imported, which improves the velocity of the system to converge to the equilibrium state. However, the terminal sliding mode control is not optimal in the convergence time, which is mainly because when the system leaves the equilibrium point, its convergence velocity is of the linear sliding mode control, which is superior to the terminal sliding mode control; and when the system approaches the equilibrium point, the convergence velocity of the terminal sliding mode control is superior to the linear sliding mode control. In order to obtain a faster convergence velocity, the nonlinear term with an exponent greater than 1 is added to Eq. (4). The nonlinear fast terminal sliding mode is designed as:

$$\mathbf{s} = \dot{\mathbf{x}} + \alpha \mathbf{x}^{m/n} + \beta \mathbf{x}^{q/p} = \mathbf{0} \quad (5)$$

Where, $\mathbf{x} \in \mathbb{R}$ is the state variable; $\alpha, \beta > 0$; p and q are positive odd numbers and $2q > p > q$; m and n are positive odd numbers and $2n > m > n$. In Eq. (5), a nonlinear term with a variable $\mathbf{x}$ index greater than 1 is added to allow the system to reach the equilibrium point from any initial state at a faster velocity.

1) Stability analysis

When the sliding mode surface $\mathbf{s} = \mathbf{0}$, the following equation may be obtained from Eq. (5):

$$\dot{\mathbf{x}} = -\alpha \mathbf{x}^{m/n} - \beta \mathbf{x}^{q/p} \quad (6)$$

Construct the Lyapunov function as:

$$\mathbf{V} = \frac{1}{2} \mathbf{x}^2 \quad (7)$$

Derive and import (6) into Eq. (7), the following will be obtained:

$$\begin{aligned} \dot{\mathbf{V}} &= \mathbf{x} \dot{\mathbf{x}} = \mathbf{x}(-\alpha \mathbf{x}^{m/n} - \beta \mathbf{x}^{q/p}) \\ &= -\alpha \mathbf{x}^{(n+m)/n} - \beta \mathbf{x}^{(p+q)/p} \end{aligned} \quad (8)$$

p and q are positive odd numbers, so $(p + q)$ is even. Similarly, $(m + n)$ is even as well. And both α and β are greater than 0, so we obtain $\dot{\mathbf{V}} < 0$. According to the Lyapunov stability principle, it is known that the system is globally stable.

2) Arrival time analysis

Suppose the initial state $\mathbf{x}(0) > 1$. The process in which the system converges from the initial state to the equilibrium point is divided into two phases according to Eq. (5).

When the system moves from $\mathbf{x}(0)$ to $\mathbf{x}(t) = 1$, this process is mainly dominated by the $\alpha \mathbf{x}^{m/n}$ item and the other item is ignored. We obtain:

$$\dot{\mathbf{x}} = -\alpha \mathbf{x}^{m/n} \quad (9)$$

$$dt = -\frac{1}{\alpha} \mathbf{x}^{-m/n} d\mathbf{x} \quad (10)$$

Perform the definite integral for Eq. (10) and obtain:

$$\int_0^{t_1} dt = \int_{\mathbf{x}(0)}^1 -\frac{1}{\alpha} \mathbf{x}^{-m/n} d\mathbf{x} \quad (11)$$

The time used in this phase is:

$$t_1 = \frac{n(1 - \mathbf{x}^{(n-m)/n}(0))}{\alpha(m - n)} \quad (12)$$

When the system converges from $\mathbf{x}(t) = 1$ to the equilibrium point, the convergence time is mainly dominated by Eq. (5), and the other function is ignored. We obtain:

$$\dot{\mathbf{x}} = -\beta \mathbf{x}^{q/p} \quad (13)$$

$$dt = -\frac{1}{\beta} \mathbf{x}^{-q/p} d\mathbf{x} \quad (14)$$

Perform the definite integral for Eq. (14) and obtain:

$$\int_0^{t_2} dt = \int_1^0 -\frac{1}{\beta} \mathbf{x}^{-q/p} d\mathbf{x} \quad (15)$$

The time used in this phase is:

$$t_2 = \frac{p}{\beta(p-q)} \quad (16)$$

In the above assumption, the convergence time is calculated by ignoring one item of the sliding mode in both phases of the convergence process. Therefore, the time required for the actual system to converge from any initial state to the equilibrium point is:

$$ts < t_1 + t_2 = \frac{n(1 - \mathbf{x}^{(n-m)/n}(0))}{\alpha(m-n)} + \frac{p}{\beta(p-q)} \quad (17)$$

Therefore, under the sliding mode (5), the system can converge from any initial state to the equilibrium within the limited time ts .

3.2. Design of Control Law

The AGV kinematic model Eq. (1) is a multi-input nonlinear system. According to Eq. (5), the fast sliding mode expression with recursive structure is designed as:

$$\begin{cases} \dot{\mathbf{s}}_1 = \dot{\mathbf{s}}_0 + \alpha_1 \mathbf{s}_0^{m_1/n_1} + \beta_1 \mathbf{s}_0^{q_1/p_1} \\ \dot{\mathbf{s}}_2 = \dot{\mathbf{s}}_1 + \alpha_2 \mathbf{s}_1^{m_2/n_2} + \beta_2 \mathbf{s}_1^{q_2/p_2} \end{cases} \quad (18)$$

Where, $\alpha_1, \beta_1, \alpha_2$ and β_2 are all real numbers greater than 0, and $p_1, q_1, p_2, q_2, m_1, n_1, m_2$ and n_2 are all positive odd numbers and satisfy $2q_1 > p_1 > q_1$, $2q_2 > p_2 > q_2$, $2n_1 > m_1 > n_1$ and $2n_2 > m_2 > n_2$.

With consideration of the lemma: for any $x \in \mathbb{R}$ with x bounded, $\phi(x) = x \sin(\arctan x) \geq 0$, when and only when $x = 0$, the equal sign is true. Combined with the AGV pose error equation (Eq. 3), when $x_e = 0$, the Lyapunov function is designed as:

$$V = \frac{1}{2} y_e^2 \quad (19)$$

Suppose $\theta_e = -\arctan(v_r y_e)$. Derive Eq. (19) and import Eq. (3), we obtain:

$$\begin{aligned} \dot{V} &= y_e \dot{y}_e \\ &= y_e (-x_e \omega + v_r \sin \theta_e) \\ &= -y_e x_e \omega - y_e v_r \sin(\arctan(v_r y_e)) \\ &= -y_e v_r \sin(\arctan(v_r y_e)) \end{aligned} \quad (20)$$

Since $y_e v_r \sin(\arctan(v_r y_e)) \geq 0$, $\dot{V} \leq 0$, it indicates that when x_e is converged to 0, and θ_e is converged to $-\arctan(v_r y_e)$, the system state y_e is converged to 0. Let:

$$\begin{cases} s_0 = x_e \\ s_1 = \theta_e + \arctan(v_r y_e) \\ s_2 = 0 \end{cases} \quad (21)$$

Import Eq. (21) into Eq. (18), and we obtain:

$$\begin{cases} \dot{x}_e + \alpha_1 x_e^{m_1/n_1} + \beta_1 x_e^{q_1/p_1} = \theta_e + \arctan(v_r y_e) \\ \dot{\theta}_e + \frac{v_r \dot{y}_e + \dot{v}_r y_e}{1 + (v_r y_e)^2} + \alpha_2 (\theta_e + \arctan(v_r y_e))^{m_2/n_2} \\ + \beta_2 (\theta_e + \arctan(v_r y_e))^{q_2/p_2} = 0 \end{cases} \quad (22)$$

Import Eq. (3) into Eq. (22), and we obtain:

$$\begin{cases} v = y_e \omega + v_r \cos \theta_e + \alpha_1 x_e^{m_1/n_1} + \beta_1 x_e^{q_1/p_1} \\ - \theta_e - \arctan(v_r y_e) \\ \omega = \frac{(\omega_r + A)(1 + v_r^2 y_e^2) + v_r^2 \sin \theta_e + \dot{v}_r y_e}{1 + v_r^2 y_e^2 + v_r x_e} \end{cases} \quad (23)$$

Eq. (23) is the path tracking control law of the AGV, where:

$$A = \alpha_2 (\theta_e + \arctan(v_r y_e))^{m_2/n_2} + \beta_2 (\theta_e + \arctan(v_r y_e))^{q_2/p_2}$$

4. Simulation Experiment

The path-tracking simulation experiment on the AGV system Eq. (1) was conducted using the control law determined by Eq. (23). The control rate parameters are $\alpha_1 = \alpha_2 = 2, \beta_1 = \beta_2 = 1, m_1 = m_2 = 7, n_1 = n_2 = 5, p_1 = p_2 = 9, q_1 = q_2 = 5$, and the sampling time is $dt = 0.01s$.

When the tracking path is a straight line, suppose that the initial pose of the AGV is $[1, -3, \pi/4]^T$ and the initial velocity $v = 0m/s, \omega = 0rad/s$, so the straight line to be tracked is $y = x$, then the expected pose of AGV to be tracked is $[x, y, \pi/4]^T$, and the expected velocity is $v_r = 1m/s$. The simulation results are as shown in Fig. (2) and Fig.(3).

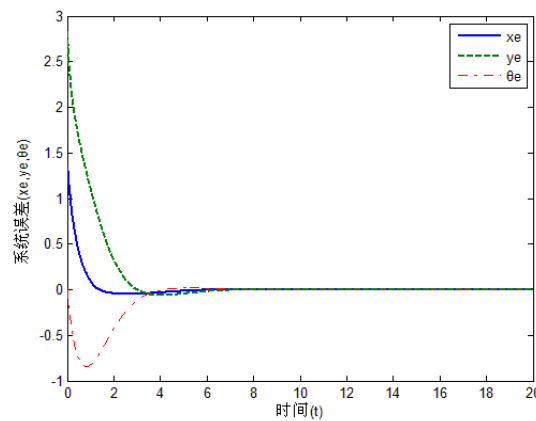


Fig.2 System Error of Straight Path Tracking

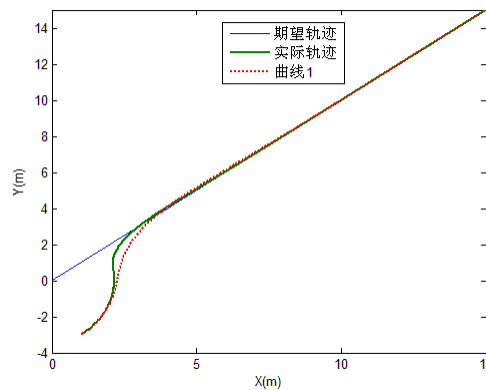


Fig.3 Tracking Trajectory of Straight Path

When the tracking path is a sine curve, suppose the initial pose of the AGV is $[1, 3, 0]^T$ and the initial velocity $v=0\text{m/s}, \omega=0\text{rad/s}$, the curve to be tracked is $y = \sin(x)$, the expected pose of AGV to be tracked is $[x, \sin(x), \arctan(\cos(x))]^T$ and the expected velocity is $v_r = 1\text{m/s}, \omega_r = -\sin(xr)/(1+(\cos(xr)^2))\text{rad/s}$. The simulation results are as shown in Fig. (4) and Fig.(5) below.

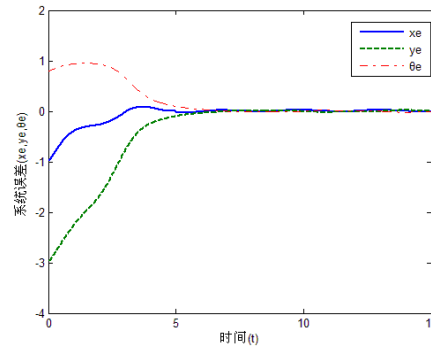


Fig.4 System Error of Sine Curve Path Tracking

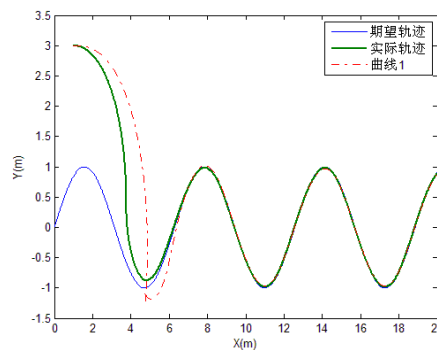


Fig.5 Tracking Trajectory of Sine Curve

From Fig. 2 and Fig. 4, it can be seen that when the system error converges from about 6s to the equilibrium point 0, the system is globally stable. And Fig. 3 and Fig. 5 indicate that the running trajectory of the AGV during the path tracking has a good smoothness. The curve 1 in Fig. 3 and Fig. 5 shows the process of path tracking under the traditional terminal sliding mode (Eq.4). It can be seen from the figure that the sliding mode (Eq.5) designed in this paper has a faster convergence velocity.

5. Summary

In this paper, the issue of AGV path tracking described with kinematics model is discussed, the sliding mode expression of the traditional terminal sliding mode control method is improved, and a new type of nonlinear sliding mode is designed. Combined with the Lyapunov method, the control law of AGV path tracking is designed, and finally, that the method can achieve the smooth and fast path tracking of AGV via the Matlab simulation is validated.

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