IOP Conf. Series: Materials Science and Engineering **322** (2018) 062015 doi:10.1088/1757-899X/322/6/062015

# Oscillation of a class of two order nonlinear systems

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**Abstract.** Nonlinear vibration plays an important role in the theory and engineering technology, this paper studies a class contains the vibration problem of two order nonlinear systems, and the elastic resistance force, using special method, calculated the parameters of the solution of this system is the form of expression, study the stability of the system.

## 1. Introduction

Great attention has been paid the nonlinear vibration problems in engineering and technology field, nonlinear vibration theory is a hot and difficult problem, research on nonlinear vibration solution form and study the stability of the solution is not divided, if you can find the solution, that is the best on the behavior of the solution. In the nonlinear vibration system, the motion of the system under the action of external force attracts more and more attention, because it plays a very important role in the theory and engineering technology.

## 2. Literature Survey

In reference to the relevant reference [1-5], the vibration problem of literature [1] is studied for a class of Semilinear systems; exact solutions of literature [2] studied a class of nonlinear systems; the [3] is studied for a class of exact solutions of -dimensional KdV (2+1) equation [4]; literature research some oscillation criteria for a class of generalized neutral Emdewn-Fowler differential equations [5]; literature study oscillation of a class of three order semilinear neutral delay differential equations, considering the regular and non regular two cases; oscillation and asymptotic behavior of literature [6] is studied for a class of three order semilinear differential equation; oscillation and correlation state on several types of literature [7-13] nonlinear system. In engineering practice, nonlinear phenomena often occur.

## 3. Proposed Method

In this paper, the stability problem of a class of two order nonlinear vibration system with practical background is studied on the basis of [1-13]:

$$y'' + \frac{3}{x}y' + \frac{4}{x^2}y = \frac{x^2}{y} \qquad (1)$$

where $\frac{3}{x}y'$ is resistance， $\frac{3}{x}$ is resistance function； $\frac{4}{x^2}y$ is elastic force， $\frac{4}{x^2}$ is elastic function， $\frac{x^2}{y}$ is external force.

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This is a two order nonlinear equation, not a constant coefficient differential equation. It is difficult to solve the equation. In order to get the analytic expression of the solution, we will do some special methods and techniques to deal with this problem:

$x = e^{-t}, \quad y = ue^{-2t}.$ （2）

Where $t$ is new variable , $u = u(t)$ is New unknown function.

So, get

$$dx = -e^{-t}dt, \quad \frac{dt}{dx} = -e^{t}.$$

$$y' = \frac{dy}{dx} = \frac{dy}{dt}\frac{dt}{dx} = (u'e^{-2t} - 2ue^{-2t})(-e^{t}) = e^{-t}(u' - 2u).$$

$$y'' = \frac{dy}{dx} = \frac{dy'}{dt}\frac{dt}{dx} = [-e^{-t}(u' - 2u) + e^{-t}(u'' - 2u')](-e^{t}) = u'' - 3u' + 2u.$$

(2) and the upper form are substituted into the equation (1), we have

$$u'' - 3u' + 2u + 3e^{t}e^{-t}(u' - 2u) + 4e^{2t}\cdot ue^{-2t} = \frac{e^{-2t}}{ue^{-2t}}.$$

After finishing, get

$$u'' = \frac{1}{u}.$$

Because

$$u'' = \frac{du'}{dt} = \frac{du'}{du}\frac{du}{dx} = u'\frac{du'}{du}.$$

Therefore, we get

$$u'\frac{du'}{du} = \frac{1}{u}, \qquad u'du' = \frac{1}{u}du, \qquad \int u'du' = \int \frac{1}{u}du,$$

$$\frac{1}{2}u'^{2} = \ln|u| + \ln c, \quad \frac{1}{2}u'^{2} = \ln|cu|, \quad cu = e^{\frac{1}{2}u'^{2}}.$$

So get

$$u = c_1 e^{\frac{1}{2}u'^{2}}.$$

where $c_1 = \frac{1}{c}$. remember $p = u'$, get

$$u = c_1 e^{\frac{1}{2}p^{2}}. \qquad (3)$$

Because

$$p = u' = \frac{du}{dt}, \quad dt = \frac{1}{p}du.$$

We have

$$t = \int \frac{1}{p}du + c_2$$

It can be obtained from (3) formula

$$t = \int \frac{1}{p}c_1 e^{\frac{1}{2}p^{2}} \cdot pdp + c_2 = c_1\int e^{\frac{1}{2}p^{2}}dp + c_2$$

The parametric solution of equation (1) can be obtained by (2) formula:

$$x = e^{-t} = \frac{1}{e^{c_1 \int e^{\frac{1}{2}p^2} dp + c_2}} . \quad y = ue^{-2t} = \frac{c_1 e^{\frac{1}{2}p^2}}{e^{2c_1 \int e^{\frac{1}{2}p^2} dp + c_2}} .$$

where $p$ is parameter. Because

$$\lim_{p \to +\infty} x(p) = \lim_{p \to +\infty} e^{-t} = \lim_{p \to +\infty} \frac{1}{e^{c_1 \int e^{\frac{1}{2}p^2} dp + c_2}} = 0 .$$

$$\lim_{p \to +\infty} y(p) = \lim_{p \to +\infty} \frac{c_1 e^{\frac{1}{2}p^2}}{e^{2c_1 \int e^{\frac{1}{2}p^2} dp + c_2}} = \lim_{p \to +\infty} \frac{c_1 p e^{\frac{1}{2}p^2}}{2c_1 e^{\frac{1}{2}p^2} e^{2c_1 \int e^{\frac{1}{2}p^2} dp + c_2}}$$

$$= \lim_{p \to +\infty} \frac{p}{2e^{2c_1 \int e^{\frac{1}{2}p^2} dp + c_2}} = \lim_{p \to +\infty} \frac{1}{4c_1 e^{\frac{1}{2}p^2} e^{2c_1 \int e^{\frac{1}{2}p^2} dp + c_2}} = 0 .$$

## 4. Conclusion

In this paper, the elastic resistance and external force, including the nonlinear system (1) in the long time range is stable, reveals the behavior of the two order nonlinear system, provides theoretical guidance on the application of the system, which provides a powerful help for further research on the vibration system theory.

## Acknowledgement

In this paper, the research was supported by Chinese Natural Science Foundation (Project No. 11361048) , Yunnan Natural Science Foundation (Project No.2017 ) and Qujing Normal University Natural Science Foundation (Project No.ZDKC2016002).

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