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Fractional Control of An Active Four-wheel-steering Vehicle

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Abstract. A four-wheel-steering (4WS) vehicle model and reference model with a drop filter are constructed. The decoupling of 4WS vehicle model is carried out. And a fractional $PI^{\lambda}D^{\mu}$ controller is introduced into the decoupling strategy to reduce the effects of the uncertainty of the vehicle parameters as well as the unmodelled dynamics on the system performance. Based on optimization techniques, the design of fractional controller are obtained to ensure the robustness of 4WS vehicle during the special range of frequencies through proper choice of the constraints. In order to compare with fractional robust controller, an optimal controller for the same vehicle is also designed. The simulations of the two control systems are carried out and it reveals that the decoupling and fractional robust controller is able to make vehicle model trace the reference model very well with better robustness.

1. Introduction

It is widely accepted that safety as well as comfort is one of the most significant goals of vehicle control. The active four-wheel-steering system (4WS) is widely applied in trucks to improve the handling stability at high speeds and the manoeuverability at low speeds. And applying the 4WS system into the intelligent vehicle systems (IVSs) can make it track navigation route [1].

Since the 4WS is developed, lots of control strategies and methods are put forward to achieve the vehicle control goals such as feedforward control, optimal control, feedback control, sliding mode control, μ synthesis, fuzzy control, H_∞ control, decoupling control, neuralnetwork control, adaptive control, and linear parameter varying system control [2, 3] etc. The relative researches have shown that the decoupling control of vehicle steering characteristics can improve the vehicle's safety, driving, even the accurate trajectory tracking because of vehicle's individual control of the lateral and yaw motions [3]. A new fractional robust control scheme based on the fractional calculus is proposed in this paper to overcome the drawbacks of parameter perturbations and other disturbance.

The subsequent parts of this paper are as follows: The 4WS vehicle model is elaborated in Section 2 as well as the reference model is presented in Section 3. A optimal controller based on the reference model and the fractional control based on the decoupling of Linear Vehicle Model are designed for gaining robustness against the system uncertainties in Section 4 and 5 respectively.The robust analysis of the linear models and the nonlinear one are presented in Section 6. Finally, the comprehensive conclusion is summarized.

2. 4WS Vehicle Model

Here only lateral motion and yaw motion are considered, thus the nonlinear vehicle model is given by equation [1]:

$$\begin{cases} m u_x \left(\dot{\beta} + \gamma\right) = F_{yf} \cos\delta_f + F_{yr}\cos\delta_r \\ I_z \dot{\gamma} = l_f F_{yf} \cos\delta_f - l_r F_{yr}\cos\delta_r \end{cases} \tag{1}$$

where m is the vehicle mass, u_x is the vehicle longitudinal velocity (which is used as the vehicle speed), and u_y is the lateral velocity, l_f is the longitudinal distance from the center of gravity to the front axle, δ_f is the front wheel steering angle and δ_r is the rear one, β is the sideslip angle and γ is the yaw rate, I_z is the moment of inertia about z axis, l_r is the distance from the center of gravity to rear axle, F_{yi} are the lateral force on the front and rear tires (i = f, r). According to Pacejka's Magic Formula, F_{yi} can be modeled as follows:

$$F_{yi} = \varphi D_i \sin(C_i \arctan(B_i (1 - E_i)\alpha_i / \varphi + E_i \arctan(B_i \alpha_i / \varphi))) \tag{2}$$

where α_i represents the sideslip angle of tire (where i means the tire number), φ represents the adhesion coefficient of road ranging from 0.2 (the slippery roads) to 1 (the dry ones).

As long as the lateral acceleration is less than 0.4g, the relationship between the lateral force and the sideslip angle becomes linear. Let x=[γ β]^T, u=[δ_f δ_r]^T, then

$$\dot{x} = Ax + Bu \tag{3}$$

$$A = \begin{bmatrix} \dfrac{l_f^2 k_f + l_r^2 k_r}{I_z u_x} & \dfrac{l_f k_f - l_r k_r}{I_z} \\ \dfrac{l_f k_f - l_r k_r}{m u_x^2} - 1 & \dfrac{k_f + k_r}{m u_x} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, B = \begin{bmatrix} \dfrac{-l_f k_f}{I_z} & \dfrac{l_r k_r}{I_z} \\ \dfrac{-k_f}{m u_x} & \dfrac{-k_r}{m u_x} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix},$$

On the basis of equation (3), the matrix transfer function is rigorously expressed as follows:

$$\begin{bmatrix} \gamma(s) \\ \beta(s) \end{bmatrix} = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \begin{bmatrix} \delta_f(s) \\ \delta_r(s) \end{bmatrix} \tag{4}$$

3. Reference model

The linear 2-DOF model with uniform circular motion is taken as the reference model [1,4]. Assuming l_{fd} is the length from CG to front axle and l_{rd} is the one to rear axle, β_d is the sideslip angle and γ_d is the yaw rate, x_d=[γ_d β_d]^T, $u_d = \delta_f$, the equation is expressed as:

$$\dot{x}_d = A_d x_d + B_d u_d \tag{5}$$

$$A_d = \begin{pmatrix} \dfrac{l_{fd}^2 k_f + l_{rd}^2 k_r}{I_z u_x} & \dfrac{l_{fd} k_f - l_{rd} k_r}{I_z} \\ \dfrac{l_{fd} k_f - l_{rd} k_r}{m u_x^2} - 1 & \dfrac{k_f + k_r}{m u_x} \end{pmatrix}, \quad B_d = \begin{pmatrix} \dfrac{-l_{fd} k_f}{I_z} \\ \dfrac{-k_r}{m u_x} \end{pmatrix}.$$

In this paper the ideal reference model is obtained by filtering the sideslip angle of the vehicle with the neutral steering characteristics. Its filtering magnitude can be adjusted in real-time and depends on the drivers' requirements. Here the transfer function for the drop filter is as follows,where η and ξ are the coefficient and the damping coefficient of the system, ω_n is the circular frequency of the system.

$$G(s) = \frac{\eta \omega_n^2}{s^2 + 2\xi\omega_n + \omega_n^2} \tag{6}$$

4. Optimal Controller Design

The optimal controller is aimed to find an optimization signal $u_c(t)$ which minimizes the performance index J described by

$$J = \int_0^\infty \left(x^T Q x + u_c^T R u_c\right) dt \tag{7}$$

where the weighting coefficient $Q \geq 0$, and the input weighting coefficient $R>0$. The control input u_c that minimizes Equation (8) is $u_c=-K_c x$, where K_c is the optimal feedback coefficient matrix given by $K_c=R^{-1}B^T P$. The positive definite matrix P is the solution of the Riccati matrix equation:

$$-PA - A^T P + PBR^{-1}B^T P - Q = 0 \tag{8}$$

5. Decoupling and Fractional Robust Controller Design

Considering the effect of the parameter uncertainty and the un-modelled dynamics, a new type of fractional controller is designed. The control strategy is mainly to revise the front and rear steering angles by the difference of state variables between the actual model and the reference model with a drop filter, the latter reduces sideslip angle of the neutral steering characteristics. Figure 1 is the control block diagram of system.

5.1. Decoupling of Linear Vehicle Model

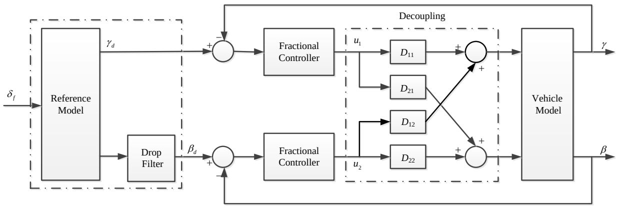

Fig.1 4WS decoupling and fractional robust control system

Here it is assumed that γ and β can be measured (in fact, β could be usually evaluated through a Kalman filter). To ensure that γ and β are controlled respectively by two independent input variables, two inputs u_1 and u_2, are introduced as shown in Figure 1. The relationship between the steering angles of front wheel and the rear one (δ_f and δ_r) and the newly introduced variables (u_1 and u_2) is as follows [5]:

$$\begin{pmatrix} \delta_f(s) \\ \delta_r(s) \end{pmatrix} = \begin{pmatrix} D_{11}(s) & D_{12}(s) \\ D_{21}(s) & D_{22}(s) \end{pmatrix} \begin{pmatrix} u_1(s) \\ u_2(s) \end{pmatrix} \tag{9}$$

Only when $\begin{pmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{pmatrix} \begin{pmatrix} D_{11}(s) & D_{12}(s) \\ D_{21}(s) & D_{22}(s) \end{pmatrix} = \begin{pmatrix} G_{11}(s) & 0 \\ 0 & G_{22}(s) \end{pmatrix}$, γ and β can be controlled by u_1 and u_2 respectively, and $D_{ij}(s)$ can be obtained.

5.2. Fractional $PI^{\lambda}D^{\mu}$ Robust Controller Design

Fractional $PI^{\lambda}D^{\mu}$ controller, which is not sensitive to the system parameter uncertainty, is robust and can be designed flexibly. The transfer function can be expressed as [6]:

$$G_{cf}(s) = \frac{U(s)}{E(s)} = K_P + \frac{K_I}{s^{\lambda}} + K_D s^{\mu} \tag{10}$$

where K_P, K_I and K_D are the proportional, integral and differential coefficient, λ and μ are the integral and differential order. When using the minimum of $\left|G_{cf}(\omega_{cg})G_{11}(\omega_{cg})\right|$ and $\left|G_{cf}(\omega_{cg})G_{22}(\omega_{cg})\right|$ as the objective function and meeting the five constraints (see reference [6]), the particle swarm optimization algorithm can be used to get these five parameters K_P, K_I, K_D, λ and μ.

6. Robust Analysis of Linear Vehicle Model

For validating the robustness of this fractional robust controller, the proposed controller is used in the vehicle model and performs a Fishhook manoeuvre as shown in Figure 2. With the same front wheel steering input and varied vehicle parameters, the yaw rates and sideslip angles of the linear and nonlinear 4WS for the optimal controller (OC) and the decoupling and fractional controller (DFC) are shown in Figures 3 and 4.

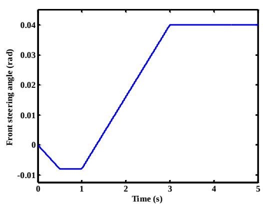

Fig.2 Fishhook manoeuvre

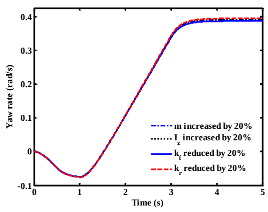

Fig.3(a) Yaw rate with OC

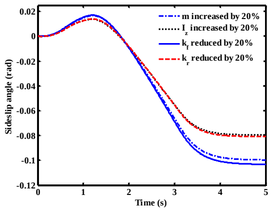

Fig.3(b) Sideslip angle with OC

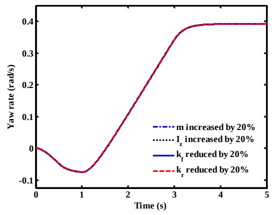

Fig.3(c) Yaw rate with DFC

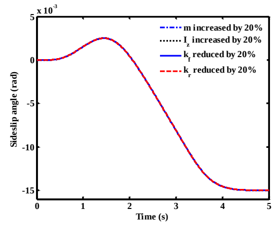

Fig.3(d) Sideslip angle with DFC

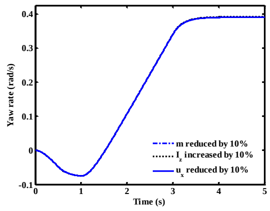

Fig.4(a) Yaw rate with OC

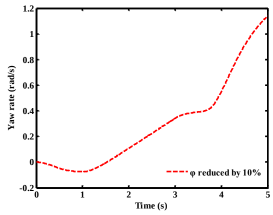

Fig.4 (b) Yaw rate with OC

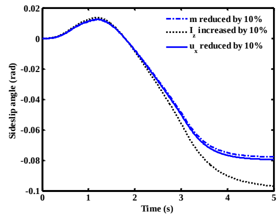

Fig.4(c) Sideslip angle with OC

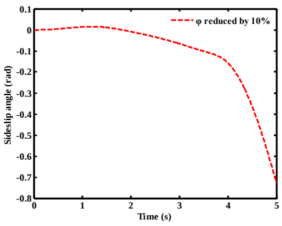

Fig.4(d) Sideslip angle with OC

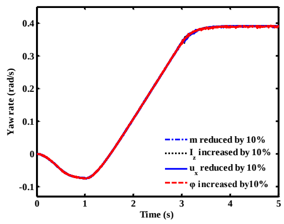

Fig.4(e) Yaw rate with DFC

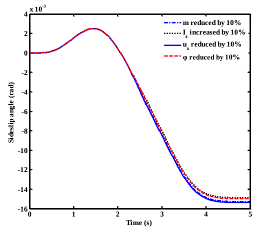

Fig.4(f) Sideslip angle DFC

As is vividly shown in Figures 3, the sideslip angles of the 4WS with the optimal controller greatly change with varied vehicle parameters, and those of the 4WS with the decoupling and fractional controller remains almost the same. Consequently the robustness of the fractional $PI^{\lambda}D^{\mu}$ controller based on the decoupling is much higher than that of optimal controller.

It is graphically shown in Figures 4 that for the optimal controller, the output yaw rates are almost the same and the sideslip angles greatly change with the varied m, I_z and u_x. Owing to the variation of the parameter φ the vehicle becomes unstable. While with the same vehicle parameters' variations, the yaw rates of the 4WS with the decoupling and fractional controller are almost the same, and the variations of the sideslip angles are extremely small. From the above sensitivity analysis it can be clearly concluded that the fractional decoupling controller has a higher robustness than the optimal controller for the nonlinear vehicle model.

7. Conclusion

In the paper, the decoupling of 4WS model is carried out and the fractional $PI^{\lambda}D^{\mu}$ controller is used to overcome the drawbacks of parameter perturbations and outer disturbance. The design of the fractional $PI^{\lambda}D^{\mu}$ controller and the numerical algorithm of the simulation are presented. The simulation results show that uncertain vehicle model can closely follow the responses of the reference model. The whole vehicle system has good robustness for outer disturbance and parameter perturbations.

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