

# A fault isolation method based on the incidence matrix of an augmented system

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**Abstract.** A new approach is proposed for isolating faults and fast identifying the redundant sensors of a system in this paper. By introducing fault signal as additional state variable, an augmented system model is constructed by the original system model, fault signals and sensor measurement equations. The structural properties of an augmented system model are provided in this paper. From the viewpoint of evaluating fault variables, the calculating correlations of the fault variables in the system can be found, which imply the fault isolation properties of the system. Compared with previous isolation approaches, the highlights of the new approach are that it can quickly find the faults which can be isolated using exclusive residuals, at the same time, and can identify the redundant sensors in the system, which are useful for the design of diagnosis system. The simulation of a four-tank system is reported to validate the proposed method.

## 1. Introduction

For improving the reliability and safety of industrial systems, model-based fault detection and isolation (FDI) is gaining increasing attention both in the scientific community and industrial applications. The structural analysis approach is an important branch of model-based FDI, which neglects the analytical form of a system model and only focuses on the structural model, i.e. only considering the relation between equations and variables [1]. In general, the structural analysis approach with no numerical problems is performed based on graph theory, which is more efficient to identify system diagnosis properties than analytical methods. Therefore, it can be used to handle large scale and complex systems and can be used in the early design stages [2].

Model-based FDI includes two important tasks: fault detection and fault isolation. Usually, fault detection is performed to check whether a fault occurrence in a system. Fault isolation is to differentiate between two possible fault occurrences. At present in the field of fault diagnosis based on structural analysis, there are two primary fault isolation approaches. The first is making use of fault signature matrix (FSM) to isolate faults [3], and the other is taking advantage of fault pairs [4].

The FSM is a binary table, which is used to characterize the diagnosability properties of a diagnosis system. In the FSM, the rows represent the set of residuals and the columns the set of faults. An  $ij$ -element of the matrix includes the pattern 1 if fault  $j$  can be detected by the  $i$ th residual, otherwise it is 0. The “signature of fault  $j$ ” is the binary word formed by the  $j$ th column of a FSM. Two faults are not distinguishable, if the corresponding signatures are identical [3]. However, the residuals in a FSM usually are obtained by calculating all of the Minimal Structurally Overdetermined (MSO) sets of a system model, where a MSO set is a subsystem model which has a redundancy degree of one, i.e. the



set of equations of the subsystem which has one more equation than unknown variables. However the associated computational complexity increases exponentially with the redundancy degree, it's a complicated and time-consuming process to get all possible MSO sets in practice [5].

The isolation approach based on fault pairs is proposed by the fault isolability representation, i.e. one fault is structurally isolable from another. In general, it is assumed that a fault  $f$  is involved in only one equation, which known as fault equation and denoted by  $e_f$ , then the isolability relations can be transformed into the incidence relations between the fault equations. If all faults are detectable in a system, the isolability relation is symmetric [4]. Then, for a system with  $n$  detected faults, more than  $n \times (n-1)/2$  times calculation is performed to derive overdetermined subsets and the isolability properties can be identified from those results. However, the calculation of this approach is still an issue for a large scale and complex system [6].

For ease of presentation, different system models are interpreted as follows. In the generic case the studied process is a stable system, and the fault free behavioral model of the system is called *original system model*. After adding some sensors to monitor the process state, a new model consists of the original system model and the sensor measurement equations, which is called *monitored system model*. In the existing structural analysis approaches, fault signals usually are neglected as know variables. Whereas, from a computational point of view, a fault signal can be regarded as an additional state variable, so an *augmented system model* can be constructed by the original system model, fault variables and the sensor measurement equations [7, 8]. During estimating the fault variables of a system model, the fault isolability properties can be found from the dependence relations of the fault variables. For simplicity, it is assumed that the sensor measuring is valid, and no sensor faults. Single fault can only violate one equation and one equation at most affected by a fault, so there is a one-to-one relationship between a fault signal and a fault equation. And  $f$  is adopted to represent both the fault signal and the fault variable in this paper. In a monitored model and in an augmented model, the equations affected by a fault  $f$  are all denoted  $e_f$ , the only difference is that in the first case  $f$  can be regarded as  $f \equiv 0$ , but in the latter  $f$  may not be 0.

As noted above, the two existing approaches have in common that it is individual fault isolates from others, which leads to large calculation. Some faults of a system are not classified with special isolability property, and the searched range of no distinguishing faults is not reduced. In addition, during identifying fault isolability they cannot find out the sensor redundancy. From the viewpoint of solving variables, a new isolation approach based on the incidence matrix of an augmented system is proposed in this paper, that can find which fault can be diagnosis by an independent residual and, at the same time, determine which sensors can be removed as redundancy, in another words, removing those sensors do not decrease the fault isolability. What is more, the results of this approach are practically valuable for fast isolating faults and optimal sensor placement.

The paper is organized as follows. Following the introduction session, a background of structural analysis for FDI is first presented in Section 2. Then, in Section 3, fault isolability properties of an augmented system are provided. In Section 4 a new approach for isolating fault is proposed and the availability of the approach is validated by means of the results of an academic example in Section 5. Finally, this paper is summed up in a conclusion in Section 6.

## 2. Background of FDI based on structural analysis

### 2.1. Bipartite graph and matching

In the field of fault diagnosis based on structural analysis, the structural model of a system can be described using a bipartite graph (or equivalently its incidence matrix), since only the relation between equations and variables is considered [2]. Let  $G(E, V, A)$  be a bipartite graph, in which  $E$  is a set of model equations,  $V$  is a set of unknown variables and  $A$  a set of edges. An edge  $(e_i, v_j) \in A$ , for  $e_i \in E$  and  $v_j \in V$ , holds for variable  $v_j$  is included in equation  $e_i$ . The corresponding incidence matrix  $M$  of the bipartite graph  $G$  is a boolean matrix where each row in the matrix represents an equation and each column a variable.  $M(i, j) = 1$  if  $(e_i, v_j) \in A$ , 0 otherwise. Then, the structure of an augmented system

incidence matrix can be regarded as a combination, which is constructed by a monitored system incidence matrix and the corresponding fault matrix (see Table 1).

Additionally, the notion of matching in a bipartite graph is the important tool for structural analysis approaches, which is used to identify the overdetermined subsystems that imply the diagnosis properties. A matching  $M$  is a subset of edges, and there is no two edges sharing the same node in  $M$ . A matching with the maximal number of edges is called a maximum matching. A vertex  $x$  is said to be covered by a matching  $M$  when there is an edge  $l \in M$  and  $x$  is incident to  $l$ . A matching is a complete matching with respect to a vertex set, if the matching covers all vertices in the set. And a matching is a perfect matching of bipartite graph  $G$ , if the matching covers every vertex of  $G$ .

## 2.2. Dulmage-Mendelsohn decomposition and algorithm

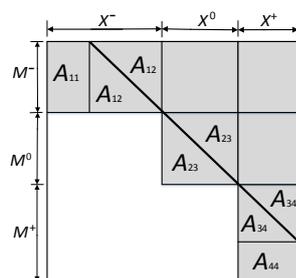
### 2.2.1. DM decomposition

The Dulmage-Mendelsohn (DM) decomposition is an important theoretical tool in the structural analysis approach [9]. That can be implemented by only permuting rows and columns of the incidence matrix to derive an upper triangular matrix. Figure 1 shows a general DM decomposition, where the gray-shaded areas include ones and zeros, but the white areas only contain zeros. Moreover, the bold line represents a maximum matching in the matrix, which shows a calculation path to be performed sequentially for some unknown variables.

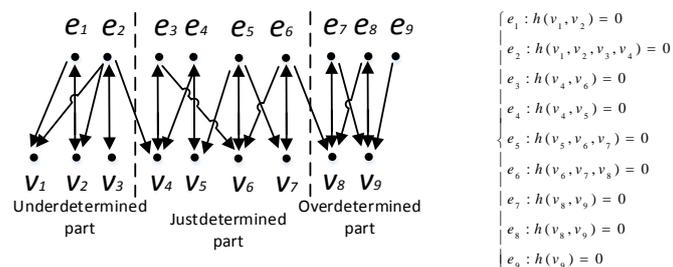
The decomposition of a model  $M$  is illustrated in Figure 1 where:  $M^+$  is the structurally overdetermined part i.e.  $|M^+| > |X^+|$  this means that it has more equations than unknown variables.,  $M^0$  is the structurally justdetermined part,  $|M^0| = |X^0|$ .  $M^-$  is the structurally underdetermined set,  $|M^-| < |X^-|$ .

It is well noted that there are two important properties on the DM decomposition for our new approach. First, it is well known that the model-based FDI is performed based on the consistency check of system redundancies. So the overdetermined part is useful for fault diagnosis since only the part includes redundancy [2] [10]. Second, there is a complete matching to the unknown variables of the overdetermined part, and the number of equations is as many as the unknown variables in the justdetermined part, then all variables in the justdetermined subsystem can be calculated unambiguously by the maximum matching.

For the convenience of presentation, the areas in a DM decomposition matrix are denoted as follows (see Figure 1). The area of unmatched variables in the underdetermined part is denoted by  $A_{11}$ ,  $A_{12}$  is the matched area in the underdetermined part,  $A_{23}$  is the justdetermined area,  $A_{34}$  is the matched area in the overdetermined part, and  $A_{44}$  is the unmatched area in the overdetermined part.



**Fig.1**DulmageMendelsohn decomposition of a model  $M$



**Fig.2** A system model and its DM decomposition

### 2.2.2. DM decomposed algorithm

Any finite-dimensional system can be divided into three main parts, namely, overdetermined, justdetermined and underdetermined parts, which can be obtained by the following algorithm [11]:

- (1) Get a maximum matching  $M$  in a bipartite graph  $G(E, V, A)$ .
- (2) Put up the directed graph  $G'$  from  $G$  by means of replacing each edge in  $M$  with bi-oriented edges, and then orienting all other edges from  $E$  to  $V$ .

- (3) Find all equation vertices which are not covered by the matching  $M$ , and calculate all descendants of the sources, then the overdetermined part  $M^+$  is composed of the descendant vertices.
- (4) Similarly, find all variable vertices which are not covered by  $M$ , and calculate all ancestors of the sinks, the underdetermined part  $M^-$  is composed of the ancestor vertices.
- (5) Finally, the justdetermined part  $M^0$  is obtained by  $M^0 = G - M^+ - M^-$ .

Figure 2 illustrates the DM decomposition of a system model in the form of a bipartite graph. It is worth mentioning that the DM decomposition result of any finite-dimensional graph is unique, which is irrelevant to the choice of a maximum matching in the diagnosis system [12]. In this paper, the decomposition is implemented with the help of the *dmperm* command in MATLAB®.

### 2.3. The basic concepts of fault diagnosis

In this section, some basic concepts are presented for explaining the properties of fault diagnosis. It is worth pointing out that those concepts are based on the fact that a fault  $f$  is regarded as a known variable.

*Definition 1 (Fault detectability)* [4]: A fault  $f$  is structurally detectable in a monitored system model  $M$ , if  $e_f \in M^+$ .

*Definition 2 (Fault isolability)* [4]: Given a monitored system model  $M$  and a set of fault  $F$  which affect the equations of  $M$ , a fault  $f_i \in F$  is structurally isolable from  $f_j \in F$ , if  $e_{f_i} \in (M \setminus \{e_{f_j}\})^+$ .

*Definition 3 (Fault diagnosability)* [13]: Given a monitored system model  $M$ , and a set of fault  $F$  which affect the equations of  $M$ , a fault  $f_i \in F$  is diagnosable if it is detectable and it holds  $e_{f_i} \in (M \setminus \{e_{f_j}\})^+$  for all  $f_j \in F \setminus f_i$ .

A conclusion can be drawn from the above definitions that fault detectability is the precondition of fault isolability. It should be emphasized that according to definition 2 some faults cannot be isolated from each other when they affect the equations in the same MSO set. So, for the sake of simplification, in this study it is assumed that the considered monitored system is an overdetermined subsystem, which redundancy degree is more than one. After transforming a monitored system model into an augmented system model, a corresponding DM decomposition can be obtained by the algorithm in section 2.2. Then a calculation ordering for some unknown variables can be derived by means of the decomposition. From the calculation ordering of a fault variable  $f$ , in other words during estimating the fault variable  $f$ , the dependence relations between  $f$  and other fault variables can be found, that implies the isolability of the faults. Therefore, in the next section some fault isolability properties are introduced from the viewpoint of solving variables.

## 3. The isolability performance of an augmented system

**Theorem 1:** *Given the DM decomposition of the incidence matrix of an augmented system model, if there is an overdetermined part, any fault variable of the model cannot exist in the overdetermined area (the areas of  $A_{34}$  and  $A_{44}$ ).*

**Proof:** Since the overdetermined part is composed of  $A_{34}$  and  $A_{44}$  in a DM decomposition, it is necessary to prove this theorem in those two areas respectively.

- (1) Any fault variable  $f$  cannot exist in the unmatched area  $A_{44}$  of the overdetermined part.

Figure 1 shows that in a DM decomposition the bold line represents a maximum matching, this means that the elements on the bold line are all 1. In this work, it is assumed that a fault only affects one equation, i.e. for a fault variable  $f$  it only appears in the corresponding equation  $e_f$ . Therefore if a fault variable  $f_1$  exists in the area  $A_{44}$ , the element of the column corresponding to variable  $f_1$  on the bold line must be 0. It is inconsistent with the DM decomposition. So, any fault variable cannot exist in the area  $A_{44}$ .

- (2) Any fault variable  $f$  cannot exist in the area  $A_{34}$

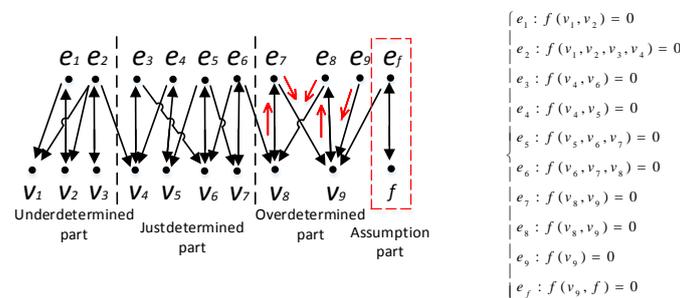
**Reduction to absurdity.** Suppose in the area  $A_{34}$  there is a fault variable  $f$ , the corresponding fault equation is  $e_f$ . Since there is a maximum matching in the DM decomposition, in a similar way with the knowledge of step 1, it is can be proved that the position  $(e_f, f)$  in the matrix must be on the bold line

of the DM decomposition, i.e. the edge  $(e_f, f)$  must belong to a maximum matching in the bipartite graph model.

According to the DM decomposition algorithm in section 2.2, the overdetermined part of a bipartite graph model is composed of all vertices, which are the descendants of all equation vertices that are not covered by a maximum matching. Since fault variable  $f$  only belongs to  $e_f$ , there are not oriented edges between  $f$  and any other equations. So, the descendants of other equations do not include variable  $f$ .

For example, in Figure 3 there is a bipartite graph model, equation vertex set  $\{e_1, e_2, \dots, e_9\}$ , and variable vertex set  $\{f, v_1, \dots, v_9\}$ , where  $f$  is the fault variable and the corresponding fault equation  $e_f$ . Suppose that edge  $(e_f, f)$  belongs to a maximum matching  $M_1$  in the model, and equation  $e_9$  is not covered by the maximum matching. As Figure 3 shows, there is not a bi-oriented edge between  $e_f$  and any other variable, for instance  $v_9$ , so the descendants of equation  $e_9$  do not include variable  $f$ . This means that edge  $(e_f, f)$  does not belong to the maximum matching  $M_1$ , and it is in contradiction with the above assumption. So, a fault variable  $f$  also cannot exist in the area  $A_{34}$ .

Then theorem 1 is proved.



**Fig.3** Assumption of a fault variable in the overdetermined part

For ease of presentation on the diagnosis property, some operations can be chose by the structure form of an augmented system incidence matrix and the solving way of a maximum matching. In the form of structure, the incidence matrix of an augmented system can be regarded as the combination about the left part — the incidence matrix of the corresponding monitored model and the right part — the fault matrix (for example in Table 1). In the matching way, a maximum matching can be obtained by giving priority to matching the state variables of the system [14]. It is worth noting that the above operations do not change the structure of the augmented model, and according to the unique result on the DM decomposition, the overdetermined part of the augmented model is also unique. Using this way to deal with augmented models, the results are also universal.

**Theorem 2:** Given the DM decomposition of an augmented system model, if there is an overdetermined part in the decomposition, it implies that there is sensor redundancy in the corresponding monitored system.

**Proof:** Since an original system is a stable system in this study, the original system is a justdetermined system. By giving priority to matching the state variables in the corresponding augmented incidence matrix, a maximum matching can be derived, where the equations and state variables of the original system model are all matched.

Theorem 1 demonstrates that fault variables do not appear in the overdetermined part. If there is an overdetermined part in the DM decomposition of the augmented system, there are some redundant equations to verify some state variables, in other words, those state variable can be evaluated without those redundant equations in the overdetermined part. Since the system behavior equations are all matched in the DM decomposition, those redundant equations must be sensor equations. This means that there are redundant sensors in the corresponding monitored system.

It is well noted that the division of the three main parts of the DM decomposition is unique, so the overdetermined part of an augmented system model is unique no matter which maximum matching is chose.

Theorem 2 is proved.

It is should be emphasized that the redundant sensors in the system can be found easily by giving priority to matching the state variable of an augmented system model.

**Theorem 3:** *Given the DM decomposition of an augmented system model. If a fault variable  $f$  exists in the area  $A_{23}$  of the justdetermined part, there is a residual  $r$  which is only sensitive to  $f$  and  $f$  is isolable from any other fault in the system.*

**Proof:** Suppose a fault variable  $f$  appears in the justdetermined part  $A_{23}$  of an augmented system model, and  $e_f$  is the corresponding fault equation. According to the properties of a DM decomposition, the variables in the justdetermined and overdetermined parts are all complete matched, this means that the variables of area  $A_{23}$  can be estimated with the known variables in  $A_{34}$  and/or  $A_{44}$ .

In this paper, single fault  $f$  is only involved in one equation. Thus, in an augmented system, there is only one entry in the corresponding column of the  $f$  being equal to 1. According to the proof of theorem 1, the position  $(e_f, f)$  of the matrix is on the maximum matching line, so there is a calculation ordering for evaluating the fault variable  $f$  which does not affect the evaluating of any other variable. Similarly, the other fault variable also does not affect the evaluation on  $f$ . The equation set of the calculation ordering for variable  $f$  is denoted by  $M_{fA}$ . So in the augmented system, the model  $M_{fA}$  is a justdetermined subsystem.

Therefore, the variable  $f$  can be calculated theoretically by a computational expression, and which do not include other fault variable. The computational expression is a fault indicator, which is a residual that only sensitive to the fault  $f$ . The residual is termed *exclusive residual* for  $f$ .

For the sake of presenting the isolability, the monitored system  $M$  is considered. In the monitored system case, the model  $M_f$  which corresponds to  $M_{fA}$  is an overdetermined subsystem since the variable  $f$  is regarded as a known variable. Then  $M_f = M_f^+$ ,  $e_f \in M_f$ . And for any other fault  $f_j$ , it holds  $e_{fj} \notin M_f$ , then  $M_f \subseteq (M \setminus \{e_{fj}\})^+$ , thus  $e_f \in (M \setminus \{e_{fj}\})^+$ . Therefore, according to definition 2, the fault  $f$  which locates in the area  $A_{23}$  is isolable from any other fault, and  $f$  is a diagnosable fault.

Theorem 3 is proved.

#### 4. A new approach for isolation fault

A new fault isolation approach can be obtained from the above theorems, which based on the DM decomposition of an augmented model. According to the faults and the corresponding fault equations, an augmented system model can be transformed from a monitored system model, and the corresponding DM decomposition can be obtained.

If there is an overdetermined part in the decomposition, the monitored system has sensor redundancy. If a fault equation  $e_f$  locates in the justdetermined part of the decomposition and the corresponding fault  $f$  is a diagnosable fault which is isolable from any other fault in the system. The pseudo-code of the corresponding new algorithm is provided as follows.

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**Algorithm:** Find redundant sensors and diagnosable faults with exclusive residuals

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Require: the augmented system model  $M_A$  and the corresponding fault equation set  $FEqu_{\Sigma}$

**Function** ReSensorAndUniFault( $M_A, FEqu_{\Sigma}$ )

UniReFEqu  $\leftarrow \phi$      % initialize the set of fault equations that the faults is isolated with exclusive residuals

UniReFault  $\leftarrow \phi$      % initialize the set of fault corresponds to UniReFEqu

ReSeEqu  $\leftarrow \phi$      % initialize the set of redundant sensor equations

$(M_A^-, M_A^0, M_A^+) \leftarrow \text{DM-decompose}(M_A)$      % the DM decomposition of  $M_A$

UniReFEqu  $\leftarrow (M_A^0) \cap FEqu_{\Sigma}$      % get the set of fault equations where the faults can be isolated with exclusive residuals

**if** UniReFEqu  $\neq \phi$  **then**

    UniReFault  $\leftarrow$  UniReFEqu     % get the corresponding faults that can be isolated with exclusive residuals

**endif**

**if**  $(M_A^+) \neq \phi$  **then**

    ReSeEqu  $\leftarrow$  the redundant equations     % get redundant sensor equations

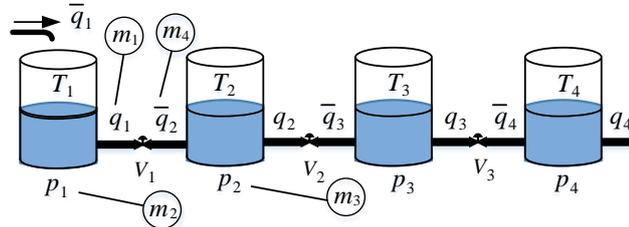
**endif**

**Return** (UniReFault, ReSeEqu)

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**5. Example**

For verifying the availability of the approach proposed in this paper, which is used to identify the fault diagnosis properties of the four-tank system in Figure 4, and then the results are checked by FSM method.



**Fig.4** Diagram of the four-tank monitored system

A four-tank system is illustrated in Figure 4, where the four tanks are denoted by  $T_1, T_2, T_3$  and  $T_4$  respectively,  $q_i$  is the outflow of tank  $T_i$ ,  $\bar{q}_i$  is the inflow of tank  $T_i$ ,  $p_i$  is the pressure in tank  $T_i$ , and the inflow  $\bar{q}_1$  is a known variable. Three valves are denoted by  $V_1, V_2$  and  $V_3$  respectively. The sensors  $m_1, m_2, m_3$  and  $m_4$  measure  $q_1, p_1, p_2$  and  $\bar{q}_2$  respectively. The leakage faults  $f_1, f_2, f_3$  and  $f_4$  are considered in the tank  $T_1, T_2, T_3$  and  $T_4$  respectively, and the leakage faults  $f_5, f_6$  and  $f_7$  affect the valves  $V_1, V_2$  and  $V_3$  respectively. The augmented model of the four-tank system is represented by the set of equations.

$$\begin{array}{llll}
 e_1: R_1q_1 = p_1 - p_2 & e_2: R_2q_2 = p_2 - p_3 & e_3: R_3q_3 = p_3 - p_4 & e_4: R_4q_4 = p_4 \\
 e_5: C_1\dot{p}_1 = \bar{q}_1 - q_1 - f_1 & e_6: C_2\dot{p}_2 = \bar{q}_2 - q_2 - f_2 & e_7: C_3\dot{p}_3 = \bar{q}_3 - q_3 - f_3 & e_8: C_4\dot{p}_4 = \bar{q}_4 - q_4 - f_4 \\
 e_9: \dot{p}_1 = dp_1/dt & e_{10}: \dot{p}_2 = dp_2/dt & e_{11}: \dot{p}_3 = dp_3/dt & e_{12}: \dot{p}_4 = dp_4/dt \\
 e_{13}: q_1 = \bar{q}_2 + f_5 & e_{14}: q_2 = \bar{q}_3 + f_6 & e_{15}: q_3 = \bar{q}_4 + f_7 & e_{16}: q_1 = m_1 \\
 e_{17}: p_1 = m_2 & e_{18}: p_2 = m_3 & e_{19}: \bar{q}_2 = m_4 & 
 \end{array}$$

Where  $C_i$  is the capacitance of tank  $T_i$ ,  $R_i$  is the flow resistance of valve  $V_i$ . It is shown that the faults  $f_1, f_2, f_3, f_4, f_5, f_6$  and  $f_7$  are involved in the fault equations  $e_5, e_6, e_7, e_8, e_{13}, e_{14}$  and  $e_{15}$  respectively.

The incidence matrix of the augmented four-tank system is shown in Table 1. It can be seen that the matrix can be regarded as a combination, which is composed of the left part that is the incidence matrix of the monitored four-tank system and the right part that is the fault matrix. The DM decomposition of the matrix is showed in Table 2. The equations  $e_5$  and  $e_{13}$  locate in the justdetermined area. According to the matching of the variables in the justdetermined part, there is mathematical expression to calculate the variables  $f_1$  and  $f_5$ , respectively, and each of the fault variables is not correlated with any other fault variable. Therefore, the faults  $f_1$  and  $f_5$  are diagnosable faults. They are isolable from any other faults, and  $f_1$  is isolable from  $f_5$  and vice versa.

Furthermore, the overdetermined part of the DM decomposition is the set of equations  $E_{A44} = \{e_1, e_{16}, e_{17}, e_{18}\}$ , and  $e_{18}$  is a redundant equation (see Table 2). Thus there is sensor redundancy in the four-tank system, and according to the properties of DM decomposition the redundant sensor equation is one of the measurement equations  $\{e_{16}, e_{17}, e_{18}\}$ .

**Table.1** Incidence matrix of the augmented four-tank system

	$q_1$	$q_2$	$q_3$	$q_4$	$p_1$	$p_2$	$p_3$	$p_4$	$\dot{p}_1$	$\dot{p}_2$	$\dot{p}_3$	$\dot{p}_4$	$\bar{q}_1$	$\bar{q}_2$	$\bar{q}_3$	$\bar{q}_4$	$f_1$	$f_2$	$f_3$	$f_4$	$f_5$	$f_6$	$f_7$
$e_1$	1	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$e_2$	0	1	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$e_3$	0	0	1	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$e_4$	0	0	0	1	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$e_5$	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$e_6$	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
$e_7$	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
$e_8$	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
$e_9$	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$e_{10}$	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$e_{11}$	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
$e_{12}$	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
$e_{13}$	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
$e_{14}$	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
$e_{15}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
$e_{16}$	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$e_{17}$	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$e_{18}$	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$e_{19}$	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0

**Table.2** DM decomposition of the incidence matrix of the augmented four-tank system

	$f_1$	$f_2$	$f_3$	$f_4$	$q_2$	$q_3$	$q_4$	$p_1$	$p_2$	$\dot{p}_1$	$\dot{p}_2$	$\dot{p}_3$	$\dot{p}_4$	$\bar{q}_1$	$\bar{q}_2$	$\bar{q}_3$	$\bar{q}_4$	$f_1$	$f_2$	$f_3$	$f_4$	$f_5$	$f_6$	$f_7$
$e_1$	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
$e_2$	0	0	0	0	0	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$e_3$	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$e_4$	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$e_{11}$	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
$e_{12}$	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0
$e_7$	1	0	0	0	0	1	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0
$e_8$	0	1	0	0	0	0	1	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0
$e_{14}$	0	0	1	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
$e_{15}$	0	0	0	1	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
$e_5$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	1	0
$e_6$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1
$e_{13}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
$e_{16}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$e_{17}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$e_{18}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$e_{19}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

For verifying the validity of the above results, FSM approach is used to calculate the fault isolability of the four-tank system and compare with those conclusions. According to the algorithm in [10], there are 21 MSO sets in the monitored four-tank system. Those MSO sets are used to generate residuals, and the transpose of the fault signature matrix of the four-tank system is presented in Table 4. As can be seen from Table 4 those residuals  $r_2, r_9$  and  $r_{10}$  are only sensitive to fault  $f_1$ , and the residual  $r_8$  is only sensitive to fault  $f_5$ . So, the faults  $f_1$  and  $f_5$  are all diagnosable fault, each of them is isolable from any other fault.

Furthermore, if removing equation  $e_{18}$  from the model of the four-tank monitored system, a revised monitored system can be obtained, and the corresponding incidence matrix is shown in Table 3. In the same way, 11 MSO sets can be derived and the transpose of its FSM is shown in Table 5. Comparing the two FSM transposes (Table 4 and Table 5), it is shown that the two monitored systems have the same isolability properties, i.e. the faults  $f_1$  and  $f_5$  are all diagnosable fault, the faults  $f_2, f_3, f_4, f_6$  and  $f_7$  are not isolable from each other. So there is sensor redundancy in the original monitored four-tank system (see Figure 4).

So the results of the new isolation approach proposed in this paper are right, and the new approach is feasible.

**Table.3** Incidence matrix of the corrected four-tank system

	$q_1$	$q_2$	$q_3$	$q_4$	$p_1$	$p_2$	$p_3$	$p_4$	$\dot{p}_1$	$\dot{p}_2$	$\dot{p}_3$	$\dot{p}_4$	$\bar{q}_2$	$\bar{q}_3$	$\bar{q}_4$
$e_1$	1	0	0	0	1	1	0	0	0	0	0	0	0	0	0
$e_2$	0	1	0	0	0	1	1	0	0	0	0	0	0	0	0
$e_3$	0	0	1	0	0	0	1	1	0	0	0	0	0	0	0
$e_4$	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0
$e_5$	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0
$e_6$	0	1	0	0	0	0	0	0	0	1	0	0	1	0	0
$e_7$	0	0	1	0	0	0	0	0	0	0	1	0	0	1	0
$e_8$	0	0	0	1	0	0	0	0	0	0	0	1	0	0	1
$e_9$	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0
$e_{10}$	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0
$e_{11}$	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0
$e_{12}$	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0
$e_{13}$	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0
$e_{14}$	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0
$e_{15}$	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1
$e_{16}$	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$e_{17}$	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
$e_{18}$	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0

**Table.4** The transpose of FSM of the four-tank system

	$r_1$	$r_2$	$r_3$	$r_4$	$r_5$	$r_6$	$r_7$	$r_8$	$r_9$	$r_{10}$	$r_{11}$	$r_{12}$	$r_{13}$	$r_{14}$	$r_{15}$	$r_{16}$	$r_{17}$	$r_{18}$	$r_{19}$	$r_{20}$	$r_{21}$	
$f_1$	0	1	1	0	0	1	0	0	1	1	1	0	0	0	0	1	1	1	1	1	1	1
$f_2$	0	0	0	1	1	1	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1
$f_3$	0	0	0	1	1	1	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1
$f_4$	0	0	0	1	1	1	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1
$f_5$	0	0	1	0	1	1	0	1	0	0	1	0	1	1	1	0	0	1	1	1	1	1
$f_6$	0	0	0	1	1	1	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1
$f_7$	0	0	0	1	1	1	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1

**Table.5** The transpose of FSM of the corrected four-tank system

	$r_1$	$r_2$	$r_3$	$r_4$	$r_5$	$r_6$	$r_7$	$r_8$	$r_9$	$r_{10}$	$r_{11}$
$f_1$	0	1	1	0	0	0	1	1	1	1	1
$f_2$	0	0	0	1	1	1	1	1	1	1	1
$f_3$	0	0	0	1	1	1	1	1	1	1	1
$f_4$	0	0	0	1	1	1	1	1	1	1	1
$f_5$	1	0	1	0	1	1	0	0	1	1	1
$f_6$	0	0	0	1	1	1	1	1	1	1	1
$f_7$	0	0	0	1	1	1	1	1	1	1	1

### 6. Conclusion

A new fault isolation approach based on augmented system structure is presented in this paper. According to the calculation correlation of the fault variables, the corresponding fault equations can be classified, which reduces the searched range of no distinguishing faults, and in the sense that it improves the isolation efficiency. After obtaining the DM decomposition of the incidence matrix of an augmented system, it can be directly determined that those faults, which corresponding fault equations locating in the justdetermined part, are diagnosable faults, i.e. each of them can be isolable from any other fault. What is more, if there is an overdetermined part in the DM decomposition, it implies that the monitored system has sensor redundancy, and the redundancy equation is a redundant sensor measurement equation, whereas, the previous isolation approaches cannot identify the sensor redundancy. In addition, it is worth mentioning that those conclusions can be obtained by only one calculation of the DM decomposition.

In this paper, the new approach is provided to increase isolation efficiency and identify the sensor redundancy at the same time, which is useful for the optimal design of a diagnosis system. However, this approach does not involve the fault isolation in the underdetermined part of the augmented system. In the subsequent paper we will try to develop a new fault isolation algorithm for identifying all of the distinguished and no distinguished faults in a system.

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