

Damage Identification of Beam with Breathing Crack under Fractal Dimension

Ying Zhao^{1,*}, Yi Chun Ren¹

¹School of Changsha University of Science & Technology, Changsha, China

*Corresponding author e-mail: 457922823@qq.com

Abstract. Damage identification has been more urgent in engineering, especially beam structure which is often damaged by ambient excitation. It is based on time-frequency domain analysis of beam with breathing crack under fractal dimension in this paper. An equivalent cantilever beam model is constructed. The stiffness cosine model with time as parameter is used to simulate the opening and closing process of crack. The most widely used box dimension in fractal dimension is selected to calculate and analyze the displacement response of beam structure. The results show that the changes of box dimensional of beam structure displacement response can determine whether there is damage or not and identify the damage degree intuitively. It is more practical for damage identification in actual engineering.

1. Introduction

As an important bearing unit of large and complex structures, beam structure has been widely applied in engineering. Generally speaking, if there is defect or crack in the beam structure, physical characteristics of the structure will change such as mass and stiffness. Therefore, it leads to changes of dynamic characteristics of the structure. Cracks of beam make the tensile stiffness of the local beam decrease at crack, thus affect the overall vibration characteristics of beam. The key to study cracks of beam is crack treatment [1]. For simulation of cracks, crack is traditionally simplified as an open crack. S.m.cheng et al. [2] propose that natural frequency of beam with breathing cracks is much smaller than that of an open crack.

The existence of cracks isn't an open state but a breathing state in fact. In 1983, Gudmunson[3] found that crack wasn't always an open state, but a periodic opening and closing process in experiment of the relationship of natural frequency among location, depth and structure of cracks. The opening and closing degrees of cracks change with structural stress state. According to this phenomenon, breaking crack model is put forward by researchers. Dr. T. Sunil Kumar [4] proposed a cosine function change model with time as parameter which identified damage degree of the cantilever beam under sinusoidal excitation by frequency domain method. In actual engineering, beam is more commonly subjected to pulse excitation and stochastic excitation. Considering damage of beam can't directly identified by using time-frequency domain under pulse excitation and random excitation, a fractal dimension theory is introduced to measure irregularity of signals.

We all know that there are many kinds of fractal dimensions. [10]. Among them, box dimension and correlation dimension are widely used. In recent years, some scholars have tried to apply fractal theory in field of civil engineering to solve practical problems[5]. Wang Buyu [7] used fractal dimension for structural damage detection. Fractal theory was used to detect and identify the damage of simply supported beam and frame structure effectively. The complexity of breathing crack model is



considered to be transferred into intuitiveness of box dimension by combining with characteristics of breathing crack model and the box dimension of beam. Box dimension is used to intuitively describe the complexity of breathing crack beam in corresponding to its displacement. Structural damage degree is identified by combined with the calculation of beam with breathing cracks under sine, random excitations. The results show that box dimension increases with the increase of damage degree. It proves that this method is effective to identify the damage of beam with breathing cracks.

2. Cantilever Beam Model With Breathing Cracks

2.1. Basic model of cantilever beam

In this paper, a simple cantilever with breathing cracks model is constructed. The vibration analysis in first modal is only considered in the vibration of cracked beam. In this analysis, schematic diagram of cantilever with breathing crack is shown in figure 1. The cantilever beam is transferred into an equivalent model of single-degree-of-freedom, as shown in Fig. 2. The opening and closing of beam with breathing cracks changes with time. Its time-varying stiffness can be simulated as a simple periodic function.

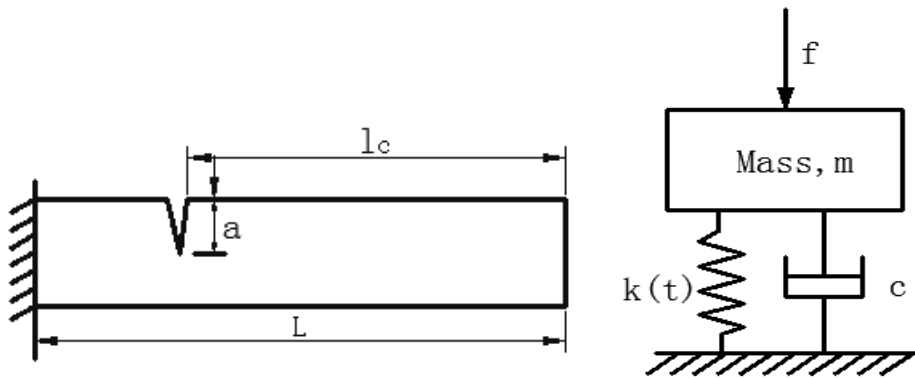


Fig. 1. cantilever beam with breathing crack

Fig. 2. single-degree-of-freedom model

2.2. Calculation of cosine crack model

Figure 3 is the load displacement curve of the crack. In the picture, P_1, P_2, P_3 represent completely open point, partially open point and completely closed point respectively. The formula of structural stiffness's expression is:

$$k = \frac{dP}{du} = k(t) \quad (1)$$

The crack state turn on load level which changes with time owing to vibration. t is an time-varying variable. Continuous stiffness of formula (1) can be resolve into Fourier polynome [9]. In this system, the first mode is used to identify the response of crack. Expression is:

$$k(t) = k_0 + k_{\Delta C} (1 + \cos(\omega t)) = k_1 + k_2 \quad (2)$$

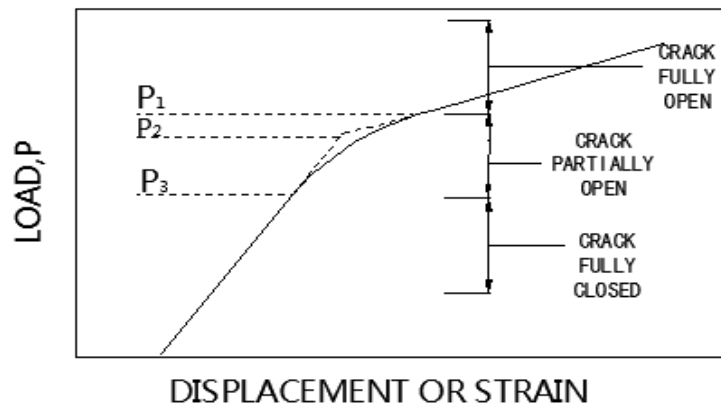


Fig. 3. Load-displacement response diagram

ω is the breathing rate of crack. When crack is completely open, $k_1 = k_0$ is the structural rigidity. The change degree of stiffness is expressed :

$$k_{\Delta c} = 1/2(k_c - k_0) \quad (3)$$

If the crack is closed, k_c is stiffness. Therefore, $k_2 = k_{\Delta c}(1 + \cos(\omega t))$. If the crack is absolutely closed, $\omega t = 2\pi n$, $n = 1, 2, 3, \dots, n$, that is $k(t) = k_c$. If the crack is completely open $\omega t = (2n - 1)\pi$, $n = 1, 2, 3, \dots, n$, that is $k(t) = k_0$. In other cases, the crack is partially closed.

Expression of dynamic equation for the following formula:

$$m\ddot{u} + c\dot{u} + k(t)u = f \quad (4)$$

Among them, f is external excitation. The broad stiffness of non-cracked beam is $k_c = 1/c = EI\pi^4/32l^3$. Stiffness of fully open crack is $k_0 = 1/c_0$ and total flexibility matrix is $c_0 = c + c_{\Delta}$. c_{Δ} is the variable of flexibility due to cracks. It is determined by the following formula:

$$c_{\Delta} = \frac{72 l_c^2 \pi (1 - \nu^2)}{Ewb^4} \varphi \quad (5)$$

w and b are width and height of beam. ν is Poisson's ratio. The function of φ is as follows:

$$\varphi = 19.60 \frac{a^{10}}{b^8} - 40.69 \frac{a^9}{b^7} + 40.07 \frac{a^8}{b^6} - 32.99 \frac{a^7}{b^5} + 20.30 \frac{a^6}{b^4} - 9.98 \frac{a^5}{b^3} + 4.60 \frac{a^4}{b^2} - 1.05 \frac{a^3}{b} + 0.63 a^2$$

a is the depth of cracks. The displacement response and acceleration response can be obtained by substituting formula (2), (3), (5) into the formula (4).

3. Calculation Of Fractal Dimension

Fractal dimension is used to describe natural objects, time series or graphic complexity or irregularity. The box counting method is the simplest and most practical method of fractal dimension to calculate the random fractal. Suppose X is nonempty bounded subset of R^n , $N(X, \varepsilon)$ represents that the maximum diameter is ε . It can also cover the minimum number of X set. The box dimension of X is defined as:

$$\dim_B X = \lim_{\varepsilon \rightarrow 0} \frac{\ln N(X, \varepsilon)}{\ln(1/\varepsilon)}$$

4. Identification of Crack By Beam Response

4.1. Numerical Simulation of Sine Excitation

Basic parameters of the cantilever beam used in this paper include :total length($L=230\text{mm}$), sectional area ($b \cdot h=20 \times 20\text{mm}$), the distance of crack and its end ($l_c=0.9L$), modulus of elasticity ($E=2.5 \times 10^3 \text{N/mm}^2$), density ($\rho =1200 \times 10^{-9} \text{Kg/mm}^3$), Poisson's ratio (0.31) and damping factor ($c = 0.15$). The natural frequency is 91HZ. The natural frequency is $\omega_1=571.769\text{r/s}$. The excitation frequency is $\omega = \omega_1 / 2$. The external force is $P=10\sin(\omega t)$. The initial displacement is $u(0) = 0 \text{ mm}$. The initial velocity is $u'(0) = 0 \text{ mm/s}$ [11]. In this article, we use MATLAB to analyze data. The displacement response solver is adopting Dormand-Prince (ode45) with variable-step. Figure 4 is a pure sine wave which shows the displacement response of the non-crack beam under sinusoidal excitation. As the cracks deepen, harmonic amplitude increases and is consistent with the results in literature[4]. FRACLAB fractal toolkit provided by Canus etc. is used for the box counting method. [12]. Table 1 shows the box dimension at crack depth ($a = 0\text{mm}, 2\text{mm}, 4\text{mm}, 6\text{mm}$ and 8mm). The crack depth increases with the box dimension. The amplitude of harmonic also increases. It proves the validity of box dimension for the damage identification of the breathing crack model.

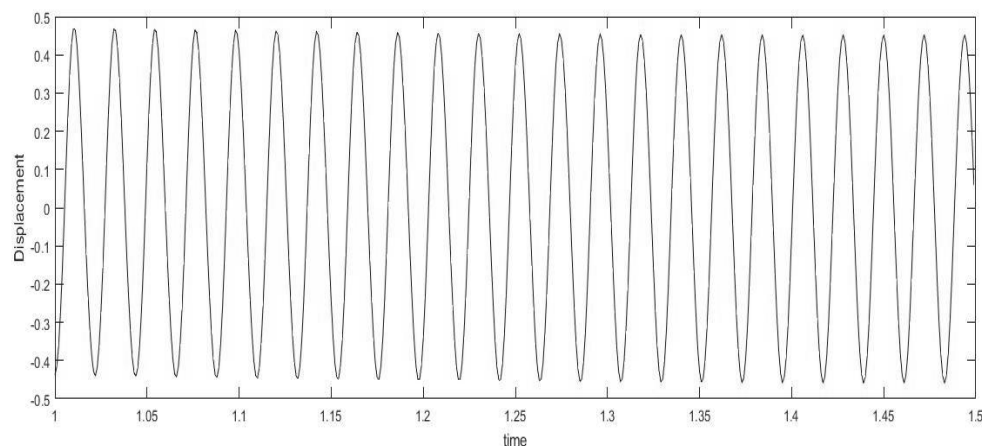


Figure 4. Displacement response of the non-crack cantilever

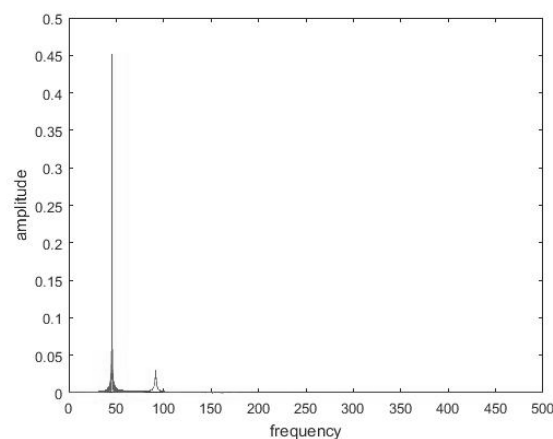


Figure 5. Frequency domain of the non-crack cantilever

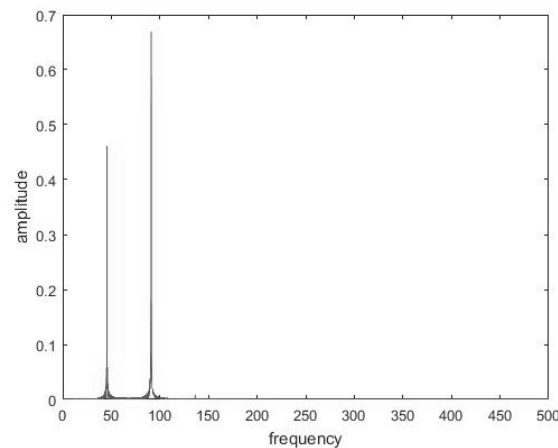


Figure 6. Cantilever frequency domain at crack depth of 8mm

Table 1. Displacement box dimension of structures with different crack depths under sinusoidal excitation

Crack depth a/mm	0	2	4	6	8
Box dimension	1.37	1.38	1.40	1.42	1.43

4.2. Numerical Structure simulation under random excitation

The random excitation is one of the most commonly used excitations in engineering. So structural damage identification under random excitation is more meaningful. The basic parameters of section 4.1 is used as the basic parameters of the cantilever beam in this section. The white noise ambient excitation is adopted in the random excitation. Figure 7 shows the time-domain response of the non-crack beam under white noise excitation. Figure 8 (a) shows the frequency domain response of the non-crack beam under white noise excitation. Fig. 8 (b) shows the frequency domain response at the crack depth of 8mm. Table 2 shows box dimension of displacement under the white noise excitation at crack depths ($a = 0\text{mm}, 2\text{mm}, 4\text{mm}, 6\text{mm}, 8\text{mm}$). The box dimension without crack is 1.35. When the crack depth is 8mm, the box dimension is 1.42. From Table 2, it can be visually seen that as the depth of crack increases, the box dimension becomes larger and larger. The box dimension can well identify the damage degree of structure under the white noise excitation.

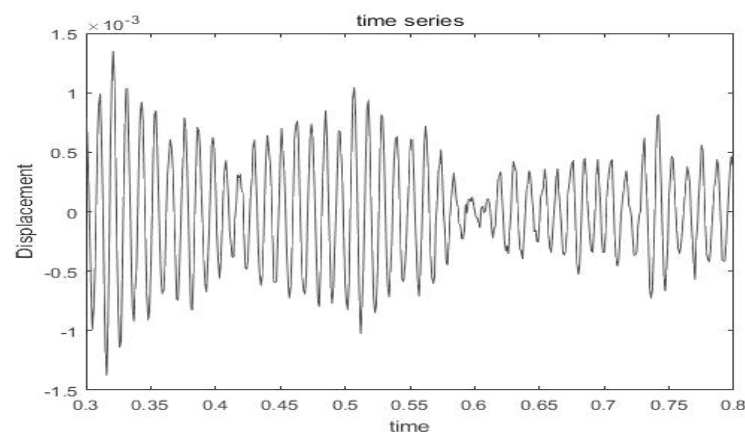


Figure 7. Time-Domain Response of Cantilever Beam Under White Noise Excitation

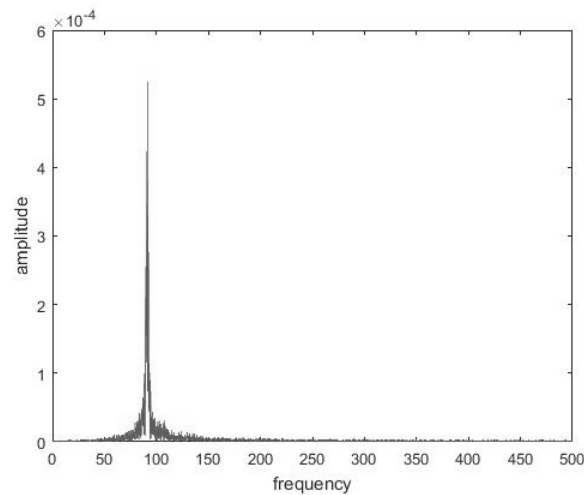


Figure 8 (a) Frequency response of the cracked cantilever under white noise excitation

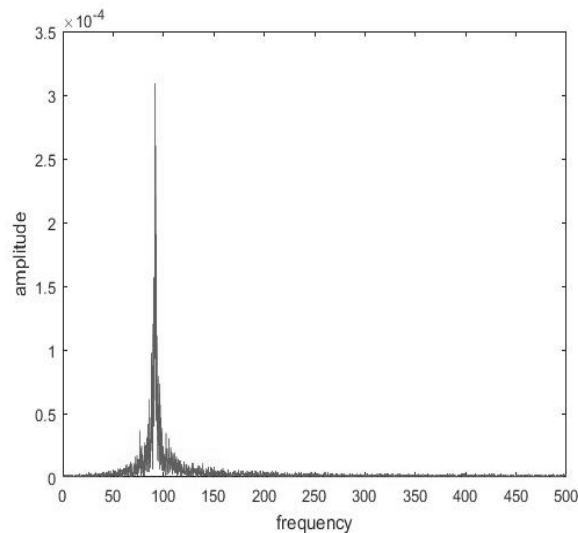


Figure 8 (b) Frequency domain response of cantilever at the crack depth of 8mm under white noise excitation

Table 2. Displacement box dimension of structures with different crack depths under white noise excitation

Crack depth a/mm	0	2	4	6	8
Box dimension	1.35	1.36	1.39	1.40	1.42

5. Conclusion

In this paper, the cantilever structure is converted into an equivalent single-degree-of-freedom model. The stiffness cosine model with time as parameter is introduced to simulate the opening and closing process of crack. The degree of damage is calculated by combining with the box dimension under the two kinds of excitations. There are following conclusions:

(1)The damage degree of beam structure can be visually recognized by the displacement frequency response of beam with breathing cracks under sine excitation the, as shown in [4]. The box dimension of displacement response is calculated and analyzed. As the crack depth increases, the box dimension becomes larger and larger.

(2)The results show that the frequency response of beam with breathing cracks under random excitation can't identify its structural damage. However, the box dimension of displacement response is calculated and analyzed. As the crack depth increases, the box dimension becomes larger and larger. It can effectively identify the degree of structural damage. It shows that the calculation of box dimension is more widely applied in the damage identification of beam structure with breathing cracks.

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