

# Memory $H_\infty$ performance control of a class T-S fuzzy system

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**Abstract.** For much nonlinear system in the control system, both the stability of the system and certain performance indicators are required. The characteristics of T-S model in fuzzy system make it possible to illustrate a great amount of nonlinear system efficiently. First and foremost, the T-S model with uncertainties and external disturbance is utilized to interpret nonlinear system so as to implement  $H_\infty$  performance control by means of fuzzy control theory. Meantime, owing to the tremendous existence of time delay phenomenon in the controlled, feedback controller with memory fuzzy state is fabricated. On the basis of Lyapunov Stability Theory, the closed-loop system becomes stable by establishing Lyapunov function. Gain matrix of the memory state feedback controller is obtained by applying linear matrix inequality methodology. And simultaneously it makes the system meet the requirement of the  $H_\infty$  performance indicator. Ultimately, the efficiency of the above-mentioned method is exemplified by the numerical computation.

## 1. Introduction

In practical engineering, as time delay of the controlled volume exists in the system, it is better to realize the performance of the system by using memory control. Since 1980s when Zames[1] first proposed  $H_\infty$  control theory, many scholars in the control field have begun to conduct research on it [2,3]. By applying the fuzzy control theory to the nonlinear system and designing fuzzy controller, the stability of nonlinear system was studied[4,5]. In recent years, many scholars have applied fuzzy control combined with many other control methods to study nonlinear systems. Liu Yi and Sun Liying[6] studied the design method of guaranteed cost controller by using parallel distribution compensation algorithm. Chen B and Xu S[7] discussed the guaranteed cost controller with memory state feedback for neutral time delay systems. Tan Chong[8] studied the robust  $H_\infty$  control problem in uncertain time-delay continuous generalized linear systems by feedback of memory state. Literature[9]and[10] research showed the design of a controller for the guarantee of cost suitable for fuzzy memory system. However, so far there has been little research on  $H_\infty$  performance of the memory T-S fuzzy systems. Therefore, the research on T-S fuzzy system with the state uncertainties and external disturbances will be expounded, and the memory state feedback controller is to be analyzed so as to achieve  $H_\infty$  performance.



## 2. Preliminaries and problem formulation

### 2.1. Relevant lemmas

**Lemma1**[12]: Let  $U, V$  and  $F$  be matrices composed by real numbers of appropriate dimensions, on the condition of  $F^T F \leq I$ , for any scalar  $\varepsilon > 0$ , we have

$$UFV + V^T F^T U^T \leq \varepsilon U U^T + \varepsilon^{-1} V^T V$$

**Lemma2**[13]: For any real vector  $x, y$  and matrices  $L, X, Y, Z$  of appropriate dimensions, if  $X, Z$  are symmetric positive definite and  $\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} \geq 0$ , we have

$$-2x^T L y \leq \inf_{x, y, z} \begin{bmatrix} x \\ y \end{bmatrix}^T \begin{bmatrix} X & Y-L \\ Y^T-L^T & Z \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

### 2.2. Problem formulation

The total fuzzy model is achieved by fuzzy aggregation [10].

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^r h_i(\theta) [(A_i + \Delta A_i)x(t) + B_i u(t) + B_{i1} \omega(t)] \\ y(t) &= \sum_{i=1}^r C_i x(t) \end{aligned} \quad (1)$$

where  $\omega(t)$  is external perturbation input.  $y(t) \in R^q$  is the output of system,  $C_i (i=1, \dots, r)$  is constant matrices

Suppose the following fuzzy controller with the function of memory state feedback is taken,.

$$u(t) = \sum_{i=1}^r h_i(\theta) [K_{1i} x(t) + K_{2i} x(t-\tau)], i=1, 2, \dots, r \quad (2)$$

where  $K_{1i}, K_{2i} \in R^{m \times n}$  are the feedback gain matrix of the  $i$ -th rule, from (2),

The closed-loop system with state feedback by (2) and (1) can be described

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^r h_i(\theta) \sum_{j=1}^r h_j(\theta) [\bar{A}_i x(t) + B_i (K_{1j} x(t) + K_{2j} x(t-\tau)) + B_{i1} \omega(t)] \\ y(t) &= \sum_{i=1}^r C_i x(t) \end{aligned} \quad (3)$$

among them  $\bar{A}_i = A_i + \Delta A_i$

In the paper, we design a fuzzy memory state feedback controller (2), which can not only make (3) asymptotically stable, but can satisfy  $H_\infty$  performance function.  $H_\infty$  performance function is defined as follows:

$$\|y(t)\|_2 < \gamma \|\omega(t)\|_2 \quad (4)$$

## 3. Main results

**Theorem:** Consider uncertain system (3), if there are some symmetric positive definite matrices  $\bar{P}, \bar{Q}, \bar{X}, \bar{Z}$ , some positive constants  $\varepsilon_i (i=1, 2, \dots, r)$  and matrix  $\bar{Y}, \bar{K}_{1j}$  and  $\bar{K}_{2j}$ , satisfying the following LMIs:

$$\begin{bmatrix} \Lambda_1 & B_i \bar{K}_{2j} - \bar{Y} & B_{li} & \tau(\bar{P}A_i^T + \bar{K}_{1j}^T B_i^T) & \bar{P}E_i^T & \varepsilon_i D_i & \bar{P}C_i^T \\ \bar{K}_{2j}^T B_i^T - \bar{Y}^T & -\bar{Q} & 0 & \tau \bar{K}_{2j}^T B_i^T & 0 & 0 & 0 \\ B_{li}^T & 0 & -\gamma^2 I & \tau B_{li}^T & 0 & 0 & 0 \\ \tau(A_i \bar{P} + B_i \bar{K}_{1j}) & \tau B_i \bar{K}_{2j} & \tau B_{li} & -\tau \bar{Z} & 0 & \tau \varepsilon_i D_i & 0 \\ E_i \bar{P} & 0 & 0 & 0 & -\varepsilon_i I & 0 & 0 \\ \varepsilon_i D_i^T & 0 & 0 & \tau \varepsilon_i D_i^T & 0 & -\varepsilon_i I & 0 \\ C_i \bar{P} & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix} < 0 \quad (5)$$

$$\Lambda_1 = A_i \bar{P} + \bar{P}A_i^T + B_i \bar{K}_{1j} + \bar{K}_{1j}^T B_i^T + \tau \bar{X} + \bar{Y} + \bar{Y}^T + \bar{Q}$$

$$\begin{bmatrix} \bar{X} & \bar{Y} \\ \bar{Y}^T & \bar{Z} \end{bmatrix} \geq 0, \quad \bar{P} \geq \bar{Z} \quad (6)$$

then, the system (3) is progressively stable and satisfies the (4).

Proof: we construct Lyapunov function candidate:

$$V(x(t)) = V_1(x(t)) + V_2(x(t)) + V_3(x(t)) \quad (7)$$

where  $V_1(x(t)) = x(t)^T P x(t)$ ,  $V_2(x(t)) = \int_{-\tau}^0 \int_{t+\beta}^t \dot{x}(\alpha)^T Z \dot{x}(\alpha) d\alpha d\beta$ ,  $V_3(x(t)) = \int_{t-\tau}^t x(s)^T Q x(s) ds$

The time derivative of  $V(x(t))$  is given by

$$\dot{V}(x(t)) = \dot{V}_1(x(t)) + \dot{V}_2(x(t)) + \dot{V}_3(x(t))$$

$$\dot{V}_1(x(t)) = \sum_{i=1}^r h_i(\theta) \sum_{j=1}^r h_j(\theta) [2x(t)^T P(\bar{A}_i + B_i K_{1j} + B_i K_{2j})x(t) + 2x(t)^T P B_{li} \omega(t) - 2x(t)^T P B_i K_{2j} \int_{t-\tau}^t \dot{x}(\alpha) d\alpha]$$

according to lemma 2, so

$$\begin{aligned} &\leq \sum_{i=1}^r h_i(\theta) \sum_{j=1}^r h_j(\theta) [x(t)^T (P\bar{A}_i + \bar{A}_i^T P + P B_i K_{1j} + K_{1j}^T B_i^T P + \tau X + Y + Y^T) x(t) \\ &\quad - 2x(t)^T (Y - P B_i K_{2j}) x(t - \tau) + \int_{t-\tau}^t \dot{x}(\alpha)^T Z \dot{x}(\alpha) d\alpha + 2x(t)^T P B_{li} \omega(t)] \end{aligned}$$

$$\dot{V}_2(x(t)) = \tau \dot{x}(t)^T Z \dot{x}(t) - \int_{t-\tau}^t \dot{x}(\alpha)^T Z \dot{x}(\alpha) d\alpha, \quad \dot{V}_3(x(t)) = x(t)^T Q x(t) - x(t - \tau)^T Q x(t - \tau)$$

when  $t \geq 0$ , by  $\dot{V}(x(t))$ , (2) and (3),

$$\dot{V}(x(t)) + y(t)^T y(t) - \gamma^2 \omega(t)^T \omega(t) \leq \sum_{i=1}^r h_i(\theta) \sum_{j=1}^r h_j(\theta) \left\{ \begin{bmatrix} x(t) \\ x(t - \tau) \\ \omega(t) \end{bmatrix}^T * \Pi * \begin{bmatrix} x(t) \\ x(t - \tau) \\ \omega(t) \end{bmatrix} \right\}$$

where

$$\Pi = \begin{bmatrix} \Lambda_2 & P B_i K_{2j} - Y & P B_{li} \\ K_{2j}^T B_i^T P - Y^T & -Q & 0 \\ B_{li}^T P & 0 & -\gamma^2 I \end{bmatrix} + \tau \begin{bmatrix} (\bar{A} + B_i K_{1j})^T \\ K_{2j}^T B_i^T \\ B_{li}^T \end{bmatrix} Z \begin{bmatrix} (\bar{A} + B_i K_{1j}) \\ K_{2j}^T B_i^T \\ B_{li} \end{bmatrix} \quad (8)$$

$$\Lambda_2 = P A_i + A_i^T P + P B_i K_{1j} + K_{1j}^T B_i^T P + \tau X + Y + Y^T + Q + C_i^T C_i$$

by  $\Delta A_i = D_i F_i(t) E_i$  [10], according to schur lemma [11] and lemma 1, we have

$$\Pi \leq \begin{bmatrix} \Lambda_2 & P B_i K_{2j} - Y & P B_{li} & \tau(A_i + B_i K_{1j})^T & E_i^T & P D_i & C_i^T \\ K_{2j}^T B_i^T P - Y^T & -Q & 0 & \tau K_{2j}^T B_i^T & 0 & 0 & 0 \\ B_{li}^T P & 0 & -\gamma^2 I & \tau B_{li}^T & 0 & 0 & 0 \\ \tau(A_i + B_i K_{1j}) & \tau B_i K_{2j} & \tau B_{li} & -\tau Z^{-1} & 0 & \tau D_i & 0 \\ E_i & 0 & 0 & 0 & -\varepsilon_i I & 0 & 0 \\ D_i^T P & 0 & 0 & \tau D_i^T & 0 & -\varepsilon_i^{-1} I & 0 \\ C_i & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix} \quad (9)$$

Pre- and post-multiplying the right side of (9) by diagonal matrix  $\{P^{-1}, P^{-1}, I, I, I, \varepsilon_i, I\}$ , let  $\bar{P} = P^{-1}$ ,  $\bar{Y} = P^{-1}YP^{-1}$ ,  $\bar{Q} = P^{-1}QP^{-1}$ ,  $\bar{X} = P^{-1}XP^{-1}$ ,  $\bar{K}_{1j} = K_{1j}P^{-1}$ ,  $\bar{K}_{2j} = K_{2j}P^{-1}$ ,  $\bar{Z} = Z^{-1}$ , at the same time, according to lemma 1, the right of (9) is equivalent to the following formula

$$\bar{\Lambda} = \begin{bmatrix} \Lambda_1 & B_i \bar{K}_{2j} - \bar{Y} & B_{li} & \tau(\bar{P}A_i^T + \bar{K}_{1j}^T B_i^T) & \bar{P}E_i^T & \varepsilon_i D_i & \bar{P}C_i^T \\ \bar{K}_{2j}^T B_i^T - \bar{Y}^T & -\bar{Q} & 0 & \tau \bar{K}_{2j}^T B_i^T & 0 & 0 & 0 \\ B_{li}^T & 0 & -\gamma^2 I & \tau B_{li}^T & 0 & 0 & 0 \\ \tau(A_i \bar{P} + B_i \bar{K}_{1j}) & \tau B_i \bar{K}_{2j} & \tau B_{li} & -\tau \bar{Z} & 0 & \tau \varepsilon_i D_i & 0 \\ E_i \bar{P} & 0 & 0 & 0 & -\varepsilon_i I & 0 & 0 \\ \varepsilon_i D_i^T & 0 & 0 & \tau \varepsilon_i D_i^T & 0 & -\varepsilon_i I & 0 \\ C_i \bar{P} & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix}$$

where  $\Lambda_1 = A_i \bar{P} + \bar{P}A_i^T + B_i \bar{K}_{1j} + \bar{K}_{1j}^T B_i^T + \tau \bar{X} + \bar{Y} + \bar{Y}^T + \bar{Q}$

by (5),  $\bar{\Lambda} < 0$ , therefore,  $\dot{V}(x(t)) + y(t)^T y(t) - \gamma^2 \omega(t)^T \omega(t) < 0, t \geq 0$ , when  $\omega(t) = 0$ , we have  $\dot{V}(x(t)) < 0$ , the (3) that achieves asymptotical stability, and for all  $\omega(t) \in L_2(0, +\infty)$ , thus  $x(t) \in L_2(0, +\infty)$ , there is  $\lim_{t \rightarrow \infty} x(t) = 0$ .

For  $\dot{V}(x(t)) + y(t)^T y(t) - \gamma^2 \omega(t)^T \omega(t)$ , on both sides of the integration, and under zero initial conditions  $V(x(t)) = 0, V(x(+\infty)) \geq 0$ , we have

$$\begin{aligned} & \int_0^{+\infty} (y(t)^T y(t) - \gamma^2 \omega(t)^T \omega(t)) dt \\ & \leq \int_0^{+\infty} (\dot{V}(x(t)) + y(t)^T y(t) - \gamma^2 \omega(t)^T \omega(t)) dt < 0 \end{aligned}$$

So by definite integral property, there is  $y(t)^T y(t) \leq \gamma^2 \omega(t)^T \omega(t)$ , equivalent to  $\|y(t)\|_2 < \gamma \|\omega(t)\|_2$ .

During the proving process, when applying lemma2, we ensure  $\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} \geq 0$ , from (6) and [10].

#### 4. Numerical examples

Considering the global fuzzy system (3) is described by the two rules, where

$$\begin{aligned} A_1 &= \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}, \quad B_1 = B_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad B_{11} = B_{12} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad D_1 = D_2 = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}, \\ E_1 = E_2 &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad C_1 = C_2 = [0 \quad 1], \quad \gamma = 1, \quad \tau = 0.2 \end{aligned}$$

From Theorem 1, one feasible solution to LMIs is computed to be

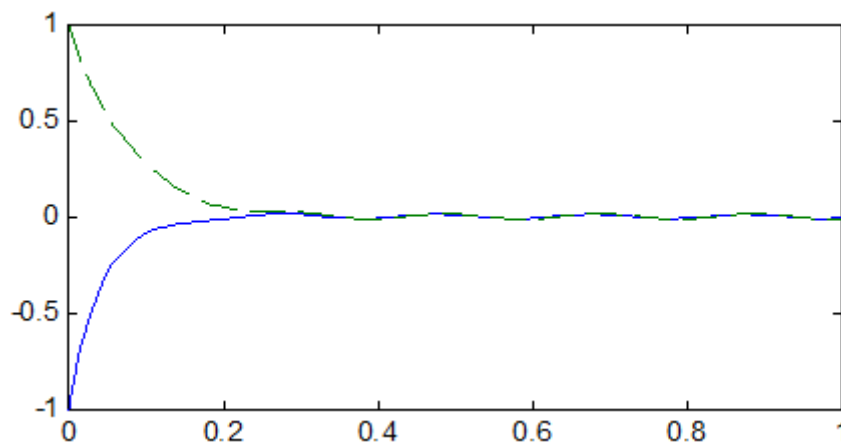
$$\bar{P} = \begin{bmatrix} 4.0710 & 0 \\ 0 & 2.4205 \end{bmatrix}, \quad \bar{Q} = \begin{bmatrix} 7.0855 & 0 \\ 0 & 2.1426 \end{bmatrix}, \quad K_{11} = \begin{bmatrix} -6.6763 & -1.6819 \\ 0 & -5.6832 \end{bmatrix}, \quad K_{21} = \begin{bmatrix} -0.0821 & 0 \\ 0 & -0.1311 \end{bmatrix},$$

$$K_{12} = \begin{bmatrix} -6.6763 & 0 \\ -0.5946 & -6.6832 \end{bmatrix}, \quad K_{22} = \begin{bmatrix} -0.0821 & 0 \\ 0 & -0.1311 \end{bmatrix}, \quad \varepsilon_1 = 1.1665, \quad \varepsilon_2 = 133.5352.$$

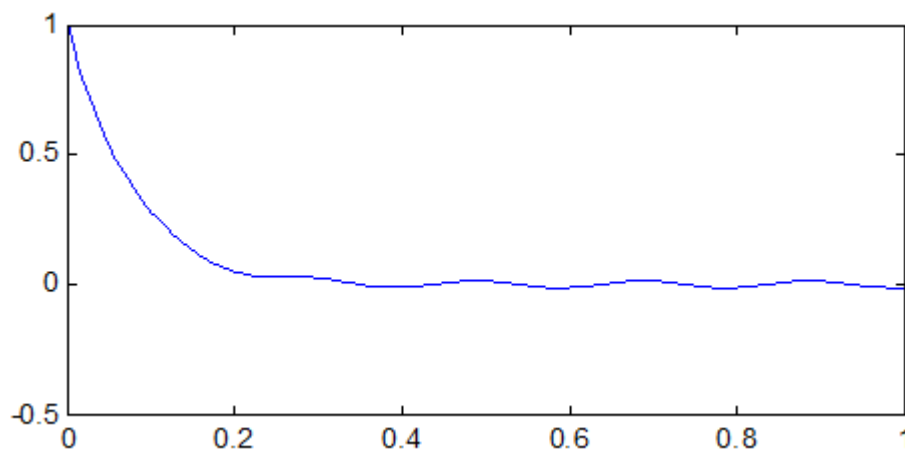
The initial conditions are given as

$$x(0) = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \quad F_1(t) = F_2(t) = \sin(\pi t) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \omega(t) = 0.5 \sin(10\pi t) * \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$

we choose the following membership functions  $\alpha_2(t) = 1 - \alpha_1(t)$ ,  $\alpha_1(t) = 1 / (1 + e^{-2x_1})$ .



**Figure 1.** The state output of the system are shown in figure obtained by Matlab, where “—” is the state  $x_1$  output, “- -” is the state  $x_2$  output.



**Figure 2.** the output of the system are shown in figure obtained by Matlab

## 5. Conclusion

Considering a class of the T-S fuzzy system with state uncertainties and external perturbation, under the condition of the uncertainty which satisfies the norm bounded, the gain matrix of memory feedback control was substantiated by designing memory state feedback control and through the stability of Lyapunov and the linear matrix inequality methodology of Matlab (LMT toolbox) so that the closed loop system becomes asymptotically stable. Meanwhile,  $H_\infty$  performance indicator was required. The methodology was proved to be effective by the examples of numerical simulation in the final part of the paper.

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