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Optimal Lease Contract for Remanufactured Equipment

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Abstract. In the last two decades, the business of lease products (or equipment) has grown significantly, and many companies acquire equipment through leasing. In this paper, we propose a new lease contract under which a product (or equipment) is leased for a period of time with maximum usage per period (e.g. 1 year). This lease contract has only a time limit but no usage limit. If the total usage per period exceeds the maximum usage allowed in the contract, then the customer (as a lessee) will be charged an additional cost. In general, the lessor (OEM) provides a full coverage of maintenance, which includes PM and CM under the lease contract. It is considered that the lessor offers the lease contract for a remanufactured product. We presume that the price of the lease contract for the remanufactured product is much lower than that of a new one, and hence it would be a more attractive option to the customer. The decision problem for the lessee is to select the best option offered that fits to its requirement, and the decision problem for the lessor is find the optimal maintenance efforts for a given price of the lease option offered. We first find the optimal decisions independently for each party, and then the joint optimal decisions for both parties.

1. Introduction

Recently, leasing an equipment is increasingly common for companies rather than to own it [1]. Over 80% of American businesses lease at least one of their equipment acquisitions, and nearly 90% say they would choose to lease again [2]. Some motivations of leasing are saving on initial investment, flexibility on equipment upgrading, and cost reduction in maintenance and inventory. Lease contracts have been studied by many researchers [3] provides a comprehensive review and the study can be done from the lessor's or lessee's perspective. From the lessor's perspective, one can study from two levels of decision problems i.e. the strategic level (issues relevant such as type and number of equipment lease, upgrade options to compensate with technological obsolescence, etc.) and the operational level (dealing with issues consisting of maintenance servicing, spare part stock, crew size, etc.).

We can view the lessee as individual households (for consumer equipments such as electronic appliances, automobiles, etc.) or businesses (for commercial or industrial equipments). The lessee wants to decide which equipment to lease for the case where several brands offered, and the best lease option from the options available for a given equipment. For the case where the study is done from the lessor and lessee point of views then a game theory formulation is needed to modelling the decision problems (See [4]).

Furthermore, the lease contracts can be grouped into two categories – one dimensional and two dimensional lease contracts. The one dimensional lease contract has only time/age limit whereas the two dimensional one has both age and usage limits. For the one-dimensional lease contract, the study have been done by many researchers but only a few works for the two dimensional lease contract.

For the one-dimensional case, the study focuses on finding an optimal maintenance policy which minimizes the total maintenance cost for the lessor, and the leased equipment considered can be a new, used or remanufactured item. The lease contract for the new lease item, can be found in [1], [5] to name a few, whilst [6], [7] examined the lease contract for used items. Finally, [8] considered lease options which include a remanufactured equipment. When the equipment is used intensively (or with high usage) per unit of time, the usage experienced affects significantly the deterioration of the equipment. This indicates the need to consider age and usage in modelling the failure and also defining the lease contract which involves two parameters –i.e. age and usage limits (called a two dimensional lease contract). We are aware only the works by [9] and Iskandar and Husniah [10] belong to this group. In [9] the period of the contract is always the same (i.e. Γ_0) with maximum usage rate r_m whilst [10] consider a two dimensional lease contract for maximum Γ_0 (age) or U_0 (usage).

In this paper, we propose a two dimensional lease contract which has a time limit but no usage limit. However, if the usage exceeds the maximum usage allowed in the contract, then the lessee has to pay some additional cost. This lease contract can be view as the extension of the lease contract studied by [9] which confine the usage rate to r_m, whilst in this paper there is no upper limit of usage rate. However, if the usage exceeding the maximum value results in some additional cost to the lessee due to more failures would likely to occur (and hence a higher maintenance cost for servicing the equipment) when the usage is beyond the maximum value. In general, OEM (as a lessor) offers not only a lease contract for a brand new equipment but also a lease contract for a remanufactured one. As the price of the lease contract for the remanufactured equipment is much lower than the price of a new one, and hence it would be a more attractive option to the companies.

The paper is organised as follows. In section 2 we give model formulation for the two dimensional lease contract studied. Sections 3 and 4 deal with model analysis and the optimal decisions for the lessor and the lessee. In section 5, we provide with a numerical example. Finally, we conclude with topics for further research in Section 6.

2. Model Formulation

In this section, we first define a new lease contract, describe failure model, formulate a PM policy and its effect on reliability, and then obtain the expected profit for a lessor and a lessee.

Notations:

$\Omega = [0,\Gamma)\times[0,\infty)$	: lease coverage region
δ_y	:Preventive maintenance level
ω	:Down time target
U_y	:Total usage in $[0,m\Gamma)$
$G(x)$	:Distribution function of downtime X_i
$m\Gamma$	:Lease contract periods, $\Gamma > 0$ and $m = 1,2,...$
y	:Usage rate
N	:Number of PM during lease contract
T_r	:The time to the first failure of the remanufactured equipment
P	:Lease contract price
C_b	:Annual cost
C_p	:Preventive maintenance cost
C_r	:Average repair cost
$F(t,\alpha_r)$ $f(t,\alpha_r)$	:Distribution function and density function for T_r

$\lambda(t,\alpha_r)$, $\Lambda(t,\alpha_r)$	:Hazard function and cumulative hazard function associated with $F(t,\alpha_r)$
$\Pi_r(m,y), \Pi_s(m,y)$	:Expected profit for a lessor, and for a lessee
$J^1(K,\delta_y), J^2(K,\delta_y)$	: expected preventive maintenance cost, and minimal repair cost

2.1 New Lease Contract

We consider that a lessee uses an equipment with a certain usage rate. A different lessee has a different usage rate. As a result, the usage rate varies from lessee to lessee. A lease contract studied is as follows. The the remanufactured equipment is leased for period of $m\Gamma$ ($\Gamma > 0$ and $m = 1,2,...$) with a maximum usage ($U_{\max}$) (e.g. number of copies/time period, km travelled/ time period or machine-hours / time period).For a given lessee (or usage rate y), if the total usage at the end of a lease period, U_y exceeds $U_{\max}$ (See Fig.1), then the lessee (or customer) will be charged an additional cost which is proportional to $\varphi = Max\{0, U_y - U_{\max}\} = Max\{0, y\Gamma - U_{\max}\}$ for one period of a lease contract. This additional cost is viewed as a compensation to the lessor as larger usage rate $(U_y > U_{\max})$ will result in more failures under lease contract and hence higher maintenance cost.

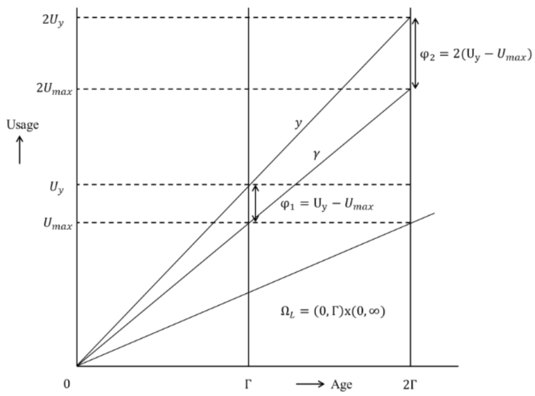

Figure 1. A lease contract with m periods ($m\Gamma, m = 2$) and the maximum usage ($U_{\max}$)

2.2. Modelling Failure

In general, most products at the end of the first life or EOU (end-of-use) have a low reliability (or their reliability is below the threshold value of reliability *R**). Remanufacturing efforts improve the reliability of the product to at least the same level of the threshold reliability as all parts have been undergone inspection to ensure that each parts meets the quality requirements. Let T_r be the time of the first failure the remanufactured product. $F(t)$ is the distribution function for T_r. If $F(t)$ is given by Weibull distribution function with $F(t,\alpha_r) = 1 - e^{-(t/\alpha_r)^\beta}$, then the reliability of the remanufactured product is $R(t,\alpha_r) = 1 - F(t,\alpha_r) = e^{-(t/\alpha_r)^\beta}$. As $R(t,\alpha_r) \geq R^*$, then we have $\alpha_r \geq t / \left\{ (\sqrt[\beta]{-\ln R^*} \right\}$. The hazard and the cumulative hazard functions associated with $F(t,\alpha_r)$ are given by $\lambda(t,\alpha_r)$ and $\Lambda(t,\alpha_r) = \int_0^t \lambda(x,\alpha_r)dx$ respectively.

Now, we consider that failure is not only influenced by age but also usage. As the remanufactured product is a repairable item then it is appropriate to modelling failures using a failure rate function. For a given customer (y), let $\lambda_y(t)$ be the conditional hazard (failure rate) function. We will use PHM (Proportional Hazard Model) whichallows to incorporate the age and usage for modelling the degradation the equipment. Using PHM, the conditional hazard function, $\lambda_y(t)$ is given by

$$\lambda_y(t) = \phi(y, y_0, \rho)\lambda_0(t, \alpha_0) \quad (1)$$

where $\phi(y, y_0, \rho) = e^{\rho(y-y_0)/y_0))}$, $\rho \geq 1$ and $\lambda_0(t, \alpha_0)$ is the base line failure rate function when the usage rate, y is equal to y_0 (a nominal usage of the equipment).

The reasons to support the form of $\phi(y, y_0, \rho)$ is as follows. Note that every equipment is designed with a nominal usage. If the lessee uses the equipment with the usage rate exceeding the nominal value, $y > y_0$, then the equipment will deteriorate faster, otherwise when $y < y_0$ then it goes slower. If the lessee uses the equipment with the usage rate exceeding the nominal value (or the maximum usage allowed in the contract), then the equipment will deteriorate faster. This in turn will result in more failures and hence repair cost under the lease contract period. As a result, the lessee will be charged an additional cost. This cost is viewed as a compensation to the lessor due to the increase in repair cost. It is considered that $\lambda_0(t, \alpha_0) = \lambda(t, \alpha_r)$ with $\alpha_0 = \alpha_r$ (or the base line failure rate function is the failure rate function influenced by age only). For the case where $y = y_o$ then $\phi(y, y_0, \rho) = 1$ and hence $\lambda_y(t) = \lambda_r(t, \alpha_r)$. If $F(t, \alpha_r)$ is a two parameter Weibull distribution function, then $\lambda(t, \alpha_r) = \beta/\alpha_r \left(t/\alpha_r\right)^{\beta-1}$. Let $T_{r|y}$ be the time to first failure of the remanufactured product a given lessee (y). The distribution function for $T_{r|y}$ is given by $F_y(t)$. Let $N_y(t)$ be the number of failures in $(0, t]$ for a given y. If all failures under the lease contract are minimally repaired and repair times are very small relativeto the mean time between failures, then $N_y(t)$ is a non-homogeneous Poisson process (NHPP) with intensity function $r_y(t) = \lambda_y(t)$. The cumulative hazard functions associated with $r_y(t)$ is given by $R_y(t) = \int_0^t r_y(x)dx$.

2.3. Modeling the PM effect

The preventive maintenance (PM) policy is defined as follows. For a truck with usage rate y, the equipment is periodically maintained at $k\tau_y$, $k = 1, 2, ..., K$. This involves k disjoint intervals - $[0, \tau_y)$, $[\tau_y, 2\tau_y)...[(K-1)\tau_y, K\tau_y)]$ in which all failures within PM period are minimally repaired, where k is an integer value. The effect of PM actions are modelled through either the reduction in (i) the intensity function or (ii) the age. Here, we model using (ii) and the virtual age of the equipment after PM at $k\tau_y$, $k = 1, 2, ..., K$ is given by $v_k = \delta k\tau_y$, $k = 1, 2, ..., K$ (or the reduction in age is $k\tau_y - \delta k\tau_y = k(1-\delta)\tau_y$). As a result, the virtual age at time t, $k\tau_y < t < (k+1)\tau_y$ is given by $v_y(t) = k\delta\tau_y + (t - \tau_y)$. The conditional hazard function at time t, $k\tau_y < t < (k+1)\tau_y$ is given by

$$r_y(v_y(t)) = \phi(y, y_0, \rho)\lambda(v_y(t), \alpha_r) \quad (2)$$

Since any failure occurring between PM is minimally repaired, then the expected total number of minimal repairs in $\left(k\tau_y, (k+1)\tau_y\right), 0 \leq k \leq N$ is given by

$$E\left[N(k\tau_y, (k+1)\tau_y\right] = \int_{k\tau_y}^{(k+1)\tau_y} r_y\{v_y(t)\}dt \quad (3)$$

where $r_y(v_y(t))$ is given in (2). As a result, we have

$$M_y(m\Gamma) = \sum_{k=0}^{K} E\left[N(k\tau_y, (k+1)\tau_y\right] \quad (4)$$

where $m\Gamma = (K+1)\tau_y$, m is the number of lease periods.

3. Simulation of The System

Under the proposed lease contract, we first obtain the expected profit (i.e. the expected total revenue - the total expected cost) for the lessor and then for the lessee.

3.1. Lessor's expected profit

The lessor's expected total cost consists of PM cost, CM cost and the penalty cost paid by the lessor due to the down time of the equipment exceeds the predetermined target ω. These costs are obtained as follows.

Preventive maintenance (PM) cost:

Let $C_p(\delta_y)$ and C_r be the cost of the k-*th* PM and the cost of each minimal repair. If $C_p(\delta_y) = C_0 + C_1(1-\delta_y)\tau_y$ as in [6] and [7] then the expected total PM cost over the lease period $(0, m\Gamma), m \geq 1$ is given by

$$J^1(K,\delta_y) = \sum_{k=1}^{N} C_p(\delta_y) = KC_0 + KC_1(1-\delta_y)\tau_y = KC_0 + KC_1(1-\delta_y)\frac{m\Gamma}{(K+1)} \tag{5}$$

where $m\Gamma = (K+1)\tau_y$.

Corrective maintenance (CM) cost:

Let C_r be the cost of each minimal repair, then the expected minimal repair cost is given by

$$J^2(K,\delta_y) = C_r\left(\sum_{j=0}^{K}\int_{k\tau_y}^{(k+1)\tau_y} r_y\{v_y(t)\}dt\right) \tag{6}$$

As a result, the lessor's expected total cost is given by $J^1(K,\delta_y) + J^2(K,\delta_y)$.

The penalty Cost:

Under the lease contract, a penalty occurs if the down time of the equipment exceeds the predetermine d target ω. Let X_i be a down time caused by the i-th failure. X_i is i.i.d with a common distribution function $G(x)$. The expected value of X_i is μ. The penalty occurswhen $X_i > \omega$ and hence, the expected penalty over $m\Gamma$ is given by

$$EP_y(m,y) = q \times y \times M_y(m\Gamma)E\left[Max\{0, X_i - \omega\}\right] = q \times y \times M_y(m\Gamma)\int_{\omega}^{\infty}[1-G(x)]dx \tag{7}$$

where q is a penalty cost per unit usage. Note that $q \leq \upsilon$ (υ is revenue per unit usage).

Expected total revenue:

The expected total revenue is the sum of the price of lease contract and some additional revenues due to the monthly usage of the equipment is greater than $U_{\max}$ (the maximum usage). The price of lease contract is dependent on the usage rate and the number of lease contract periods. The price of the lease contract for the first period is P. The lessor will give an incentive if the equipment is leased for more than one periods (or $m\Gamma, m \geq 2$). If the equipment is leased for more than one periods, then the lessee will get a discount price given by

$$P(m) = P\left\{e^{-\mu m}/e^{-\mu}\right\} = P\left\{e^{-\mu(m-1)}\right\} \tag{8}$$

where $0 \leq \varphi < 1$ and m represents a discount parameter and the number of lease periods, respectively. Note: We use a different form of $P(m)$ as in [8]. For a lessee's usage rate y, the additional revenues earned by the lessor at the end of period $m\Gamma, m = 1, 2...$ given by

$$\Phi(m,y) = C_d\sum_{j=1}^{m} Max\{0, (y \times j\Gamma - jU_{\max})\}, \tag{9}$$

where C_d is the additional cost charged (e.g. \$/km or \$/page copied). $\Phi(m,y)$ is viewed as some additional revenues for the lessor. As a result, we have the expected profit given by

$$\Pi_r(K,\delta_y) = P(m) + \Phi(m,y) - EP_y(m,y) - J^1(K,\delta_y) - J^2(K,\delta_y) - mC_b^r \tag{10}$$

3.2. Lessee's expected profit

The lessee's expected profit is equal to **t**he expected total revenue minus the total expectedcost, that will be obtained as follows. The total expected cost consists of (i) the price of lease contract paid and (ii) some additional charges due to total usage exceeds the maximum usage allowed in the contract.

Lease contract price:

For a given $U_{\max}$ (maximum of usage rate allowed), the price of a lease contract depends on the number of lease periods (m) chosen by the lessee, and it is given by $P(m)$. A more periods leased results in a lower price of the lease contract as in eq. (8). This in turn gives an incentive for the lessee to lease more than 1 period.

Expected total revenue:

Let υ be the revenue per unit of usage (e.g. \$/km, \$/page copied). The total revenue earned by the lessee, R_y consists of the revenue generated by the equipment and some compensation, $EP_y(m)$ paid by the lessor due to the down time of the equipment exceeds the predetermined target ω .It is given by

$$R_y(m,y) = y \times \upsilon \times \left(m\Gamma - E\left[D\right]\right) + EP_y(m) \tag{11}$$

where $\Gamma - E\left[D\right]$ is total time available, $E\left[D\right]$ is the expected total downtimes over the contract periods ($m\Gamma$), given by $E\left[D\right] = M_y \times \mu$, M_y is the expected number of failures over $m\Gamma$ and μ is the expected downtime caused by each failure.

Expected additional charges:

The lessee incurs some additional costs when a monthly usage is greater than $U_{\max}$. The additional cost, $\Phi(m,y)$ is given by (9). As a result, the expected profit is given by

$$\Pi_s(m,y) = y \times \nu \times \left(m\Gamma - E\left[D\right]\right) + EP_y(m) - P(m) - \Phi(m) \tag{12}$$

4. Optimization

We first consider the case where the optimization is carried out independently from the viewpoint of each party, and then the situation where the lessor and the lessee would like to achieve a win-win solution (or the optimization is done simultaneously from both parties).

Case 1a: **[Lessor's viewpoint]**

The lessor offers lease contracts for a fixed price which covers all maintenance costs, administration cost and a profit margin. As mentioned previously, the lease contract provides a full coverage of maintenance (including PM and CM). In order to maximise its profit, the lessor tries to improve the maintenance performance such that to minimize the maintenance costs, which is conditional on the lessee's usage rate. The nominal usage rate can be used to estimate the lessee's usage rate. Hence, the optimization problem is given as follows.

$$\begin{aligned} &Max\left\{\Pi_r(K,\delta_y)\right\} = Max\left\{P(m) + \Phi(m) - EP_y(m) - J^1(K,\delta_y) - J^2(K,\delta_y) - mC_b^r\right\} \\ &s/t: 0 \le \delta_y < 1; K = 1,2,... \end{aligned} \tag{13}$$

Case 1b: **[Lessee's viewpoint]**

The equipment is used to generate revenues or to support a lessee's business process. In both cases, the equipment plays a key role to achieve the lessee's revenue target. Of interest to the lessee is to find the optimal usage rate and the number of lease periods maximising its profit. We view that each lessee has a different optimal usage rate. Hence, the optimization is done from a given lessee seeking for y and m that maximize its profit. As a result, the optimization problem is given by

$$\begin{aligned} &Max\left\{\Pi_s(m,y)\right\} = Max\left\{y \times \nu \times \left(m\Gamma - E\left[D\right]\right) + EP_y(m) - P(m) - \Phi(m)\right\} \\ &s/t: y_{\min} \le y < y_{\max}; m = 1,2,...L/\Gamma \end{aligned} \tag{14}$$

where L is a life cycle of the equipment.

Case 2: **[Joint Optimization]**

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Here, we assume that both parties **(**the lessor and the lessee)would like to cooperate or to achieve a win-win solution. In other words, the optimization is done jointly from both parties. This joint optimization is expected to give more profit for each party, which is at least the same as the profit earned if the optimization is done independently (Case 1). This incentive would give a strong motivation for both parties to cooperate. As a result, the optimization problem is given by

$$Max\left\{\Pi_r(K,\delta_y)+\Pi_s(m,y)\right\} \quad (15)$$

$$s/t:\Pi_r(N,\delta_y)\geq\pi_r,\ \ \Pi_s(m,y)\geq\pi_s\ y_{\min}\leq y<y_{\max};m=1,2,...L/\Gamma,0\leq\delta_y<1;K=1,2,...$$

Where π_s, π_r represent the profit obtained in Case 1 (when the optimization is done separately from each viewpoint). If $\tilde{\pi}_s$ and $\tilde{\pi}_r$ represent the optimal profit resulting from (15) for the lessor and the lessee, respectively, then $\tilde{\pi}_s\geq\pi_s$ and $\tilde{\pi}_r\geq\pi_r$.

5. Numerical Example

We consider the following parameter values as shown in Table 1.

Table 1. Parameter values used

Parameter	Value	Parameter	Value
ρ	1	y	1.5x 60000
m	1	y_0	1 x 60000
β	2.5	C_r	1
β_2	0.5	C_0	1.5
α_r	6	C_1	0.25
α_2	0.25	v	0.8
R	0.8	q	$0.8v$
C_b	10	C_d	$0.7v$
δ	0.027	K	1
U_{max}	60000	Γ	12
ω	4	τ	$\frac{\Gamma}{(K+1)}$

Table 2. Results for Case 1 and Case 2

Case 1: Individual Optimization			
Lessor		Lessee	
δ^*	0.668	y	1.5
$y=y_0$	1		
Profit	354.705	Profit	308.614
Case 2: Joint Optimization			
Lessor		Lessee	
δ^*	0.452	y	1.5
Profit	527.560	Profit	357.632

Table 2 shows optimal solutions for cases 1 and 2. In Case 1, the optimization is carried out independently for each party. Profit obtained is 354.705 for the Lessor, and 308.614 for the Lessee, whilst the profit resulting from the joint optimization (Case 2) is 527.560 for the Lessor, and 357.632 for the Lessee. This is as expected since (i) the Lessor performs a more effective PM (i.e. with lower δ^* (=0.452)) and the PM in turn will decrease the number of failures and hence the total maintenance cost and downtime, and (ii) the usage increases from 60000 to 90000 (this causes the profit for the Lessor increases significantly).

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6. Conclusion

In this paper we have studied a two-dimensional lease contract for equipment in which the lessee will be charged some additional cost if the usage at the end of the contract exceeds the maximum usage allowed. We find the optimal solution independently for each party and then seek for the joint optimization for both parties. The profit obtained from joint optimization is at least the same as the optimal profit obtained independently. One can model the decision problems for the lessor and the lessee using a Stakelberg game theory formulation the lessor as leader and the lessee as follower, or a Nash game theory formulation. Another topic is to modelling failures using the twodimensional approach (or using a bivariate distribution function). These two topics are currently under investigation.

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