IOP Conf. Series: Materials Science and Engineering **317** (2018) 012019 doi:10.1088/1757-899X/317/1/012019

Mathematical modeling of a process the rolling delivery

Mikhail A. Stepanov, Andrey A. Korolev

PhD. of Tech. Sci., Assistant Professor, Moscow State University of Civil Engineering
Postgraduate student, Moscow State University of Civil Engineering

Abstract. An adduced analysis of the scientific researches in a domain of the rolling equipments, also research of properties the working material. A one of perspective direction of scientific research this is mathematical modeling. That is broadly used in many scientific disciplines and especially at the technical, applied sciences. With the aid of mathematical modeling it can be study of physical properties of the researching objects and systems. A research of the rolling delivery and transporting devices realized with the aid of a construction of mathematical model of appropriate process. To be described the basic principles and conditions of a construction of mathematical models of the real objects. For example to be consider a construction of mathematical model the rolling delivery device. For a construction that is model used system of the equations, which consist of: Lagrange's equation of a motion, describing of the law conservation of energy of a mechanical system, and the Navier – Stokes equations, which characterize of the flow of a continuous non-compressed fluid. A construction of mathematical model the rolling deliver to let determined of a total energy of device, and therefore to got the dependence upon the power of drive to a gap between of rolls. A corroborate the hypothesis about laminar the flow of a material into the rolling gap of deliver.

1. Introduction

The most important condition of development the modern industry, including of the building domain, this is the rise of intensification manufacture. This is can achieved with the aid of apply of the high-productivity technological equipments.

The delivery and transportation of a material this is a one of basic operation of technological process to manufacture of the building articles. The modern equipments the delivery of a material to realize by next mechanisms and devices: band conveyer, screw, rolling, plate conveyer, scraper conveyer, pneumatic, hydraulic and other kinds [1, 2].

The most widespread type of delivery and transporting device this is the rolling mechanism. It is to be different by simple construction, high reliability, minimum of power consumption.

The scientific researches of the rolling devices were considered in the works [3-5]. Was a study the basic technical and technological parameters this is type of equipments. Was determined the forces on rolls and torques, the power of drive and productivity of devices.

The properties of different kinds the working material, which to pass through rolls, were study in the works [6-8]. Was considered viscous-elastic and viscous-plastic properties of material, was determined rheological characteristics, was compiled equations of the flow, was determined effective viscosity.

On this basis it can be done a conclusion about necessity of a construction of mathematical models the rolling devices on purpose to study their physical properties.

IOP Conf. Series: Materials Science and Engineering **317** (2018) 012019 doi:10.1088/1757-899X/317/1/012019

2. The basic principles of a construction of mathematical model

A one of the method of scientific researches different objects to serve of a construction of mathematical model. Mathematical modeling a very most important article of research for everything special and applied disciplines. A special meaning that is has in physics and technical sciences [9-12].

In order to construct of mathematical model of a real object (at present case of mechanical system – the rolling deliver) necessary [13-16]:

1. formulate of hypothesis, that is to determine of the laws by which to work the researching object, and to determine the forces acting on the working parts, also to determine of the basic (fundamental) physical laws, which necessary to apply for present research;
2. work out a equations, which to describe with the aid of mathematics the researching process or object, that is direct a construction of mathematical model;
3. solution of mathematical equations.

The real object of research can have several different of mathematical models. This is connected with possibility research of different properties a system, also the choice of the type model. Moreover, a research of complex the real object with the aid of different mathematical models to can substantially to expand of the knowledge about occurring process and raised precision of research.

Also not exclude the use of the same mathematical model (similar differential equations) for a research of different the real object. At that to appear possibility of a substitution the researching objects on another. Therefore, cannot constructed again of mathematical model, and correctly choice and apply already well-known models.

The most important the requirement, which to produced of mathematical model, this is adequacy [15, 17], that is correct accordance of the real object. There under to be understood:

1. correct qualitative description of the properties the researching object;
2. correct quantitative description this is the properties.

A quantitative prediction not always demonstrated enough precisely. First of all, this is noticeably at social and biological sciences. Sometimes them necessary neglected and at technical sciences because of considerably to complicate research of object.

An initial stage of a construction any mathematical model this is to receipt the most complete conception about the modeling object. In the case of mechanical system, at present case of the rolling deliver, to determine the basic parts, from which consist the researching device, their joint work and action, the forces acting on the working parts and on a work material.

The next stage of a construction of mathematical model this is formulated of mathematical problem. The problem can be concrete, that is determination of any parameter the researching system, or to express of the indefinite form, that is research any process, occurring inside system, or a conduct of system in general and so on.

Formulate of mathematical problem for a research of the rolling devices at the indefinite form: «Optimization of a process the rolling delivery of viscous-plastic materials».

At present example to choice as parameter optimizations of power of drive the rolling device. Further to study in detail of mathematical model this is parameter can be specified and changed.

3. The construction of mathematical model the rolling device

As the initial system of the equations of mathematical model the rolling deliver of viscous-plastic the building materials, for example the clay for ceramic articles, to accept Lagrange’s equation of a motion for the rolling device and the Navier – Stokes equations of the flow non-compressed fluid for a material. During a construction of mathematical model the flow of a material considered as laminar.

In accordance with principle of the least action every mechanical system to move likes this as integral of an action equal:

$$S = \int_{t_1}^{t_2} L(r,v,t)dt \quad (1)$$

IOP Conf. Series: Materials Science and Engineering **317** (2018) 012019 doi:10.1088/1757-899X/317/1/012019

where $L(r,v,t)$ - function of Lagrange with specified, accordingly, the coordinates of points system r, velocity of points system v at determinate the moment of the time t.

Integrated function (1), and have a number of transformations, to get Lagrange's equation of a motion:

$$\frac{d}{dt}\cdot\frac{\partial L}{\partial v}-\frac{\partial L}{\partial r}=0 \tag{2}$$

Or another mode:

$$\frac{d}{dt}\cdot\frac{\partial L}{\partial v}=\frac{\partial L}{\partial r} \tag{3}$$

The Navier – Stokes equations for volumetric the flow of non-compressed viscous fluid to have three components [18, 19], which in general kind can be written down the following mode:

$$\rho\cdot\left(\frac{\partial v}{\partial t}+\sum_{i=1}^{3}v_i\frac{\partial v}{\partial x_i}\right)=-grad\ p+\mu\cdot\left(\sum_{i=1}^{3}\frac{\partial^2 v}{\partial x_i^2}\right)+F \tag{4}$$

where ρ - density of a material, kg/m^3;
v - the velocity, m/s;
$grad\ p$ - gradient of the pressure, Pa;
μ - viscosity of a material, $Pa\cdot s$;
F - the external force, N.

In order that the flow of non-compressed viscous fluid in a gap the rolling device to be continuously, necessarily to observe of a condition non-tearing of the stream. That is to determine of equation Saint-Venant, expressing the principle of mass conservation in the cases setting of the motion, and has the following mode:

$$\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}+\frac{\partial v_z}{\partial z}=0 \tag{5}$$

where v_x, v_y, v_z - the components of velocity, m/s.

Now necessary to solve this is system of equations, in order that to find of parameters, which to be choice as optimization, at present case this is the power of drive the rolling device.

4. The solution of mathematical model

The distribution of viscous non-compressed fluid in a gap between by rolls to be occur into two direction: on axis X, as showed on figure 1, that is on length of the stream, and on axis Y, that is on width of a sheet (on figure 1 not showed). The sizes on axis Z, that is on thickness of a sheet, to be substantially less and them in further calculations can be neglect.

IOP Conf. Series: Materials Science and Engineering **317** (2018) 012019 doi:10.1088/1757-899X/317/1/012019

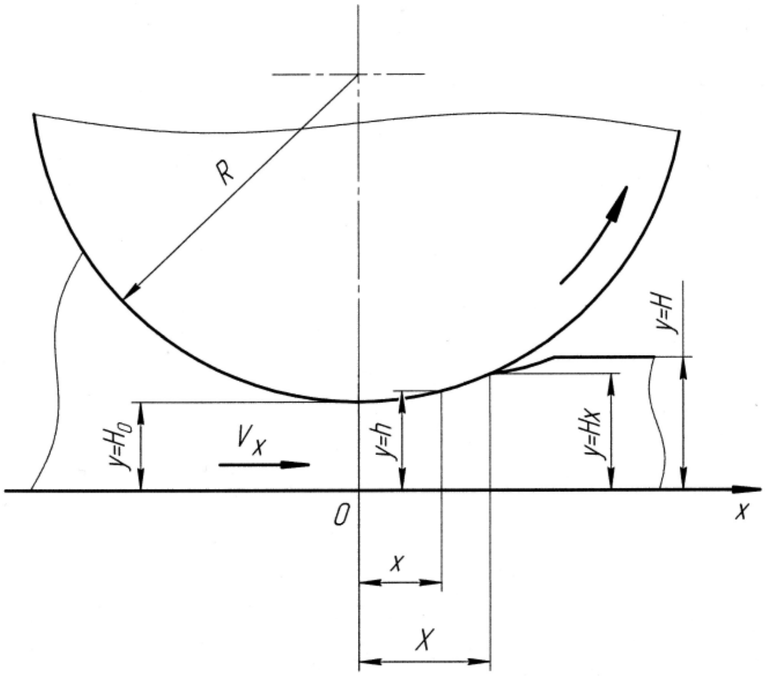

Figure 1. The flow of viscous fluid into a gap of rolls.

Thereupon that a function of Lagrange (equation 1) this is a differential equation of a motion, that is to be the law conservation of energy [17]:

$$L = \frac{m \cdot v^2}{2} - U(r) = E \tag{6}$$

where m – the mass, kg;
v – the velocity, m/s;
$U(r)$ – the potential energy of system, Joule;
E – a total energy of system, Joule.

In the case of flat-parallel the flow of viscous fluid the equation of a total energy to accept the following mode:

$$E = \frac{m \cdot \left(v_x^2 + v_y^2\right)}{2} - U(x, y) \tag{7}$$

where v_x, v_y – the components of velocity on axis X and Y, m/s.

The Navier – Stokes equations of a motion of continuously non-compressed fluid also simplify in connection with introduce into allowing, and to be contain the two components, which to accept the following mode:

$$\rho \cdot \left(\frac{\partial v_x}{dt} + v_x \cdot \frac{\partial v_x}{\partial x} + v_y \cdot \frac{\partial v_x}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu \cdot \left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} \right) + F_x \tag{8}$$

$$\rho \cdot \left(\frac{\partial v_y}{dt} + v_x \cdot \frac{\partial v_y}{\partial x} + v_y \cdot \frac{\partial v_y}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu \cdot \left(\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} \right) + F_y \tag{9}$$

where ρ - density, kg/m^3;
v_x, v_y – the components of velocity on axis X and Y, m/s;

p - the pressure, Pa;

μ - viscosity of a material, $Pa \cdot s$;

F_x, F_y – the external forces on axis X and Y, N.

Accordingly changed the equation of the non-tearing of Saint-Venant and to accept the following mode:

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0 \tag{10}$$

where v_x, v_y – the components of velocity on axis X and Y, m/s.

The solution of the equation (7) to give a chance to got the most precisely of a total energy of system the rolling deliver, and therefore the most precisely to calculate the power of drive the rolling device. A system of equations (8 – 10) to characterize of flat-parallel the flow of viscous fluid into a gap of rolls delivery device.

In the work [4] adduced the values of experimental researches the rolling device for high-plastic viscous materials with plastic's number more than 25%. Was a study delivery of pieces for different the gaps between of rolls. Was a got the factual values of the power of drive device. A comparison of the calculating and experimental values (figure 2) showed insignificant divergence of the results (on the average not more than 5%).

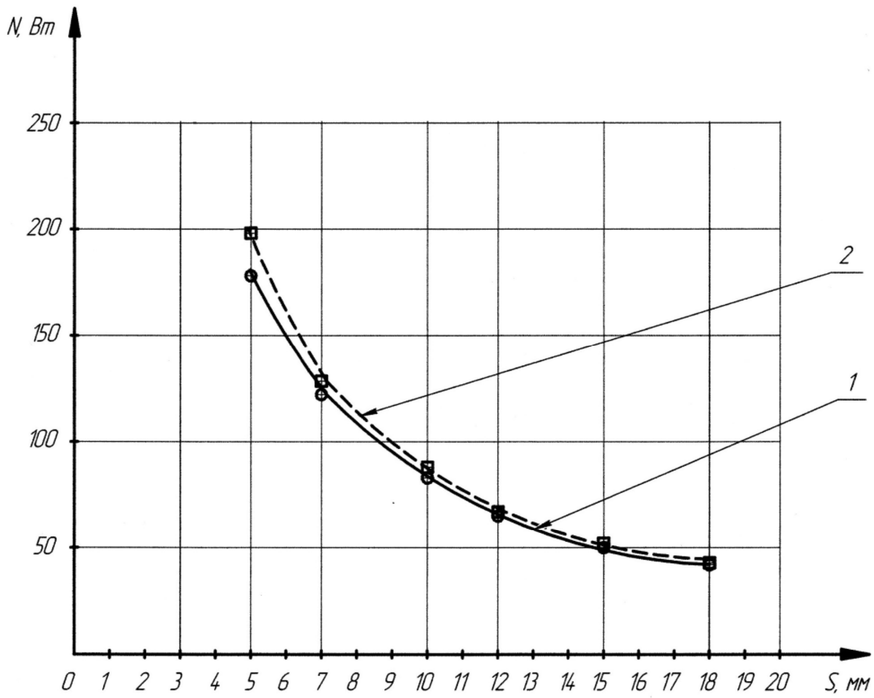

Figure 2. The diagram of dependence upon power to a gap between of rolls:
1. – factual power; 2. – calculated power.

IOP Conf. Series: Materials Science and Engineering **317** (2018) 012019 doi:10.1088/1757-899X/317/1/012019

5. Conclusion

- A construction of mathematical model to let considered everything factors, which influence on experiment, will choice of the optimum variant and to minimize of quantity the researching factors;
- A construction of mathematical model to let determined of dependence upon power of drive the rolling device to a gap between of rolls, which to be express by exponential function;
- Mathematical model to corroborate the hypothesis about laminar the flow of a material into the rolling gap of deliver, which enough precisely described with the aid of the Navier – Stokes equations;
- A construction of mathematical model the rolling deliver to let determined of a total energy of device and therefore the most precisely to calculate the power of drive and to reduce of energy costs.

References

[1] Uvarov V A, Karpachyov D V 2008 *Machines for technological transporting of the building materials and articles* Belgorod: Publishing house of BGTU, pp. 156.
[2] Uvarov V A, Stepanov M A, Koshkaryov E K 2013 *Machines for technological transporting of the building materials and articles* MGSU, pp. 216.
[3] Lukyanov N A, Stepanov M A, Korolev A A 2014 *Mech. Constr.* **1** 8–11.
[4] Lukyanov N A, Stepanov M A, Korolev A A 2014 *Mech. Constr.* **12** 24–7.
[5] Shrubchenko I V, Hurtasenko A V, Sharapov R R, Duyun T A, Shchetinin N A 2015 Mod. Appl. Sci. **9(1)** 195.
[6] Lukyanov N A, Stepanov M A, Korolev A A 2013 *Vestnik MGSU,* **5** 43–8.
[7] Gridchin A M, Yadykina V V, Trautvain A I, Sharapov R R, Zhukova A A 2014 Res. J. Appl. Sci. **9(12)** 1053–8.
[8] Sharapov R R, Kapyrin P D, Lozovaya S Yu, Yadykina V V, Agarkov A M 2016 *Research deducting efficiency of the inertial hub with adjustable parameters* MATEC web conf 5-th International scientific conference «Integration, partnership and innovation in construction science and edication», **86** 7.
[9] Andreev V K 2015 *Mathematical models of continuum mechanics* Lan, pp. 231.
[10] Arnold V I 2000 *Mathematical methods of classic mechanics* Editorial URSS, pp. 408.
[11] Blekhman I I, Myshkis A D, Panovko Ya G 2007 *Applied mathematics: theme, logic, specific of approach With examples from mechanics* Publishing house LKI, pp. 376.
[12] Meyer W J 2004 *Concepts of mathematical modeling* – N Y – London: Dover publications, pp. 448.
[13] Zeldovich Ya B, Myshkis A D 2014 *The elements of applied mathematics* LENAND, pp. 592.
[14] Myshkis A D 2007 *Applied mathematics for the engineers* Phys Math Lit, pp. 688.
[15] Myshkis A D 2014 *The elements of theory mathematical models* LENAND, pp. 200.
[16] Tykhonov A N, Samarsky A A 1999*The equations of mathematical physics* Publishing house of MGU, pp. 798.
[17] Fedotkin I M 2015 *Mathematical modeling of technological processes* Book-house «LIBROCOM», pp. 416.
[18] Temam R, Miranville A 2005 *Mathematical modeling in continuum mechanics* Cambridge: Cambridge University Press, pp. 320.
[19] Turenko A V 2004 *A calculation of clay-reprocessing equipments and the presses of plastic forming for a manufacture of ceramic the building articles* MGSU, pp. 114.