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The Strength Analysis of Differential Planetary Gears of Gearbox for Concrete Mixer Truck

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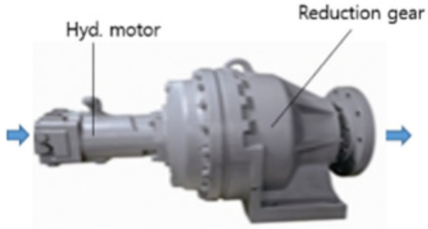
Abstract. The power train of mixer gearbox for concrete mixer truck includes differential planetary gears to get large reduction ratio for operating mixer a drum and simple structure. The planetary gears are very important part of a mixer gearbox where strength problems namely gear bending stress, gear compressive stress and scoring failure are the main concern. In the present study, calculating specifications of the differential planetary gears and analyzing the gear bending and compressive stresses as well as scoring factor of the differential planetary gears gearbox for an optimal design of the mixer gearbox in respect to cost and reliability are investigated. The analyses of actual gear bending and compressive stresses of the differential planetary gears using Lewes & Hertz equation and verifications of the calculated specifications of the differential planetary gears evaluate the results with the data of allowable bending and compressive stress from the Stress-No. of cycles curves of gears. In addition, we also analyze actual gear scoring factor as well as evaluate the possibility of scoring failure of the differential planetary gear.

1. Introduction

A concrete mixer is a device that homogeneously combines cement, aggregate such as sand or gravel, and water to form concrete. A typical concrete mixer uses a revolving drum to mix the components. The drum is traditionally made of steel but on some trucks as a weight reduction measure, fiberglass has been used. Special concrete transport trucks are made to transport and mix concrete up to the construction site.

(a) Photograph of mixer truck



(b) Gearbox

Figure 1. Drum capacity, 8m³ grade concrete mixer truck and the mixer gearbox with hydraulic motor.

The concrete mixer truck with drum capacity, 8m³ grade concrete mixer truck and the mixer gearbox with hydraulic motor has been shown in figure 1. Table 1 indicates the specifications of the mixer gearbox that calculated by developed program. Mixer gearbox is driven by a hydraulic motor, as an important device to rotate the mixer drum and to convert the required torque and rotational speed.

Table 1. Specifications of the mixer gearbox.

Hydraulic. motor Max. input torque/speed	Gear ratio	Max. output torque
397N·m/1,320rpm	132:1	52,400 N·m

In spite of the increasing initial torque resulting to the inertia moment increases of output section, the compound differential planetary gear system applies the rotating motion that makes the mixer drum run smoothly which consists of the sun gear, the differential planetary gear and two ring gears. Gear teeth are damaged due to the lack of fatigue strength, compound planetary gears for mixer and severe operating conditions of a concrete mixer truck that can become a problem. The schematic diagram of an analytical model of mixer gear box has been shown in figure 2.

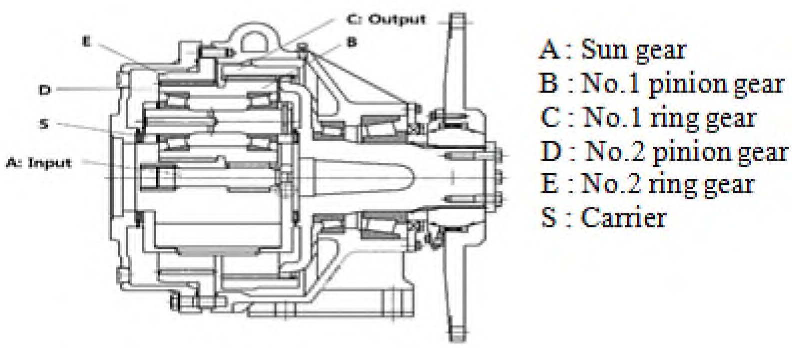


Figure 2. Schematic diagram of analytical model.

Several investigations have been reported, as cited by D.E. Imwalle et al [2]. D.L. Seager et al [3] in their paper established load distribution calculation of the planetary gears. F. Cunliffe et al [4] analyzed the dynamic tooth loads in epicyclic gears for planetary gears. Castellani G. et al [5] also cited the gear strength analysis method. Coy, J.J. et al [6] further emphasized the dynamic capacity and surface pressure durability life of spur and helical gears. Oda et al [7] similarly stressed the effect of bending endurance strength for addendum modification of spur gears and it was likewise investigated. There is also an inclusion of a typical bending strength calculation of planetary gears AGMA 218.01[8] and Gear Handbook by D.W. Dudley [9] that shows the bending strength calculation method of planetary gears.

In this study, developing the gear specifications calculation program and producing detailed specifications of the differential planetary gear system for mixer gearbox are based on Gear Handbook by D.W. Dudley [9]. Developing the stress analysis program of differential planetary gear system by Lewes [1-5] & Hertz [6, 9] equation and analyzing the safety factor of gear bending and compressive stresses considering required life time of mixer gearbox and the S/N curve are presented in the Gear Handbook by D.W. Dudley.

The predictive validity has also been verified with respect to the developed programs. Figure 3 shows the equation system solving with gear specifications calculation and strength analysis of the differential planetary gear system for mixer gearbox. It has also been verified the predictive validity with respect to the development and estimation of the scoring failure for differential planetary gears by scoring factor analysis.

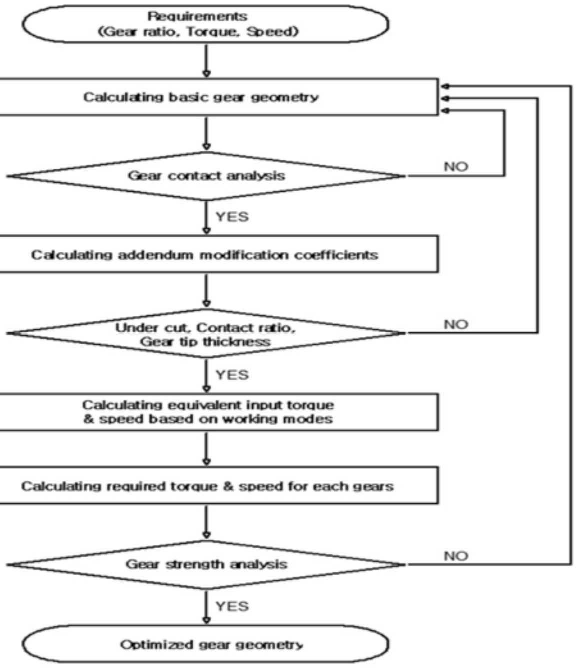


Figure 3. Equation system solving with gear specifications calculation and stress analysis.

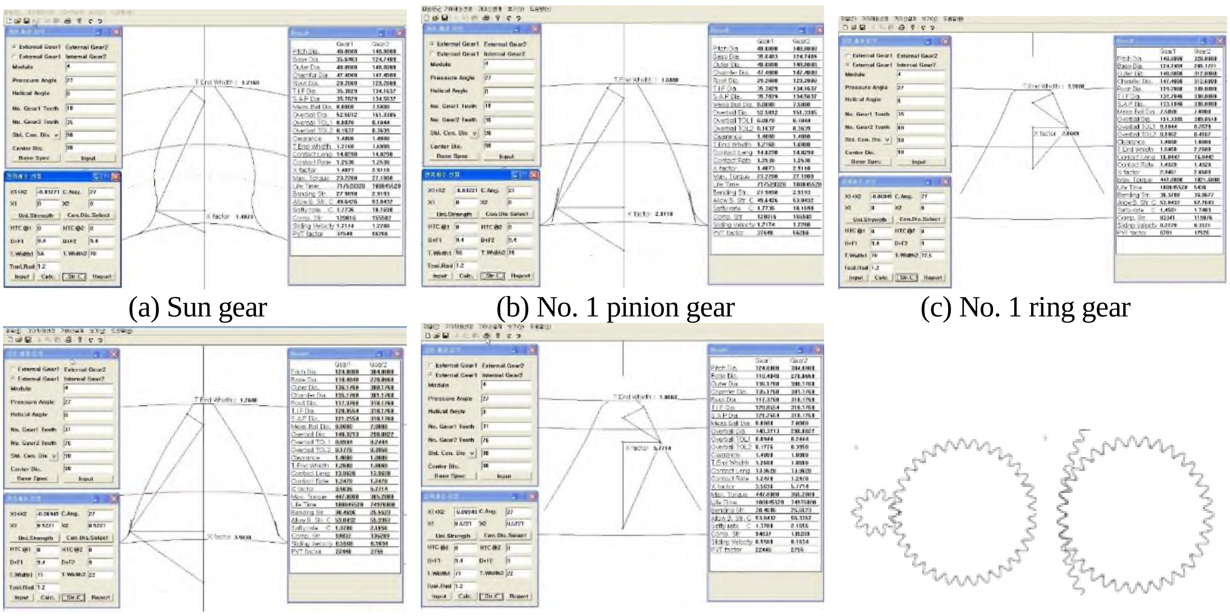
2. Material and data analysis

2.1. Calculation of gear specifications

Table 2 shows the specifications of the planetary gears of mixer gearbox after calculation. Figure 4 presents the results of the gear specification calculation program.

Table 2. Specifications of the planetary gear system.

Item	Sun gear(A)	No.1pinion gear(B)	No. 1 ring gear(C)	No. 2 pinion gear(D)	No. 2 ring gear(E)
Module	4	4	4	4	4
Pressure angle(°)	27	27	27	27	27
Helix angle(°)	0	0	0	0	0
No. gear teeth	10	35	80	31	76
Tooth mod. factor	0	0	0	+0.5220	+0.5220
Pitch dia.	40	140	320	124	304
Outside dia.	48	148	312	136.176	300.176
Over pin measurement	$52.501^{-0.087}_{-0.163}$ ($\Phi 8$)	$151.338^{-0.104}_{-0.196}$ ($\Phi 7.5$)	$309.851^{+0.416}_{+0.257}$ ($\Phi 7$)	$140.320^{-0.094}_{-0.177}$ ($\Phi 8$)	$298.081^{+0.395}_{+0.244}$ ($\Phi 7$)
Face width	56	70	72.5	71	22
Backlash	0.117 ~ 0.220		0.184 ~ 0.314	0.184 ~ 0.314	
Center distance	90		90	90	



(a) Sun gear (b) No. 1 pinion gear (c) No. 1 ring gear
(d) No. 2 pinion gear (e) No. 2 ring gear (f) Gear tooth contact

Figure 4. The results of the gear specifications calculation program.

2.2. Input equivalent torque/ rotation speed analysis

The required service period of life for a concrete mixer truck is 15 years with the vehicle operation rate of 70%, operating time is set 12 hours for a day, based on the total 28,400 hours, as shown in table 3.

Table 3. Operating mode and the required life period.

Working mode	Frequency of use (%)	Working time (h)	Input		Duty cycle	Cycle ratio
			Torque (N·m)	Speed (rpm)		
Input concrete	4	1,136	189.9	1320	89971200	0.125391849
Driving	41	11,644	189.9	264	184440960	0.257053291
Normal working	12	3,408	241.7	660	134956800	0.188087774
Maximum working	1	284	284.8	132	2249280	0.003134796
Driving	38	10,792	52.4	264	170945280	0.238244514
Washing	4	1,136	52.4	1980	134956800	0.188087774
Total	100	28,400	-	-	717,520,320	1

Equivalent mean torque for the average equivalent load of mixer reducer, T_{mi} is as follows:

$$T_{mi} = \left[\frac{\sum N_i t_i T_i^n}{\sum N_i t_i} \right]^{\frac{1}{n}} \tag{1}$$

where T_i is working torque, N is rotating speed, t is working time, n is power index (n=20.8)
Equivalent mean rotating speed for the average rotating speed of mixer reducer, N_{mi} is as follows:

$$N_{mi} = \left[\frac{\sum N_i t_i}{\sum t_i} \right] \tag{2}$$

where, N_{mi} is equivalent rotating speed for the average equivalent rotating speed, N_i is rotating speed, t_i is working time.

From the equation (1) and (2), the equivalent mean torque/rotating speed was calculated as 227.6N.m/421.08rpm.

2.3. Calculation of torque and number of rotation

From schematic diagram in figure 2, the gear ratio of mixer reducer calculated by relative speed diagram method [10] is as follows:

$$\Upsilon = \left\{ \frac{1 + Z_E Z_B / Z_A Z_D}{1 - Z_E Z_B / Z_D Z_C} \right\} \tag{3}$$

The number of rotation for each planetary gear calculated by relative speed diagram method is as follows:

$$N_B = N_D = \left\{ \frac{Z_A Z_C (N_A - N_C)}{Z_B (Z_A + Z_C)} \right\} \tag{4}$$

$$N_C = N_A / \Upsilon \tag{5}$$

$$N_S = \left\{ \frac{Z_A Z_D N_A}{Z_E Z_B + Z_A Z_D} \right\} \tag{6}$$

From the above equations, the torque and rotation speed is shown in table 4.

Table 4. Torque and number of rotation (N·m/rpm).

T_A/N_A (Torque/Number of rotation of sun gear)	227.6 / 421.08
T_B/N_B (Torque/Number of rotation of No.1 pinion gear)	4380.6(265.6) / 106.13
T_C/N_C (Torque/Number of rotation of No.1 ring gear)	10,012.5 / 3.19
T_S/N_S (Torque/Number of rotation of carrier)	30,264.9 / 43.95
T_D/N_D (Torque/Number of rotation of No.2 pinion gear)	4,380.6 / 106.13
T_E/N_E (Torque/Number of rotation of No.2 ring gear)	3,579.7 / 43.95

where, Z_A is the number of teeth of sun gear, Z_B is the number of teeth of No.1 pinion gear, Z_C is the number of teeth of No.1 ring gear, Z_D is the number of teeth of No. 2 pinion gear, Z_E is the number of teeth of No. 2 ring gear.

2.4. Calculation of gear bending stress

The actual gear bending stress equation by Lewes formula is as follows:

$$S = \frac{29{,}400\pi T}{N_a F X Z} \tag{7}$$

where, S is the actual gear bending stress (N/mm^2), T is torque on gears (N·m), N_a is the contact length of action (mm), F is the face width of gear (mm), X is the Lewes bending factor (mm), Z is the number of teeth in gear.

Allowable gear bending stress equation by Gear Handbook of D.W Dudley and AGMA Standard 218.01 [8] including gear bending S/N curve is as follows:

$$Sab = \frac{C_1}{N_F^{\frac{1}{20.8}}} \tag{8}$$

where, Sab is the allowable gear bending stress (N/㎟), N_F is the No. of cycles, C_1 is the coefficient.

2.5. Calculation of gear compressive stress

The actual gear compressive stress *P* (N/mm^2) applied at the tip of the planetary gears based on contact formula of Hertz is as follows:

In the case of external gear contact, the actual gear compressive stresses of sun gear and pinion gear are,

$$P_S = 19.43\sqrt{\frac{2\pi T_S \times CDSIN\alpha}{A_S(CDsin\Phi - A_S)\times F_c \times N_a \times Z_S}} \tag{9}$$

$$P_P = 19.43\sqrt{\frac{2\pi T_S \times CDSIN\alpha}{A_P(CDsin\Phi - A_P)\times F_c \times N_a \times Z_S}} \tag{10}$$

In the case of internal gear contact, the actual gear compressive stresses of pinion gear and ring gear are,

$$P_P = 19.43\sqrt{\frac{2\pi T_P \times CDSIN\alpha}{A_P(CDsin\Phi + A_P)\times F_c \times N_a \times Z_P}} \tag{11}$$

$$P_R = 19.43\sqrt{\frac{2\pi T_P \times CDSIN\alpha}{A_R(A_R - CDsin\Phi)\times F_c \times N_a \times Z_P}} \tag{12}$$

where, α is the normal pressure angle, Φ is the transverse pressure angle, T is the torque on driving gear (N·m), F_c is the active face width in contact (mm), Z is the No. of gear teeth, CD is the operating center distance, Na is the contact length of action (mm), A=, OR is the outside radius of gear, BR is the base radius of gear.

Allowable gear compressive stress equation by Gear Handbook of D.W Dudley and AGMA Standard 218.01 including gear compressive S/N curve is as follows:

$$Sac = \frac{C_2}{N_F^{\frac{1}{6.8433}}} \tag{13}$$

where, *Sac* is the allowable gear compressive stress (N/mm^2), N_F is the No. of cycles, C_2 is the coefficient.

2.6. Calculation of Gear scoring factor

Gear scoring factor [4] is based on Gear Handbook of D.W Dudley. To predict scoring failure of differential planetary gears, *PVT* (N·m/s·mm) is as follows: In the case of external gear contact, the actual gear scoring factor of sun gear and pinion gear are,

$$PVT_S = \frac{\pi N_S}{0.08525}\left(\frac{Z_P + Z_S}{Z_P}\right)(A_S - PR_S \times SIN\Phi)^2 \tag{14}$$

$$PVT_P = \frac{\pi N_P}{0.08525}\left(\frac{Z_P + Z_S}{Z_S}\right)(A_P - PR_P \times SIN\Phi)^2 \tag{15}$$

In the case of internal gear contact, the actual gear compressive stresses of pinion gear and ring gear are,

$$PVT_P = \frac{\pi N_P}{0.08525}\left(\frac{Z_R - Z_P}{Z_R}\right)(A_P - PR_P \times SIN\Phi)^2 \tag{16}$$

$$PVT_R = \frac{\pi N_P}{0.08525}\left(\frac{Z_R - Z_P}{Z_R}\right)(PR_R \times SIN\Phi - A_R)^2 \tag{17}$$

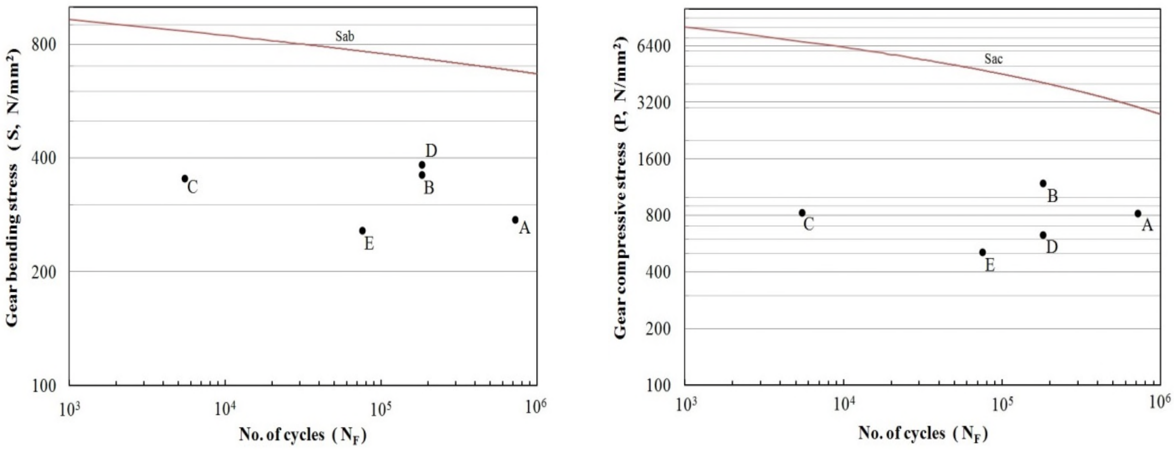
where, P is the actual compressive stress (N/mm^2), V is the sliding velocity (m/sec), T is the contact length from pitch point to contact point (mm), N is the speed of gears (rpm), Z is the number of gear teeth, Φ is the operating pressure angle.

Gear scoring factor, *PVT* must not exceed 124,019(N/sec·mm) to prevent scoring failure under HRC 60, carburized gears in mineral oil condition.

2.7. The results of gear bending and compressive stress analysis

The calculating actual gear bending and compressive stresses of planetary gear system for mixer gearbox and considering allowable gear bending and compressive stresses as well as the findings of the safety factors and verification of the problems of gear strength for the calculated specifications of the planetary gear system in mixer gearbox have been presented in this paper precisely. Figure 5(a) shows the results of gear bending stress analysis and figure 5(b) shows the results of gear compressive stress analysis of planetary gear system consisted of five gears (A to E) in mixer gearbox.

It can be shown that actual gear bending & compressive stresses of the differential planetary gears are under the allowable gear bending and compressive stresses in these S/N curves. Thus, calculation results are set safely and have verified as valid.



(a) Bending stress (b) Compressive stress

Figure 5. The results of gear stress analysis

2.8. The results of Gear Scoring factor analysis

Table 5 shows the results of calculated actual scoring factor, PVT and judge the safety of gear scoring failure considering the limit value, 124,019(N/sec·㎜).

Table 5. Calculated actual gear scoring factor.

	Sun gear	No. 1 pinion gear		No. 1 ring gear
Max. torque(N·m)	227.6	4,380.6		10,012.5
Speed(rpm)	421.08	106.13		3.19
Actual scoring factor(N·m/sec·mm)	3,031.2	5,741.6	15,708.0	1,085.6

(a) Sun gear + No. 1 pinion gear + ring gear

	No.2 pinion gear	No.2 ring gear
Max. torque(N·m)	4,380.6	3,579.7
Speed(pm)	106.13	44.00
Actual scoring factor(N·m/sec·mm)	23,733.7	163.7

(b) No. 2 pinion gear + No.2 ring gear

3. Conclusion

In this study, actual gear bending and compressive stresses of the differential planetary gears using Lewes & Hertz equation and the calculated specifications of differential planetary gears of mixer gearbox for 8m^3 grade concrete mixer truck have been analyzed and the results with the data of allowable bending and compressive stress from the Stress-No. of cycles curves of gears, based on Gear Handbook of D.W. Dudley and AGMA Standard 218.01 have been evaluated. The outcomes of this study can be summarized as:

(1) The strength of the differential planetary gears and the developed programs have been verified, in respect of the result of gear bending and compressive stress analysis of calculated specifications of differential planetary gears of mixer gear box for 8m^3 grade concrete mixer truck.

(2) In respect of the result of actual scoring factor analysis of calculated specifications of the differential planetary gears, scoring failure was not predicted.

(3) The developed programs calculating the specifications and analyzing the gear bending and compressive stresses and gear scoring factor of the planetary gear system for gearbox are expected to be effectively utilized. And future research on more excellent planetary gear system of the various reducers for construction machines is expected to be still performed.

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