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Modeling of the motion of automobile elastic wheel in real-time for creation of wheeled vehicles motion control electronic systems

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Abstract. Modeling of the motion of the elastic wheel of the vehicle in real-time is used in the tasks of constructing different models in the creation of wheeled vehicles motion control electronic systems, in the creation of automobile stand-simulators etc. The accuracy and the reliability of simulation of the parameters of the wheel motion in real-time when rolling with a slip within the given road conditions are determined not only by the choice of the model, but also by the inaccuracy and instability of the numerical calculation. It is established that the inaccuracy and instability of the calculation depend on the size of the step of integration and the numerical method being used. The analysis of these inaccuracy and instability when wheel rolling with a slip was made and recommendations for reducing them were developed.
It is established that the total allowable range of steps of integration is 0.001..0.005 s; the strongest instability is manifested in the calculation of the angular and linear accelerations of the wheel; the weakest instability is manifested in the calculation of the translational velocity of the wheel and moving of the center of the wheel; the instability is less at large values of slip angle and on more slippery surfaces. A new method of the average acceleration is suggested, which allows to significantly reduce (up to 100%) the manifesting of instability of the solution in the calculation of all parameters of motion of the elastic wheel for different braking conditions and for the entire range of steps of integration.
The results of research can be applied to the selection of control algorithms in vehicles motion control electronic systems and in the testing stand-simulators

1. Purpose

The analysis of the inaccuracy and the instability of the results of numerical calculation of motion parameters in real-time of elastic wheel rolling with a slip and development of recommendations for reducing them.

2. Introduction and relevance

Modeling of the motion of the elastic wheel of the vehicle in real-time is used in the tasks of constructing different models in the creation of wheeled vehicles motion control electronic systems, in the creation of automobile stand-simulators etc. [1-13]. During the motion of the vehicle wheel the lateral force is almost always present [9, 11]. Therefore, the models of his motion must take into account the phenomenon of slip, by which the motion of the elastic wheel is accompanied in the presence of lateral force. The accuracy and the reliability of simulation of the parameters of the wheel motion in real-time when rolling with a slip within the given road conditions are determined not only by the choice of the model, but also by the inaccuracy and instability of the numerical calculation. It is established that the inaccuracy and instability of the calculation depend on the size of the step of integration and the numerical method being used [11]. The authors of the article found that the inaccuracy and instability of the calculation depend on the size of the step of integration and the numerical method being used.

3. Problem statement

To achieve the purpose of research it is necessary to determine the model of the motion of a wheel, numerical methods and steps of integration.

Lateral force can be defined using the slip angle, calculated by the deformation theory [1, 9]:

$$P_y \approx \frac{\mathrm{tg}(\delta)\cdot C_{ty}\cdot (l_c/2)}{1 \pm \sin(\alpha)}; \tag{1}$$

where P_y – the lateral force, δ – the slip angle; C_{ty} – the coefficient of the tyre lateral stiffness, l_c – the length of a tyre contact patch with the road, α – the wheel camber angle.

The sign in the denominator is defined by the correspondence between the tilt of the wheel and lateral force directions.

The value of the lateral force is related to the value of proportion of the static friction coefficient in the longitudinal direction [1, 9]:

$$f_{stx} = \begin{cases} \sqrt{f_{st}^2 - \left(\dfrac{P_y}{P_z}\right)^2}; \text{ if } f_{st}^2 - \left(\dfrac{P_y}{P_z}\right)^2 \geq 0; \\ 0; \text{ if } f_{st}^2 - \left(\dfrac{P_y}{P_z}\right)^2 < 0. \end{cases} \tag{2}$$

where P_z – normal wheel load, f_{st} – static friction coefficient.

$$\begin{cases} M_{bi} = \begin{cases} f_{st}\cdot P_z \cdot R \cdot (t_i/T) \text{ if } M_{bi} \leq f_{st}\cdot P_z\cdot R\cdot (t_i/T); \\ f_{st}\cdot P_z\cdot R \text{ if } M_{bi} > f_{st}\cdot P_z\cdot R\cdot (t_i/T); \end{cases} \\ \dot{\omega}_i = \dfrac{1}{J}\cdot\left(-M_{bi} + P_z\cdot R\cdot \varphi_{x\ i-1}\right); \\ \omega_i = \omega_{i-1} + \dot{\omega}_i \cdot dt; \\ \dot{v}_i = -\varphi_{x\ i-1}\cdot g; \\ v_i = v_{i-1} + \dot{v}_i\cdot dt; \\ s_{xi} = 1 - \dfrac{\omega_i \cdot R}{v_i}; \\ CUSF_i = f(s_{xi}); \\ \varphi_{xi} = f_{sl}\cdot s_{xi} + CUSF_i \cdot f_{st\,x}(1 - s_{xi}); \\ L_{xi} = L_{xi-1} + v_i \cdot dt; \\ L_{yi} = L_{xi-1}\cdot tg(\delta). \end{cases} \tag{3}$$

where i – the step number; t – the current value of time; M_b – the braking moment applied to the wheel; R – the dynamic radius of the wheel; T – the time of braking moment change; $\dot{\omega}$, ω – angular

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acceleration and wheel velocity; $\dot{v}$, v – linear acceleration and velocity of the wheel axis; J – the inertia moment of the wheel; g – acceleration of free fall; $CUSF$ – the coefficient of use of static friction in the contact patch; φ_x – the friction coefficient between wheel and road in the longitudinal direction; s_x – coefficient of longitudinal wheel sliding; f_{sl} – sliding friction coefficient (φ_x if s_x=100%); f_{st} – static friction coefficient; $f_{st\,x}$ – static friction coefficient in the longitudinal direction; L_x– motion of the center of mass of a wheel in the longitudinal direction, L_y – motion of the center of mass of a wheel in the transverse direction.

For the analysis of the inaccuracy and the instability of the numerical calculation the model of the single wheel at braking (3) is used, as it is the most unstable mode. The parameters of the motion of a wheel are defined by solving the well-known system of differential equations and the constraint equations [5, 6, 8, 11] using the models of CUSF [1] for calculating the values of the friction coefficient.

Currently the researchers, who work in this field, apply the following integration steps: 0.001 – 0.005s. These steps are selected on the basis of the manifesting of the instability of the solution. Therefore, to achieve the purpose of the research the following integration steps: 0.001s, 0.002s, 0.003s, 0.004s and 0.005s are being used, and the implicit Euler method. At the present time it is the most common as being a relatively simple and accurate method.

4. Problem solving

Baseline data and constant values accepted for the solution of system (3) using the expressions (1) and (2) are presented in the table 1.

Table 1. Baseline data and constant values accepted for the solution of system (3).

Designation	Denotation	Denomination
Coefficient of a tyre radial stiffness	C_{tz}	210 N/mm
Coefficient of a tyre lateral stiffness	C_{ty}	275 N/mm
Length of a tyre contact patch with the road	l_c	223 mm
Wheel camber angle	α	0º
Normal wheel load	P_z	4000 N
Acceptable normal wheel load	$[P_z]$	6000 N
Time of braking moment change	T	0,5 s
Mass of the tyre	m_t	9,3 kg
Mass of the wheel disc	m_{belt}	7,247 kg
Linear and angular acceleration at the initial moment of time	$\dot{\omega}_0, \dot{v}_0$	0
Translational velocity of the wheel axle at the initial moment of time	v_0	10 m/s
Angular velocity of the wheel at the initial moment of time	ω_0	31,84 s-1

Table 1. (continued).

Designation	Denotation	Denomination
Coefficient of the longitudinal slip of a wheel at the initial moment of time	s_{x0}	0
Coefficient of friction between wheel and road in the longitudinal direction at the initial moment of time	φ_{x0}	0
Braking moment at the initial moment of time	M_0	0
Motions of the center of the wheel in the longitudinal and the transverse directions at the initial moment of time	L_{x0}, L_{y0}	0
Constant coefficients for calculation of CUSF	$a,b,c,d,e,$ f,g,h,i,j	a = 0.001814; b = 26.747630; c= -324.541748; d=2137.928850; e= -8375.670586; f =20260.588666; g=-30442.87972; h= 7611.479368; i=-13822.359721; j = 2929.705537

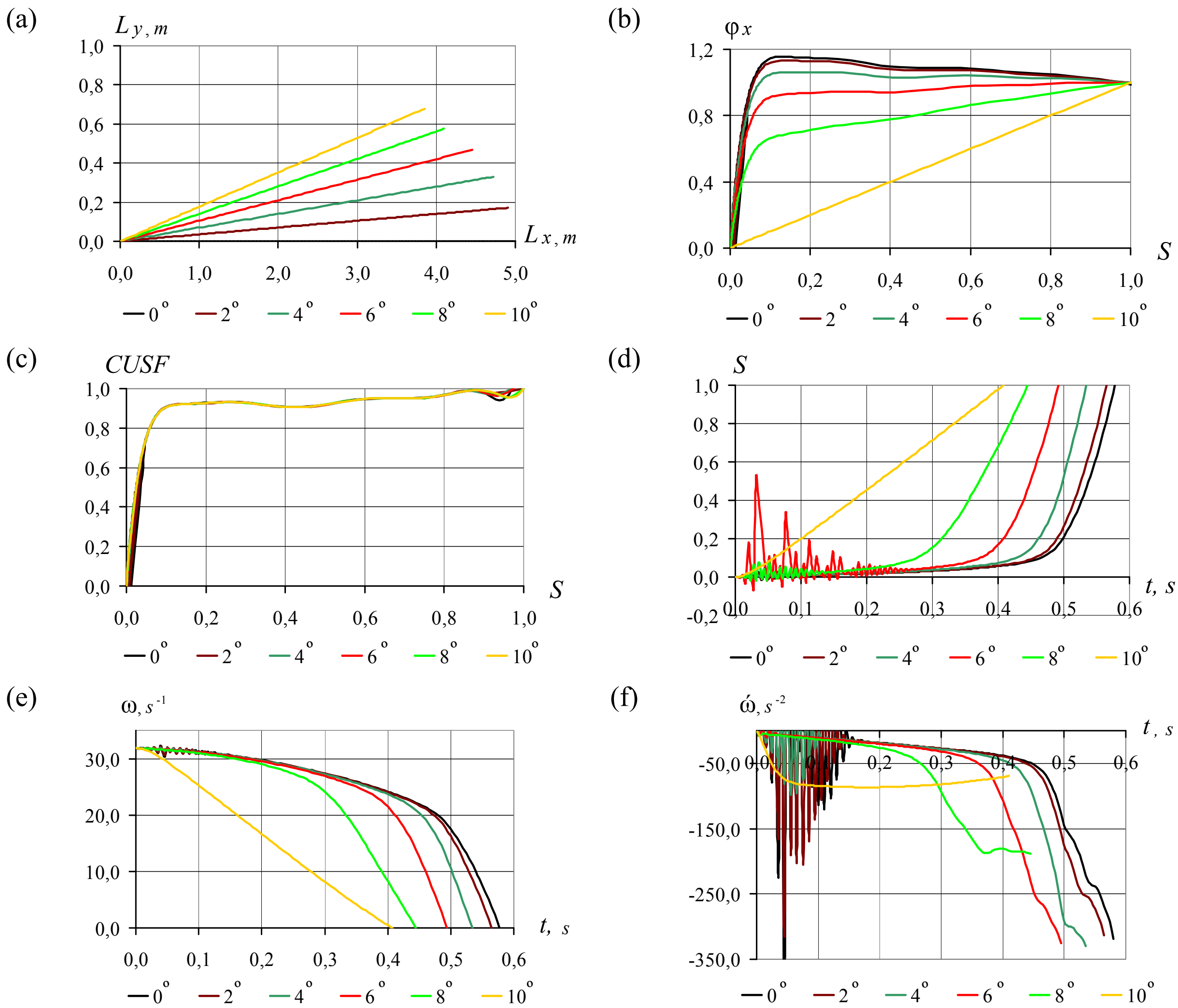
In the present study the calculations of parameters of braking of a vehicle wheel (3) for different integration steps, road surfaces and various values of lateral force (slip angle) using the implicit Euler method and using the method of average accelerations (system 4) were carried out. The equation that the system 4 differs from the system 3 is shown in grey below on the system 4.

$$\begin{cases} M_{bi} = \begin{cases} f_{st} \cdot P_z \cdot R \cdot (t_i/T) \text{ if } M_{bi} \le f_{st} \cdot P_z \cdot R \cdot (t_i/T); \\ f_{st} \cdot P_z \cdot R \text{ if } M_{bi} > f_{st} \cdot P_z \cdot R \cdot (t_i/T); \end{cases} \\ \dot{\omega}_i = \dfrac{\dot{\omega}_{i-1} + \dfrac{1}{J} \cdot \left(-M_{bi} + P_z \cdot R \cdot \varphi_{x\ i-1}\right)}{2}; \\ \omega_i = \omega_{i-1} + \dot{\omega}_i \cdot dt; \\ \dot{v}_i = -\varphi_{x\ i-1} \cdot g; \\ v_i = v_{i-1} + \dot{v}_i \cdot dt; \\ s_{xi} = 1 - \dfrac{\omega_i \cdot R}{v_i}; \\ CASF_i = f(s_{xi}); \\ \varphi_{xi} = f_{sl} \cdot s_{ix} + CASF_i \cdot f_{st\ x}(1 - s_{ix}); \\ L_{xi} = L_{xi-1} + v_i \cdot dt; \\ L_{yi} = L_{xi-1} \cdot tg(\delta). \end{cases} \quad (4)$$

Calculations were done for the following conditions:

- surfaces: «dry asphalt concrete » (f_{sl} = 1 and f_{st}=1.28) and «wet asphalt concrete » (f_{sl} = 0.7 and f_{st} = 0.93);
- steps of integration: 0.001s, 0.002s, 0.003s, 0.004s and 0.005s;
- slip angles: 0º, 2º, 4º, 6º, 8º and 10º.

Figure 1 presents an example of the calculation of parameters of braking using the system 4 for the following conditions: the integration step 0.005s, the surface coefficient: f_{sl}=1, f_{st}=1.28 and the method of average accelerations.



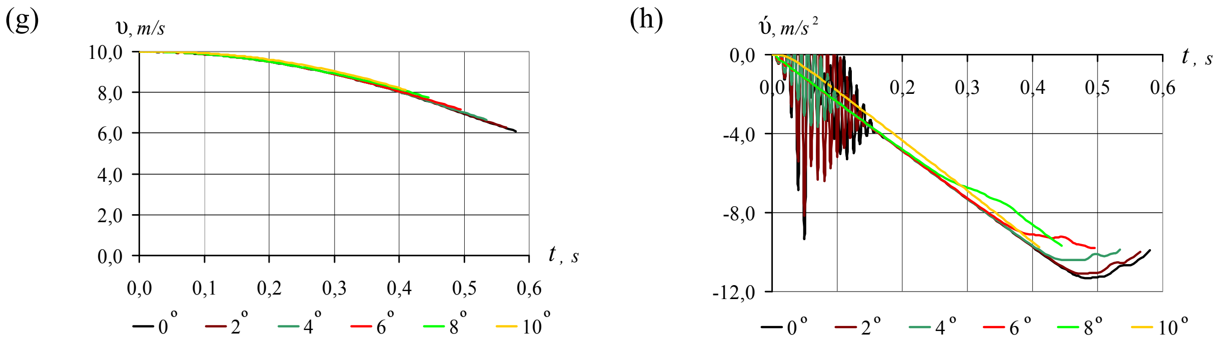


Figure 1. The motion parameters of vehicle wheel in the braking mode (integration step 0.005 s, surface coefficients: f_{sl} = 1, f_{st}=1.28, the method of average accelerations):
(a) – the dependence L_y of L_x; (b) – the dependence φ_x of s_x; (c) – the dependence $CUSF$ of s_x; (d) – the dependence s_x of t; (e) – the dependence ω of t; (f) – the dependence $\dot{\omega}$ of t; (g) – the dependence v of t; (h) – the dependence $\dot{v}$ of t

On the basis of the performed analysis of the influence of calculation method on the stability of the calculation results of motion parameters of vehicle tires, overall it is found that the application of the method of average acceleration will significantly reduce the manifestation of the instability of the solution in the calculation of all motion parameters of the slowing down vehicle wheel for different conditions.

The application of this method allows to calculate the required parameters without the instability of the solution with the larger value of the integration step up to 0.004 s and more on all surfaces with different values of coefficients of static and sliding friction.

5. Practical significance

The results of research can be applied to the selection of control algorithms in vehicles motion control electronic systems and in the testing stand-simulators.

6. Conclusions

1) in the tasks, related to the real-time calculating of motion parameters of the vehicle elastic wheel during braking mode the total allowable range of integration steps is 0.001..0.005s;

2) the instability never manifests in the calculation of all motion parameters of a wheel for all slip angles when braking on all road surfaces when the integration step is 0.001s using numerical methods of first and higher orders. In other cases, the instability is manifested;

3) the strongest instability is manifested in the calculation of the angular and linear accelerations of the wheel;

4) the weakest instability is manifested in the calculation of the translational speed of the wheel and motions of the wheel center in longitudinal and transverse directions;

5) the amplitude and the duration of the interval of manifestation instability is less for large values of slip angle;

6) the amplitude and the duration of the interval of manifestation instability is less on more slippery surfaces;

7) the application of the suggested new method of average acceleration allows to significantly reduce (up to 100%) the manifestation of instability of the solution in the calculation of all parameters of motion of the elastic wheel for different braking conditions and entire range of steps of integration.

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