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The mathematical model of dynamic stabilization system for autonomous car

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Abstract. Leading foreign companies and domestic enterprises carry out extensive researches and developments in the field of control systems for autonomous cars and in the field of improving driver assistance systems. The search for technical solutions, as a rule, is based on heuristic methods and does not always lead to satisfactory results. The purpose of this research is to formalize the road safety problem in the terms of modern control theory, to construct the adequate mathematical model for solving it, including the choice of software and hardware environment. For automatic control of the object, it is necessary to solve the problem of dynamic stabilization in the most complete formulation. The solution quality of the problem on a finite time interval is estimated by the value of the quadratic functional. Car speed, turn angle and additional yaw rate (during car drift or skidding) measurements are performed programmatically by the original virtual sensors. The limit speeds at which drift, skidding or rollover begins are calculated programmatically taking into account the friction coefficient identified in motion. The analysis of the results confirms both the adequacy of the mathematical models and the algorithms and the possibility of implementing the system in the minimal technical configuration.

1. Introduction

At present, practically all leading foreign companies and domestic enterprises carry out extensive researches and developments in the field of control systems for autonomous cars and in the field of improving driver assistance systems [1]. The search for technical solutions, as a rule, is based on heuristic methods and does not always lead to satisfactory results. Thus, the known electronic stability control (ESC) and velocity control system (VCS) have non-removable deficiencies, which does not allow to effectively use their capabilities in the road and climate conditions of Russian Federation.

The electronic stability programs use cyclic operation (like ABS) with separate brake control. The cyclic operation initiates sharp decrease in braking efficiency and stability stabilization on uneven surfaces such as "washboard", on paving stones, as well as on ice and snow-covered surfaces. On sandy surfaces, when driving in a track, the rollover speed is lower than the drift speed even for cars with low center of gravity.

For vehicles with high center of gravity such as SUVs, minivans the situation occurs on smooth asphalt surfaces. For these vehicles, dynamic stabilization systems were developed based on predicting the rate of rollover, skidding and drift [2,6]. However, the lack of technology to identify the top values of the friction coefficients in changing driving conditions required an underestimation of their estimates and, accordingly, the predicted speeds of skidding and drift.

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The purpose of the paper is to formalize the problem of improving vehicle's stability in terms of modern control theory, working-out adequate mathematical model for its solution, including the choice of software and hardware.

2. The formulation of vehicle's stability problem

At the conceptual level, vehicle's stability problem is reduced to predicting the excess of the limit speeds of skidding, drifting and rollover and to formation the control actions on traction and brakes, preventing the occurrence of these events.

At the content level, vehicle's stability problem follows from the solution of heading angle Ψ_m differential equation:

$$\dot{\Psi}_m = \omega_m + \Delta\omega_m, \tag{1}$$

where ω_m is the yaw rate of a car at a turn;

$\Delta\omega_m$ is the additional component of yaw rate at wheel drifts.

The solution of (1) with allowance $\omega_m = b^{-1} V_m \, \Psi_c$ in the discrete time can be represented in the form:

$$\Psi_m\,(k) = \Psi_m\,(k-1) + b^{-1}\int_{t_{k-1}}^{t_k} V_m\,(\tau)\Psi_c\,(\tau)d\tau + \int_{t_{k-1}}^{t_k}\Delta\omega_m d\tau. \tag{2}$$

The increment of heading angle $\Delta\Psi_m\,(k)$ at the k^th^ step is:

$$\Delta\Psi_m\,(k) = \Psi_m\,(k) - \Psi_m\,(k-1) = b^{-1}\int_{t_{k-1}}^{t_k} V_m\,(\tau)\Psi_c\,(\tau)d\tau + \int_{t_{k-1}}^{t_k}\Delta\omega_m d\tau. \tag{3}$$

In the case of rear wheels drift (oversteering), $\Delta\omega_m$ coincides in sign with Ψ_c and increases the increment of the heading angle. In the case of front wheels drift (understeering), $\Delta\omega_m$ has the opposite sign Ψ_c and reduces the module of heading angle increment. Additional rotation with yaw rate $\Delta\omega_m$ occurs around the center of front wheels axle at rear wheels drift and around center of rear wheels axle at front wheels drift. The reason for these phenomena is the excess of the centrifugal force acting on the front and rear wheels, the frictional forces values in the lateral direction.

In the case of rollover, the torque created by the centrifugal force is greater than the returning torque, caused by the force of gravity acting on the center of mass. The centrifugal force is proportional to the square of the speed, which makes it possible to reduce the conditions for preventing drifts and overturning to the system of inequalities for the speed:

$$\begin{cases} -V_{Lm0} \le V_m \le V_{Lm0}; \\ -V_{Lm1} \le V_m \le V_{Lm1}; \\ -V_{Lm2} \le V_m \le V_{Lm2}, \end{cases} \tag{4}$$

where V_{Lm0} is rollover limit speed ($\mathrm{m \cdot s^{-1}}$);

V_{Lm1} is the limit speed of front wheels drift ($\mathrm{m \cdot s^{-1}}$);

V_{Lm2} is the limit speed of rear wheels drift ($\mathrm{m \cdot s^{-1}}$).

Combining inequalities (4) into one allows one to reduce the problem of preventing these events to the problem of dynamic stabilization:

$$V_{Lm}^L \le V_m \le V_{Lm}^U, \tag{5}$$

where $V_{Lm}^U = \min[V_{Lm0}, V_{Lm1}, V_{Lm2}]$; $V_{Lm}^L = \max[-V_{gr0}, -V_{gr1}, -V_{gr2}]$.

Obviously the fulfillment of inequality (5) with the corresponding V_{Lm}^U and V_{Lm}^L means that inequalities (4) will also be fulfilled, in this case, drift and rollover events will not occur.

From the standpoint of modern control theory, the solution of the dynamic stabilization problem on a finite time interval $(t_1 \div t_2)$ is estimated from the value of control quality quadratic functional, taking into account functional limitations on the technical and economic indicators of the system that implements control:

$$Q(t_2) = \int_{t_1}^{t_2} C_1 \, [V_m(\tau) - V_{Lm}^U(\tau)]^2 \, d\tau + \int_{t_1}^{t_2} C_2 \, [V_m(\tau) - V_{Lm}^L(\tau)]^2 \, d\tau \Rightarrow \min \quad , (6)$$

under $U \in U_{allow}$, $R = (R_H, R_S)^T \in R_{allow}$; $q_i\ (R) \le q_{i\ allow}$, $1 \le i \le l$, where

$$C_1 = \begin{cases} 0, if\ V_m < V_{Lm}^U; \\ C_1^* \gg 0, if\ V_m \ge V_{Lm}^U; \end{cases}$$

$$C_2 = \begin{cases} 0, \text{если}\ V_m > V_{Lm}^L; \\ C_{2i}^* \gg 0, \text{если}\ V_m \le V_{Lm}^L; \end{cases}$$

R is the vector of technical solutions;

R_H and R_S are vectors of hardware (H) and software (S) solutions;

R_{allow} is allowable set of technical solutions;

$q_i\ (R)$ is i^{th} component of technic and economic indicators vector;

$q_{i\ allow}$ is allowable value of the i^{th} component.

The number of basic technical and economic indicators include the levels of power consumption, the influence of external factors, universality, fault tolerance, operating costs and the cost of hardware and software package.

The best solution to the problem in the proposed formulation is the algorithm for generating control actions that ensures dynamic stabilization of the object state vector, implemented in a software and hardware environment that satisfies the system of limitations on technical and economic indicators.

3. Results of constructing the mathematical model

The mathematical model of dynamic stabilization system includes equations of linear movement and limit speeds, equation system for wheel speeds on the turn, virtual sensor algorithms for car speed, turn angle, the additional component of yaw rate, for identification top values of friction coefficients between wheels and road surface, for control brakes by actuator.

3.1. The limit speeds of front, rear wheels drift and car rollover

The number of limit speeds in the general case includes next rates: rollover speed (V_{Lm0}), front drift speed (V_{Lm1}), rear drift speed (V_{Lm2}), tire cord breaking speed (V_{Lm3}), driving wheels slip speed (V_{Lm4}), speed of brake overheating (V_{Lm5}), traffic sign speed limit (V_{Lm6}), the speed of front obstacle (V_{Lm7}), driving speed with “limited use” spare tire (V_{Lm8}), driving speed at insufficient tire pressure (V_{Lm9}), etc.

The upper limit of safe speed is defined as $V_{Lm}^U = \min[V_{Lm0}, \dots, V_{Lmn}]$, and the lower limit is defined as $V_{Lm}^L = \max[-V_{Lm0}, \dots, -V_{Lmn}]$. For the positive direction of motion with $V_m \ge 0$ only the upper limit $V_{Lm}^U \ge 0$ has physical meaning.

The equation of rollover speed V_{Lm0} for horizontal surface is obtained from sufficient equilibrium condition (equality of tipping and returning torques):

$$V_{Lm0} = \sqrt{0.5abgh_m^{-1}|\Psi_C^{-1}|}, \quad (7)$$

where a and b are accordingly track and wheelbase of the car (m);

h_m is the height of car’s mass center (m);

g is the acceleration of gravity ($\mathrm{m \cdot s^{-2}}$);

Ψ_C is turn angle (rad).

The equation of front drift speed V_{Lm1} is obtained from III Newton Law (equality of frictional forces on front wheels in lateral direction and half of total centrifugal force):

$$V_{Lm1} = Re\sqrt{2[m_{12}gb - R_d\, a_{dT}]k_{sq}|\Psi_c^{-1}|}\ (\mathrm{m \cdot s^{-1}}), \tag{8}$$

where $m_{12} = (m_1 + m_2) \cdot m_0^{-1}$ is relative mass distribution on front wheels (m_1 and m_2) to the total car's mass m_0 ;

R_d is the dynamic radius of driving wheels (m);

a_{dT} is traction-braking acceleration ($\mathrm{m \cdot s^{-2}}$);

k_{sq} is top value of wheel frictional coefficient in lateral direction.

The equation of rear drift speed V_{Lm2} is also obtained from III Newton Law (equality of frictional forces on rear wheels in lateral direction and half of total centrifugal force):

$$V_{Lm2} = Re\sqrt{2[m_{34}gb + R_d\, a_{dT}]k_{sq}|\Psi_c^{-1}|}\ (\mathrm{m \cdot s^{-1}}), \tag{9}$$

where $m_{34} = (m_3 + m_4) \cdot m_0^{-1}$ is relative mass distribution on rear wheels (m_3 and m_4) to the total car's mass m_0 .

Traction-braking acceleration a_{dT} for (8) and (9) is defined from equation of linear movement:

$$a_m = a_{dT} - k_x\, m_0^{-1} V_m^2 - k_{rf} g - \alpha_T\, g, \tag{10}$$

where $a_m = \mathrm{d}V_m / \mathrm{d}t$ is longitudinal acceleration of car's mass center ($\mathrm{m \cdot s^{-2}}$);

k_x is the coefficient of frontal aerodynamic drag ($\mathrm{N \cdot m^{-2} \cdot s^2}$);

k_{rf} is the coefficient of rolling resistance between tires and road;

α_T is pitch angle (rad).

The values of tire lateral friction coefficient k_{sq} are related to the values of tire longitudinal friction coefficient k_{sd} by Kamm circle [3,7]:

$$(k_{sq}^2 + k_{sd}^2) \leq (k_s^*)^2\ , \tag{11}$$

where k_s^* is the top (maximum) value of resultant friction coefficient.

In this case, the value k_{sq} for one wheel is equal to:

$$k_{sq} = k_s^* \sqrt{1 - \left(\frac{k_{sd}}{k_s^*}\right)^2}\ , \tag{12}$$

where $k_{sd} = A \cdot m_0 \cdot a_{dT} \cdot F_N^{-1}$;

A is the distribution coefficient of tractive-braking forces per wheel;

F_N is the normal component of the force per wheel.

For the small values of module a_{dT} and A: $k_{sd} \ll k_s^*$ and $k_{sq} \approx k_s^*$.

The concept of optimal weighting follows from the equations of drift speed limits. The maximum speed at a turn for $V_{Lm1} < V_{Lm0}$ and $V_{Lm2} < V_{Lm0}$ is limited by $V_{min} = \min[V_{Lm1}, V_{Lm2}]$ and it's maximum value is achieved when $V_{Lm1} = V_{Lm2}$.

This condition is met when:

$$m_{12}gb - R_d\, a_{dT} = m_{34}gb + R_d\, a_{dT}.$$

Taking into account that $m_{34} = 1 - m_{12}$ we obtain:

$$m_{12} = a_{dT} R_d \; g^{-1} b^{-1} + 0.5 \text{ and}$$

$$m_{34} = 0.5 - a_{dT} R_d \; g^{-1} b^{-1}.$$

For Mercedes-Benz E-Class $b = 2.833$ m, $R_d = 0.3$ m, $a_{dT} = 3 \text{ m} \cdot \text{s}^{-2}$ at second gear we obtain $m_{12} = 0.53$; $m_{34} = 0.47$ which coincides with the technical data of these models.

3.2. Virtual sensors for turn angle and the additional component of yaw rate

The indirect measurements of turn angle Ψ_C and additional component $\Delta\omega_m$ of yaw rate allow to exclude from the system specialized sensors of steering wheel angle and angular velocity sensor. The indirect measurements are based on the incorrect problem solution of calculation variables in the equation system (accepted indexation of variables: 1, 3 correspond to the front and rear wheels of the port side; 2,4 correspond to the front and rear wheels of starboard):

$$\begin{cases} V_1 = V_m + 0.5ab^{-1}V_m \Psi_c + \Delta V_{s1} + 0.5a \cdot \Delta\omega_m; \\ V_2 = V_m - 0.5ab^{-1}V_m \Psi_c + \Delta V_{s2} - 0.5a \cdot \Delta\omega_m; \\ V_3 = V_m + 0.5ab^{-1}V_m \Psi_c + \Delta V_{s3} + 0.5a \cdot \Delta\omega_m; \\ V_4 = V_m - 0.5ab^{-1}V_m \Psi_c + \Delta V_{s4} - 0.5a \cdot \Delta\omega_m, \end{cases} \tag{13}$$

where ΔV_{s1}, ΔV_{s2}, ΔV_{s3}, ΔV_{s4} are longitudinal wheel slip speeds;
V_1, V_2, V_3, V_4 are linear wheel speeds.

The transformation of this problem to the correct one is possible when introducing additional determining conditions corresponding to the properties of the object [4,8,10]. So for the pair of wheels, the difference $V_i - V_j = \Delta V_{ij}$ $(i, j = 1{,}2; 3{,}4; 1{,}4; 3{,}2)$ is equal:

$$\Delta V_{ij} = ab^{-1}V_m \Psi_c + \left(\Delta V_{si} - \Delta V_{sj}\right) + a \cdot \Delta\omega_m. \tag{14}$$

Speed estimation for car's center of gravity is $\widehat{V}_m = 0.5\left(V_i + V_j\right) = V_m + 0.5\left(\Delta V_{si} + \Delta V_{sj}\right)$ and in the case $\Delta V_{si} = \Delta V_{sj} = 0$: $\widehat{V}_m = V_m$.

The solution (14) with respect to Ψ_c taking into account the accepted assumptions is:

$$\Psi_c = \Delta V_{ij} \, a^{-1} b V_m^{-1} - b V_m^{-1} \Delta\omega_m. \tag{15}$$

Taking $\widehat{\Psi}_c = \Delta V_{ij} \, a^{-1} b V_m^{-1}$, we obtain that:

$$\begin{cases} \widehat{\Psi}_c = \Psi_c + b \cdot V_m^{-1} \cdot \Delta\omega_m; \\ \Delta\omega_m = b^{-1} \cdot V_m \cdot \left(\widehat{\Psi}_c - \Psi_c\right). \end{cases} \tag{16}$$

Taking into account the additional defining properties of the object in the part of the value and sign $\Delta\omega_m$ in the form of conditions:

$$\begin{cases} \text{If } V_m < \min\left[V_{Lm1}, V_{Lm2}\right], \text{ then } \Delta\omega_m = 0; \\ \text{If } V_{Lm1} > V_{Lm2} \text{ and } V_m > V_{Lm2}, \text{then } sgn\left(\Delta\omega_m\right) = sgn\left(\widehat{\Psi}_c\right); \\ \text{If } V_{Lm2} > V_{Lm1} \text{ and } V_m > V_{\text{rp1}}, \text{then } sgn\left(\Delta\omega_m\right) = -sgn\left(\widehat{\Psi}_c\right), \end{cases} \tag{17}$$

under $\Psi_c = \Psi_{Lm} sgn(\widehat{\Psi}_c)$ we obtain:

$$\Delta\widehat{\omega}_m = \begin{cases} 0, \text{if } V_m < V_{Lm1} \text{ and } V_m < V_{Lm2}; \\ b^{-1}V_m \left[\widehat{\Psi}_c - \Psi_{Lm2} sgn\left(\widehat{\Psi}_c\right)\right], \text{if } V_{Lm1} > V_{Lm2} \text{ and } V_m > V_{Lm2}; \\ -b^{-1}V_m \left[\widehat{\Psi}_c - \Psi_{Lm1} sgn\left(\widehat{\Psi}_c\right)\right], \text{if } V_{Lm2} > V_{Lm1} \text{ and } V_m > V_{Lm1}. \end{cases} \tag{18}$$

$\Psi_{Lm} = \min[\Psi_{Lm1}, \Psi_{Lm2}]$; $\Psi_{Lm1} = 2[m_{12}gb - R_d\, a_{dT}]k_{sq}V_m^{-2}$;
$\Psi_{Lm2} = 2[m_{34}gb + R_d\, a_{dT}]k_{sq}V_m^{-2}$.

3.3. Dynamic stabilization system

The dynamic stabilization system (DSS) includes ABS speed sensors, brake pedal actuator and control board with the program of processing and control.

The braking deceleration a_T at the step is formed as $[V_{m\,e} - V_{Lm\,e}] \cdot \Delta T^{-1}$, where $V_{m\,e}$ is extrapolated speed value of the car; $V_{Lm\,e}$ is the extrapolated value of the limit speed $V_{Lm\,e} = \min[V_{Lm0\,e}, V_{Lm1\,e}, V_{Lm2\,e}]$; $V_{Lm0\,e}$, $V_{Lm1\,e}$, $V_{Lm2\,e}$ are extrapolated with respect to the angle Ψ_c values of rollover speed, front drift speed and rear drift speed. To compensate for the delay of about 1 s, introduced by the brake actuator, the extrapolation time is set at 2 s.

Control actions are formed by power keys with PWM signals and *PWM_T* duty ratio in accordance with the calibrated characteristic $PWM\_T = f(a_T)$ of the brake system with the actuator. Differentiation of the control action is performed programmatically to give the cascade connection of the actuator-braking system the proportional element properties [5,9,11]. The maximum deceleration value a_{Tmax} is calculated depending on the identifiable top value of the sliding friction coefficient in order to prevent wheel blocking during braking. In the motion mode with activated adaptive cruise control, the set speed V_{mz} is defined as $V_{mz} = \min[V_{mz0}, V_{Lm0}, V_{Lm1}, V_{Lm2}, \dots, V_{Lm9}]$ and possible excesses $V_{m\,e}$ over $V_{Lm\,e}$ can’t be significant.

3.4. Experimental results

DSS research tests were carried out for LADA Kalina car with electric traction in FSUE “NAMI” including NAMI’s Testing Centre.

The total mass of the car with driver and passenger is $m_0 = 1280$ kg, mass distribution on front wheels is $m_{12} = 0.5$, the track is $a = 1.42$ m; the wheelbase is $b = 2.461$ m, the height of car’s mass center is $h_m = 0.65$ m, $k_x = 0.51\ \mathrm{N \cdot s^{-2} \cdot m^2}$, winter studless tires are Michelin X-Ice 175/65 R14 82Q.

Test drives were conducted in a circle with the diameter of about 20 m on wet and dry asphalt, as well as during the passage of right and left turns with the radius of about 10 m with a change in the course angle by 90°. Figure 1 shows the time diagrams of car motion with disabled DSS at $k_s^* = 0.58$.

The analysis of the test results shows that the resulting turn angle Ψ_c, determined from the speeds of rear wheels with a fixed rudder position, decreases modulo as car speed increases, which corresponds to the side-drift effect due to non-zero lateral sliding of the wheels. The virtual sensor fixes positive $\Delta\omega_m$ corresponding to the drift of front wheels at speed close to $8\ \mathrm{m \cdot s^{-1}}$, positive a_{dT} and lateral acceleration (a_q) greater than $6\ \mathrm{m \cdot s^{-2}}$, which does not contradict the theoretical data.

Figure 2 shows the time diagrams of change car parameters at motion with enabled DSS, $k_s^* = 0.58$.

The analysis of the results shows that the actuation of the braking system occurs during predicted excess of the limit speed, which ensures the absence of non-zero $\Delta\omega_m$, speed stabilization at the level of $7\ \mathrm{m \cdot s^{-1}}$and lateral acceleration stabilization at the level of $6\ \mathrm{m \cdot s^{-2}}$.

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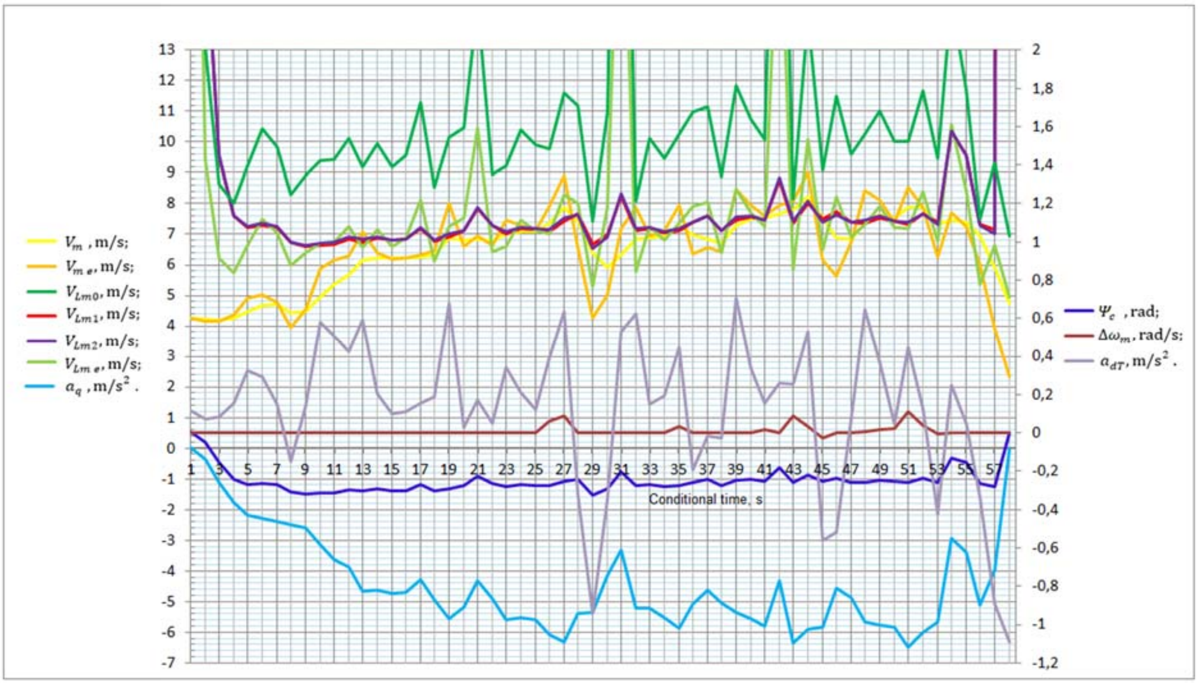

Figure 1. The time diagrams of car motion with disabled DSS on a circle.

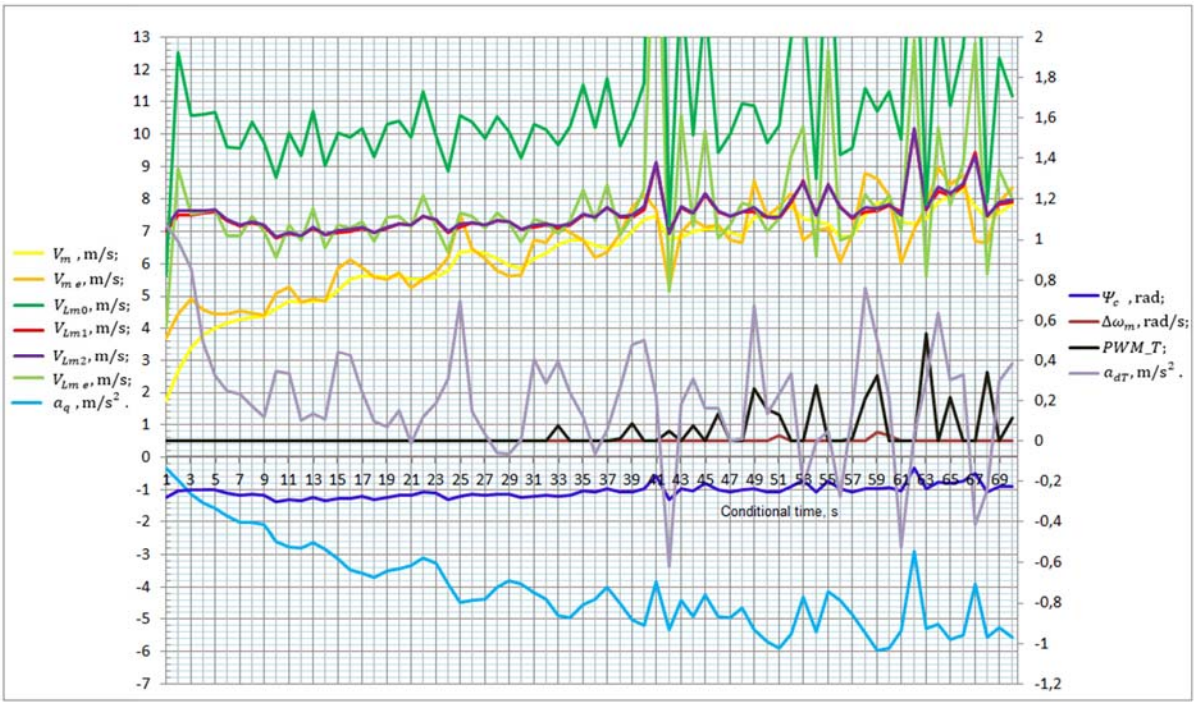

Figure 2. The time diagrams of car motion with enabled DSS on a circle.

4. Conclusion

The dynamic stabilization problem for autonomous car has been formulated in terms of minimizing the quadratic functional for control quality, taking into account functional limitations on technical and economic indicators.

The analysis of conducted research confirms both the adequacy of the mathematical models, the algorithms and the possibility of implementing the system in the minimal configuration of technical means.

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