

Airborne data measurement system errors reduction through state estimation and control optimization

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Abstract. The paper discusses the problem of airborne data measurement system errors reduction through state estimation and control optimization. The approaches are proposed based on the methods of experiment design and the theory of systems with random abrupt structure variation. The paper considers various control criteria as applied to an aircraft data measurement system. The physics of criteria is explained, the mathematical description and the sequence of steps for each criterion application is shown. The formula is given for airborne data measurement system state vector posterior estimation based for systems with structure variations.

1. Introduction.

Airborne data measurement systems (DMS) are widely used in aviation. Irrespective of DMS type, sensor spectral band, design features, etc., one of major vehicle DMS requirements is the obtaining of reliable information on current situation, as well as the timeliness of information.

Airborne DMS has the following parameters changing in a wide range: alternating vibration g-loads, temperature, humidity and pressure due to various environmental conditions of vehicle operation. In addition, DMS is susceptible to interference in the spectral range of its sensors and to jamming.

The paper proposes approaches to improve onboard DMS functioning for two cases: a fault-free properly functioning DMS and a system with errors. The causes of these errors can be both natural – ageing and wear – and intentional – jamming. DMS state recognition in abovementioned conditions is a very urgent problem in radar, navigation and communication areas. Besides, it is advisable to solve this problem without normal system operation interruption.

Let us consider the solution to the problem by the example of an airborne radar system.

2. Statement of problem

We consider an active airborne radar with passive response and an airborne digital computer as parts of an aircraft DMS. The aircraft task is to locate air vehicles, probably equipped with radar jammers. Because of ageing, wear and other adverse factors there might be gradual negative changes in DMS characteristics and its component features. The airborne radar may receive not only useful return signals but also deception signals, generated by jammers. It is essential to recognize the aircraft DMS



state, specifically, to find out whether DMS functions in a routine mode or in conditions of abnormal measurements due DMS faults or jamming. DMS functioning in regimes of abnormal measurements is associated with possible changes of system structure.

3. Method of system state recognition in inaccurate measurements

The joint aircraft and airborne radar state n -dimensional \mathbf{X} -vector characterizes center-of-mass motion kinematics, the dynamics of motion relative to the center-of-mass, radar and target relative positioning. It also includes airborne radar basic states such as angular misalignment φ between aircraft axis and target direction; line of sight angular rate ω , aircraft velocity relative to the target V and distance to the target D .

In addition, aircraft constructive element bending vibrations, environmental parameters, etc. can be taken into account.

Random inputs and outputs are typical for airborne radars. Let's consider an airborne radar observation model. In this case, a generalized measurement model can be presented in the form of m -dimensional vector \mathbf{Z} and random measurement noise vector \mathbf{N}

$$\mathbf{Z}(t) = \mathbf{C}\mathbf{X}(t) + \mathbf{D}(t)\mathbf{N}(t)$$

where \mathbf{C} - measurement matrix; $\mathbf{N}(t)$ - m -dimensional centered Gaussian white measurement noise vector with correlation function $\mathbf{K}_N(t, t_1) = \mathbf{Q}(t)\delta(t - t_1)$; $\mathbf{Q}(t)$ - measurement noise intensity matrix; $\delta(t - t_1)$ - Dirac function, $\mathbf{D}(t)$ - noise amplification matrix. In cases of failures and abnormal measurements caused by other reasons the matrix $\mathbf{D}(t) \gg \mathbf{I}$; for uninformative measurements measurement matrix \mathbf{C} - zero matrix, $\mathbf{D}(t)$ - unity matrix; in deception jamming environment vector $\mathbf{X}(t)$ is influenced by the jamming signals.

The generalized object model for joint aircraft and airborne radar state vector we use in the form of a variable structure model [1]

$$\mathbf{X}_{k+1} = \Phi_{k, k+1}^{(s)} \mathbf{X}_k + \mathbf{B}_k^{(s)} \mathbf{U}_k + \mathbf{F}_k^{(s)} \xi_k, \quad (1)$$

where \mathbf{X}_k - joint aircraft and airborne radar state vector; $\Phi_{k, k+1}^{(s)}$ - aircraft and radar state transition matrix; k - discrete current time; s - index, corresponding to the aircraft and radar structure number, active in k time moment; $\mathbf{F}_k^{(s)}$ - specified matrices which components are the functions of measurement vector; \mathbf{U}_k - aircraft control vector relative to a selected target; ξ_k - centered discrete Gaussian noise vector with correlation function matrix $\mathbf{K}_\xi(k, h) = \mathbf{G}_k \delta_{kh}$, h - time moment different from k ; \mathbf{G}_k - aircraft state noise intensity matrix; δ_{kh} - Kronecker delta.

For variable structure airborne radar measurements can be written in the form [3]

$$\mathbf{Z}_k = \mathbf{C}_k^{(s)}(\boldsymbol{\mu}_k, \boldsymbol{\gamma}_k) \mathbf{X}_k + \mathbf{N}_k^{(s)} \zeta_k; \quad (2)$$

$$\boldsymbol{\mu}_k = f_{k-1}^{(s)}(\boldsymbol{\mu}_{k-1}, \boldsymbol{\gamma}_k), \quad \mu_0 = \hat{\mu}_0, \quad k = \overline{1, K}, \quad (3)$$

where \mathbf{Z}_k - measurement vector; $\mathbf{C}_k^{(s)}(\boldsymbol{\mu}_k, \boldsymbol{\gamma}_k)$ - non-random measurement matrix that is a function of $\boldsymbol{\mu}_k$ и $\boldsymbol{\gamma}_k$, parameters, defining measurement conditions in s -th structure; $\boldsymbol{\mu}_k$ - measured parameters set matrix; $\boldsymbol{\gamma}_k$ - measurement process control vector with limitations

$$\boldsymbol{\gamma}_k \in \Gamma_K, \quad g(\boldsymbol{\mu}_k) \leq \bar{g}. \quad (4)$$

In (2)-(4) function $f_{k-1}^{(s)}(\cdot)$, $g(\boldsymbol{\mu}_k)$, \bar{g} value and Γ_K set - are specified initial parameters; $\mathbf{N}_k^{(s)}$ - specified matrix; ζ_k - measurement noise vector; i.e. centered discrete Gaussian noise vector with correlation function matrix $\mathbf{K}_\zeta(k, h) = \mathbf{Q}_k \delta_{kh}$, \mathbf{Q}_k - measurement noise intensity matrix; $\mu_0, \hat{\mu}_0$ -

respectively, measured parameters and their estimates at the initial moment of normal system operation.

In countermeasures environment measurement vector errors might be taken into account using measurement control procedure, in this case radar measurement vector components are formally presented in the form

$$\varphi + \Delta\varphi; \quad \omega + \Delta\omega; \quad V + \Delta V; \quad D + \Delta D,$$

that is

$$X = X + \Delta X. \quad (5)$$

It is assumed that ΔX components gradually change from zero to maximum values, which are individual for every parameter. Let's consider measurement control problems for specific control types $\{\gamma_k\}$ [3].

4. Measurement control problems. Measurement program (mode) selection

In measurement program selection γ_k and μ_k are scalar parameters. Γ_k set consists of two elements: $\Gamma_k = \{0, 1\}$, we assume $\gamma_k = 1$, if at the moment k the measurement is made; $\gamma_k = 0$, if there is no measurement. Then equation (3) for s -th structure can be written as

$$\mu_k = \mu_{k-1} + \gamma_k, \quad \mu_0 = 0$$

with limitations

$$\mu_K = \sum_{k=1}^K \gamma_k \leq K_\Sigma,$$

where K_Σ – a specified number of measurements.

In equation (2)

$$C_k^{(s)}(\mu_k, \gamma_k) = \gamma_k C_k^{(s)},$$

where $C_k^{(s)}$ – sensitivity of measurement channel in s -th structure.

5. Measured parameter combination selection

When selecting parameters for measurement in the equation (3) we formally write down $\mu_k = \gamma_k$, and measurement matrix in (2)

$$C_k^{(s)}(\mu_k, \gamma_k) = \mu_k.$$

Thus, matrix control $\gamma_k \in \Gamma_K$ specifies the combination of measured parameters, and Γ_k set is a potentially possible parameter combination.

6. Onboard DMS position (flight path) selection

In some cases, we may enhance measurement system efficiency through better operating conditions. For example, we may form the aircraft flight path in order to reduce airborne radar measurement errors.

In such a case the model (1) – (3) can be written as:

$$X_{k+1} = \Phi_{k, k+1}^{(s)} X_k + B_k^{(s)}(X_k) U_k + F_k^{(s)} \xi_k; \quad (6)$$

$$Z_k = C_k^{(s)}(X_k, U_k) X_k + N_k^{(s)} \zeta_k; \quad (7)$$

$$C_k^{(s)}(X_k, U_k) = C_{0k}^{(s)}(U_k) + \sum_{j=1}^n c_{jk}^{(s)}(U_k) X_k, \quad (8)$$

where $C_{0k}^{(s)}(\mathbf{U}_k)$ - nonlinear functions; $c_{jk}^{(s)}$ - statistical linearization coefficients for centered state variables. $C_{0k}^{(s)}$ and $c_{jk}^{(s)}$ are calculated according algorithms [4] with the use of Gaussian approximation of posterior probability density [5] and depend on \mathbf{U}_k control inputs, posterior mathematical expectations $\hat{\mathbf{X}}_{jk}^{(s)}$ and noise intensity matrices \mathbf{G}_k and \mathbf{Q}_k .

The additional component in the model (6) – (8) is the control optimization procedure \mathbf{U}_k in accordance with a pre-validated criterion. For example, if the measurement control purpose is just filtering quality improvement, then we shall consider average losses as an “information” criterion

$$\mathbf{I}(\hat{\mathbf{X}}_k) = M[\Psi(\mathbf{X}_k, \hat{\mathbf{X}}_k)] = \sum_{s=1}^S \int_{-\infty}^{\infty} \Psi(\mathbf{X}_k, \hat{\mathbf{X}}_k) \hat{p}_k(\mathbf{X}) d\mathbf{X}; \quad (9)$$

$$\Psi(\mathbf{X}_k, \hat{\mathbf{X}}_k) = \sum_{j=1}^n [\mathbf{X}_{jk} - \hat{\mathbf{X}}_{jk}]^2, \quad (10)$$

where $M[\cdot]$ – mathematical expectation operation; $\Psi(\mathbf{X}_k, \hat{\mathbf{X}}_k)$ – quadratic loss function; $\hat{p}_k(\mathbf{X})$ – posterior probability density of state vector \mathbf{X} .

If the purpose of measurement control is the simultaneous filtering quality enhancement and airborne measurement characteristics improvement through flight path optimization, then the problem of aircraft control should be solved according to a generalized “information and geometric factor” criterion

$$\mathbf{J}(\hat{\mathbf{X}}_k) = \alpha \mathbf{I}(\hat{\mathbf{X}}_k) + \beta \mathbf{L}(\hat{\mathbf{X}}_k); \quad (11)$$

$$\mathbf{L}(\hat{\mathbf{X}}_k) = [\hat{\mathbf{X}}_k - \mathbf{X}_{\text{opt}}]^2, \quad (12)$$

where α and β – coefficients weighting the requirements for \mathbf{X} vector filtering accuracy and optimal positioning of airborne radar relative to a selected target. With that $\mathbf{I}(\hat{\mathbf{X}}_k)$ is determined according to (9), (10); \mathbf{X}_{opt} – aircraft state vector providing optimal measurement conditions for a selected target.

The solution of this problem necessitates aircraft position control in accordance with the rule

$$\mathbf{U}_k = \begin{cases} \mathbf{U}_{\text{max}}, & \text{при } \mathbf{U}_k \geq \mathbf{U}_{\text{max}}; \\ \mathbf{U}_{\text{o.y}}, & \text{при } \mathbf{U}_k < |\mathbf{U}_{\text{max}}|; \\ -\mathbf{U}_{\text{max}}, & \text{при } \mathbf{U}_k \leq -\mathbf{U}_{\text{max}}, \end{cases}$$

where $\mathbf{U}_{\text{o.y}}$ is a control input determined according to the methodology [6], ensuring that airborne radar location is a compromise between two goals reflected in the criterion (11). \mathbf{U}_{max} – control input maximum value.

For simultaneous stability in the intersensor interference environment and the required filtering quality an airborne radar optimization should be carried out in accordance with the following criterion

$$\mathbf{F}(\hat{\mathbf{X}}_k) = \mathbf{I}(\hat{\mathbf{X}}_k) + M(\hat{\mathbf{X}}_k, \mathbf{Z}_k, \mathbf{U}_k); \quad (13)$$

$$M(\hat{\mathbf{X}}_k, \mathbf{Z}_k, \mathbf{U}_k) = \int_{-\infty}^{\infty} \Psi(\mathbf{X}_k, \hat{\mathbf{X}}_k) \hat{p}_k(\mathbf{X}) d\mathbf{X} \int_{-\infty}^{\infty} \phi_k^{(s)}(\mathbf{X}, \mathbf{Z}, \mathbf{U}) \hat{p}_k^{(s)}(\mathbf{X}) d\mathbf{X}; \quad (14)$$

$$\phi_k^{(s)}(\mathbf{X}, \mathbf{Z}, \mathbf{U}) = \sum_{l, q=1}^n \frac{\bar{Q}_{lq}^{(s)}}{|\mathbf{Q}^{(s)}|} [\mathbf{Z}_k - \mathbf{C}_{lk}^{(s)}(\mu_k, \gamma_k) \mathbf{X}_k] [\mathbf{Z}_k - \mathbf{C}_{qk}^{(s)}(\mu_k, \gamma_k) \mathbf{X}_k], \quad (15)$$

where l, q – indices of airborne radar state vector components; $\bar{Q}_{lq}^{(s)}$ – algebraic complement of element Q_{lq} in the determinant $|\mathbf{Q}^{(s)}|$ of measurement system noise matrix. In this case, the control becomes two-level [7]. The first-level control is carried out according to the criterion (14) for each s -th structure, and results in the selection of the best s number. At the second level, the problem of filtering quality improvement is solved in accordance with the criterion (9).

The estimation of a DMS state vector is based on the posterior probability density $\hat{p}_k^{(s)}(\mathbf{X})$ of an aircraft state vector in the s -th structure which is determined by the Bayes's rule based on the prior probability density $p_k^{(s)}(\mathbf{X})$ and \mathbf{Z}_k measurement:

$$\hat{p}_k^{(s)}(\mathbf{X}) = \frac{p_k^{(s)}(\mathbf{X}) \exp[-0,5 \phi_k^{(s)}(\mathbf{X}, \mathbf{Z}, \mathbf{U})]}{\sum_{s=1}^S \int_{-\infty}^{\infty} p_k^{(s)}(\mathbf{X}) \exp[-0,5 \phi_k^{(s)}(\mathbf{X}, \mathbf{Z}, \mathbf{U})] d\mathbf{X}}, \quad (16)$$

7. Conclusion

Thus, the problem of reduction an airborne DMS errors is discussed for two mostly typical cases:; operational flight with properly functioning system and DMS use in jamming environment. In the first case the problem is solved traditionally based on the criterion (9), the aircraft flight path optimization may also be introduced in accordance with (12) and minimizing the criterion (11); in the second case – it is necessary to determine possible jamming effects according to the criterion (14) and with that to provide the required filtering quality in accordance with the criterion (13). The problem solution is based on the joined aircraft-DMS state vector $\{\mathbf{X}_k, s_k\}$ posterior probability estimation using the theory of systems with random abrupt structure variation [7].

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