

Nonparametric method for failures diagnosis in the actuating subsystem of aircraft control system

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Abstract. In this paper we design a nonparametric method for failures diagnosis in the aircraft control system that uses the measurements of the control signals and the aircraft states only. It doesn't require a priori information of the aircraft model parameters, training or statistical calculations, and is based on analytical nonparametric one-step-ahead state prediction approach. This makes it possible to predict the behavior of unidentified and failure dynamic systems, to weaken the requirements to control signals, and to reduce the diagnostic time and problem complexity.

1. Problem setting for fault diagnosis

Let the models of a non-failure and failure aircraft dynamic be represented in the state space as

$$x_{i+1} = Ax_i + Bu_i, \quad (1)$$

$$x_{j+1}^f = Ax_j^f + B_f u_j, \quad (2)$$

where $i = \overline{0, l-1}, j = \overline{l, l+1, \dots}$ are the discrete times before and after the occurrence of failures; l is the instant a failures occur; x, x^f are the non-failure and failure aircraft state vectors of length n_x ; u is the control vector of length; A, B are the matrices of eigen-dynamics and control efficiency; $B_f = BF$ is the control efficiency matrix of the failure aircraft; $F = \text{diag}[f(1) f(2) \dots f(n_u)]$ is the matrix of failures, $f(*) = 1$ for the non-failure control channels, $0 \leq f(*) < 1$ for the failure control channels. It is necessary, based on the measurements of control signals u and states x only, to diagnose the failures in aircraft control system by determining the residuals in aircraft control efficiency parameters $\Delta B = B - B_f$. These residuals can be very useful for subsequent failure compensation by means of a aircraft control system reconfiguration [1–2].

There are two different approaches to solution of this problem [3–8]. The first approach is based on identification of B and B_f directly from (1)–(2) with subsequent computation of ΔB . The main challenge of this approach is the unidentifiability of aircraft model parameters without special test signals and a priory information [9–11]. So, the simplest way to solve failures diagnosis problem is to use parametric one-step-ahead prediction approach with the help of known estimations \hat{A}, \hat{B} :

$$\hat{x}_{i+1} = \hat{A}x_i + \hat{B}u_i, \quad (3)$$



$$\hat{x}_{j+1}^f = \hat{A}x_j^f + \hat{B}u_j. \quad (4)$$

Then we can calculate the difference between real (2) and predicted (4) failure aircraft state values

$$\Delta x_{j+1}^f = x_{j+1}^f - \hat{x}_{j+1}^f = Ax_j^f - \hat{A}x_j^f + B_f u_j - \hat{B}u_j = (A - \hat{A})x_j^f + (B_f - \hat{B})u_j. \quad (5)$$

If the aircraft model parameters are known exactly, then $\hat{A} = A$, $\hat{B} = B$, $\Delta \hat{A} = 0$, $\Delta \hat{B} = \Delta B$, and ΔB due to (5) can be easily derived from simple equation

$$\Delta \hat{B}u_j = \Delta x_{j+1}^f. \quad (6)$$

The main problem of the parametric prediction methods is that they require a priori information about the model parameters, which may be uncertain or even completely undefined.

2. The solution of failures diagnosis problem by the nonparametric prediction method

Widely known nonparametric prediction methods (neural networks, cellular automata, Markovian, chaotic, etc.) require preliminary training/tuning for a particular system or based on statistical algorithms, which use a sufficiently large sample of measurement data for the statistical properties extraction [12–15]. Therefore, such methods are not applicable, for example, for maneuverable aircraft evolution prediction. However, the main disadvantage of the known non-parametric prediction methods is that they can't be easily applied to solve one-step-ahead state prediction problems for systems with failures. In this paper we propose to solve the problem by analytical nonparametric prediction method, which is not subject to model errors, do not use statistical calculations, do not require training, and can be used to predict unidentified and failure dynamic systems.

Let's write the one-step-ahead prediction expressions (3)–(4) in the matrix form

$$\hat{X}_{i+1} = \hat{A}X_i + \hat{B}U_i, \quad \hat{X}_{j+1}^f = \hat{A}X_j^f + \hat{B}U_j, \quad (7)$$

where $U_i = [u_i \dots u_{i+h}]$, $U_j = [u_j \dots u_{j+h^f}]$, $X_i = [x_i \dots x_{i+h}]$, $X_j^f = [x_j^f \dots x_{j+h^f}^f]$, $\hat{X}_{i+1} = [\hat{x}_{i+1} \dots \hat{x}_{i+1+h}]$, $\hat{X}_{j+1}^f = [\hat{x}_{j+1}^f \dots \hat{x}_{j+1+h^f}^f]$. Note that, in contrast to the modeling problems (1)–(2), the one-step-ahead prediction problems (7) can be written by the single seamless block matrix expression

$$\begin{bmatrix} \hat{X}_{i+1} & \hat{X}_{j+1}^f \end{bmatrix} = \begin{bmatrix} \hat{A} & \hat{B} \end{bmatrix} \begin{bmatrix} X_i & X_j^f \\ U_i & U_j \end{bmatrix}. \quad (8)$$

Let's rewrite (8) to the form of parameters estimation equation

$$\begin{bmatrix} \hat{A} & \hat{B} \end{bmatrix} \begin{bmatrix} X_k \\ U_k \end{bmatrix} = \hat{X}_{k+1}, \quad (9)$$

where $k = i$ before and $k = j$ after the occurrence of failures; $X_k = [x_k \dots x_{k+h^*}]$, $\hat{X}_{k+1} = [\hat{x}_k \dots \hat{x}_{k+h^*}]$, $U_k = [u_k \dots u_{k+h^*}]$; $h^* = h$ for $k = 0:l-1$ and $h^* = h^f$ for $k \geq l$.

It's known [2, 16, 17], that any linear matrix equation of the form $YC = D$ with known matrices C and D is solvable for Y if and only if the solvability condition holds

$$D\bar{C}^R = 0, \quad (10)$$

and all set of its solutions is defined by a formula

$$X = \begin{bmatrix} D\tilde{C}^R & \Theta \end{bmatrix} \begin{bmatrix} \tilde{C}^L \\ \bar{C}^L \end{bmatrix} = D\tilde{C} + \Theta\bar{C}^L, \quad (11)$$

where Θ is an arbitrary matrix; \bar{C}^L , \bar{C}^R are the left and right zero divisors of maximal rank (matrices for which conditions $\bar{C}^L C = 0$, $C \bar{C}^R = 0$ are satisfied); \tilde{C}^L , \tilde{C}^R are the left and right unity divisors ($\tilde{C}^L C \tilde{C}^R = I$); $\tilde{C} = \tilde{C}^R \tilde{C}^L$ is the generalized inverse matrix defined by a canonical decomposition

$$C = \begin{bmatrix} \tilde{C}^L \\ \bar{C}^L \end{bmatrix}^{-1} \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{C}^R & \bar{C}^R \end{bmatrix}^{-1}.$$

Subject to (10) the equation (9) is solvable if and only if

$$\hat{X}_{k+1} \begin{bmatrix} X_k \\ U_k \end{bmatrix}^R = 0. \quad (12)$$

The condition (12) is fulfilled for any moment of time and can be used to solve the one-step-ahead state prediction problem. Let's write the estimation equation (9) in form:

$$[A \ B] \begin{bmatrix} x_k & \dots & x_{k+h^*-1} & x_{k+h^*} \\ u_k & \dots & u_{k+h^*-1} & u_{k+h^*} \end{bmatrix} = [\hat{x}_{k+1} \ \dots \ \hat{x}_{k+h^*} \ \hat{x}_{k+h^*+1}], \quad (13)$$

then the solvability condition (12) can be written as an expression

$$\begin{bmatrix} \hat{x}_{k+1} & \dots & \hat{x}_{k+h^*} & \hat{x}_{k+h^*+1} \end{bmatrix} \begin{bmatrix} x_k & \dots & x_{k+h^*-1} & x_{k+h^*} \\ u_k & \dots & u_{k+h^*-1} & u_{k+h^*} \end{bmatrix}^R = 0, \quad (14)$$

where h^* is the minimum number of observations for which the right zero divisor has the form of a column vector:

$$\begin{bmatrix} x_k & \dots & x_{k+h^*-1} & x_{k+h^*} \\ u_k & \dots & u_{k+h^*-1} & u_{k+h^*} \end{bmatrix} \begin{bmatrix} r_k(1) \\ \vdots \\ r_k(h^*) \\ r_k(h^*+1) \end{bmatrix} = 0. \quad (15)$$

Hence the condition (14) takes the form

$$\begin{bmatrix} \hat{x}_{k+1} & \dots & \hat{x}_{k+h^*} & \hat{x}_{k+h^*+1} \end{bmatrix} \begin{bmatrix} r_k(1) \\ \vdots \\ r_k(h^*) \\ r_k(h^*+1) \end{bmatrix} = 0, \quad (16)$$

that can be rewritten as an equation

$$\hat{x}_{k+h^*+1} r_k(h^*+1) = - \begin{bmatrix} \hat{x}_{k+1} & \dots & \hat{x}_{k+h^*} \end{bmatrix} \begin{bmatrix} r_k(1) \\ \vdots \\ r_k(h^*) \end{bmatrix}. \quad (17)$$

From (17) the next value of the state vector can always be uniquely defined by expression

$$\hat{x}_{k+h^*+1} = -[\hat{x}_{k+1} \dots \hat{x}_{k+h^*}] \begin{bmatrix} r_k(1) \\ \vdots \\ r_k(h^*) \end{bmatrix} / r_k(h^* + 1), \quad (18)$$

where the invertibility of $r_k(h^* + 1)$ is guaranteed according to (15) by the condition

$$\begin{bmatrix} x_k & \dots & x_{k+h^f-1} \\ u_k & \dots & u_{k+h^f-1} \end{bmatrix} \begin{bmatrix} r_k(1) \\ \vdots \\ r_k(h) \end{bmatrix} \neq 0. \quad (19)$$

Expression (18) is indeed a universal one-step-ahead prediction problem solution in the absence of errors in the model parameters ($\hat{A} = A$, $\hat{B} = B$) for both non-failure (3) and failure (4) cases.

Then to diagnose the failures we have to make several simple steps.

1. For $k=1:h$ we need to collect non-failure observations to fulfill the condition (15).
2. For $k=h+1:l$ we predict the non-failure aircraft dynamics \hat{X}_k by (18) with $h^* = h$. Note that in the absence of failures the condition $\hat{x}_k = x_k$ is always satisfied for non-failure case.
3. For $k=l+1$ the predicted state does not equal to real one $x_{l+1} \neq \hat{x}_{l+1}$, so we can conclude that there is a failure in the aircraft control system.
4. For $k=l+1:l+h^f$ we predict the failure aircraft dynamics \hat{X}_k^f by (18) with $h^* = h^f$, and estimate the difference $\Delta X_{k+1}^f = X_{k+1}^f - \hat{X}_{k+1}^f$.
5. The last step is to solve the equation of the form (6):

$$\Delta B U_k = -\Delta X_{k+1}^f. \quad (20)$$

Due to (10) the equation (20) is solvable if and only if

$$\Delta X_{k+1}^f \bar{U}_k^R = 0, \quad (21)$$

and due to (11) the set of all its solutions is defined by a formula

$$\Delta B = \Delta X_{k+1}^f \tilde{U}_k + \Theta \bar{U}_k^L, \quad (22)$$

where Θ is an arbitrary matrix. The fulfillment of the solvability condition (21) is always guaranteed in the absence of disturbances and can be used to control the computations.

From (22) it follows that in order to obtain a unique failures diagnosis problem solution

$$\Delta B = \Delta X_{j+1}^f \tilde{U}_j, \quad (23)$$

it's necessary and sufficient to ensure the linear independence of rows in the control matrix ($\bar{U}_j^L = 0$), that is required for all failures diagnosis methods [18, 19].

Figure 1 shows the implementation scheme of the proposed failures diagnosis method.

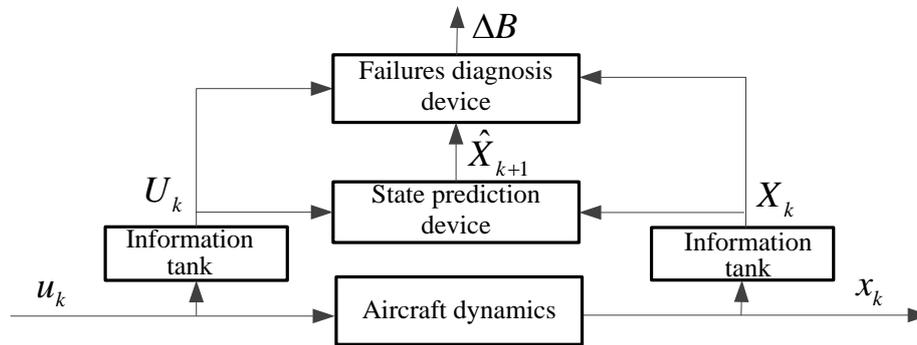


Figure 1. Failures Diagnosis Scheme.

3. Conclusion

The designed method refers to model-based nonparametric methods based on the analysis of input and output signals only and has the following advantages: it doesn't require a priori information about the model parameters, thus there are no any parametric errors; it's applicable for failures diagnosis in unidentifiable dynamic systems; it can reduce the diagnostic time and problem complexity due to no need of identifying the eigen-dynamics parameters; it weakens the requirements to control signals by ensuring only the independence of the controls with no need of exciting all oscillation modes of model.

Acknowledgments

This work is supported by Russian Foundation for Basic Research (17-08-01445-a, 17-01-00251-a)

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