

Optimal design of geometrically nonlinear shells of revolution with using the mixed finite element method

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Abstract. The article is concerned with a methodology of optimal design of geometrically nonlinear (flexible) shells of revolution of minimum weight with strength, stability and strain constraints. The problem of optimal design with constraints is reduced to the problem of unconstrained minimization using the penalty functions method. Stress-strain state of shell is determined within the geometrically nonlinear deformation theory. A special feature of the methodology is the use of a mixed finite-element formulation based on the Galerkin method. Test problems for determining the optimal form and thickness distribution of a shell of minimum weight are considered. The validity of the results obtained using the developed methodology is analyzed, and the efficiency of various optimization algorithms is compared to solve the set problem. The developed methodology has demonstrated the possibility and accuracy of finding the optimal solution.

1. Introduction

The proposed methodology of optimal design of shells allows designing the shells with variable thickness, e.g., ribbed shells, shells with a discrete inner layer, etc. In this paper the shell is regarded as structurally orthotropic.

Stress-strain state of shell for the solution of optimization problem was determined using the Galerkin method [1] in mixed finite element formulation [2, 3]. This method allows to do without the construction procedure of the functional of problem and to solve equations of problem in the form in which they are written. The authors were obtained in an explicit form of the matrix and vectors for the finite element shell. Their advantage is that the numerical integration is not used, which improves the accuracy of calculations. Another of the advantages of using the mixed formulation (as indicated by, e.g., Streng [2]) - reduction of the condition number of the matrix of the finite element, in comparison with the classical method of finite elements in displacements, which also positively effect on the accuracy of the calculation results. The methodology for determining the stress-strain state of geometrically nonlinear shells of revolution is described in detail in [3-5].

2. Description of the methodology

The problem of optimal design of the shell was considered as the problem of shell volume minimization with strength, stability and displacement constraints:

$$\bar{V}(\bar{x}) = 2\pi \sum_{i=1}^N \int_{-1}^1 h_i(\rho) \rho \left(1 + \left(\frac{df(\rho)}{d\rho} \right)^2 \right)^{1/2} d\xi \rightarrow \min, \\ p - p_{cr}(\bar{x}) \geq 0,$$



$$\begin{aligned} w_{\max}(\bar{x}) - w_u &\leq 0, \\ \sigma_{\max}(\bar{x}) - R &\leq 0. \end{aligned} \quad (1)$$

Such parameters as shell shape parameters, thickness distribution, etc. can be selected as components of the vector of the design variables \bar{x} .

In equation (1) p is the load imposed on the shell; $p_{\text{cr}}(\bar{x})$ is the critical load; $w_{\max}(\bar{x})$ is the maximum deflection of the shell; w_u is the ultimate permissible deflection; $\sigma_{\max}(\bar{x})$ is the maximum equivalent stresses in the shell; R is the ultimate allowable stress.

The component of the vector of maximum equivalent stresses ($\sigma_{\max}(\bar{x})$) are calculated as:

$$\begin{aligned} \sigma_{\max} = & \left[\left(\frac{N_\alpha}{h(\alpha, \beta)} + \frac{6M_\alpha}{h(\alpha, \beta)^2} \right)^2 + \left(\frac{N_\beta}{h(\alpha, \beta)} + \frac{6M_\beta}{h(\alpha, \beta)^2} \right)^2 - \right. \\ & \left. - \left(\frac{N_\alpha}{h(\alpha, \beta)} + \frac{6M_\alpha}{h(\alpha, \beta)^2} \right) \cdot \left(\frac{N_\beta}{h(\alpha, \beta)} + \frac{6M_\beta}{h(\alpha, \beta)^2} \right) + 3 \cdot \left(\frac{S}{h(\alpha, \beta)} + \frac{6H}{h(\alpha, \beta)^2} \right)^2 \right]^{1/2} \end{aligned} \quad (2)$$

The values $p_{\text{cr}}(\bar{x})$, $w_{\max}(\bar{x})$ and $\sigma_{\max}(\bar{x})$ are determined at each step of the optimization algorithm using the Galerkin method in mixed finite element formulation [4 - 6].

The problem of optimal design with constraints is reduced to the problem of unconstrained minimization using the penalty functions method. Penalty function r_k takes the form proposed by Mischke [6]:

$$\bar{V}_k(\bar{x}) = V(\bar{x}) + r_k d \sum_{i=1}^N b_i \cdot (\bar{x}_c - \bar{x})^2 + s \rightarrow \min, \quad (3)$$

where, $V(\bar{x})$ is the objective function of the problem with constraints; \bar{x}_c is the vector of design parameters values corresponding to any point in design space, where all constraints $g_i(\bar{x})$ hold (in the domain of feasible solutions); N is the number of constraints; d is the scale multiplier to reduce functions $V(\bar{x})$ and $g_i(\bar{x})$ to same order; b_i is the parameter equal to 1 if the constraint $g_i(\bar{x})$ is satisfied and equal to 0 if it is not satisfied; r_k is the penalty parameter for the k -th minimization problem (iteration) selected from the following sequence: 1, 100, 10,000, 1,000,000; s is the parameter used in some problems to define a barrier on the boundary of feasible solutions ($s=0$ within the domain of feasible solutions and $s=\text{const}>0$ outside this domain); $k=1..M$ – number of iteration; M is the number of iterations.

The problem of optimal design in form as in equation (3) can be solved any of classical algorithms of unconstrained minimization (e.g. method of steepest descent, grid search method, method of conjugate gradients, etc.).

3. Test problems

To check the validity of the results obtained using the developed methodology and to compare the effectiveness of different algorithms of unconstrained optimization, two test problems of optimal design of a shallow axisymmetric shell of revolution were solved. Shell dimensions: the shell base radius $a=3$ [m], rising height $f_0=0.45$ [m]; shell material characteristics: $E=2.1 \cdot 10^{11}$ [Pa], $\nu=0.3$, $R=210$ [MPa]; ultimate strain $w_u=0.01$ [m]. The shell is closed at the top. The shell is fixed rigidly to the support. The load is distributed uniformly on the shell; its intensity is $p=10$ [kPa]. The shell was divided evenly into 20 finite elements along the generatrix. The position the of the finite elements

nodes along the shell generatrix was calculated by formula: $f(\rho) = f_0 \rho^Z$, where Z is the shell shape parameter, ρ is the radial coordinate.

The 1st test problem is to determine the shape and thickness of the shell of minimum weight (design variables are Z and h) under the assumption that the shell thickness is the same over its entire surface.

The 2nd test problem is to determine the optimum thickness distribution for a minimal weight shell (design variables are h and hI) under the assumption that the thickness varies linearly from the centre (hI) to the edges (h) and does not change in the circumferential direction.

4. Checking the results

The diagrams of changes of objective and penalty functions depending on the node parameter values were plotted (see figure 1 - 5 for the 1st problem, and figure 6 for the 2nd problem).

For problem 1 the boundary of permissible values with stability, strain and strength constraints has the form of a parabola with the vertex at the values $Z = 1.5 - 2$ (figure 1 - 5). Taking into account the descending direction of the volume function, it is possible to determine visually the position of the objective function minimum on the boundary of the domain of permissible values when $Z = 1 - 1.7$ (see figure 5).

The results of the 1st optimization problem solution using the developed methodology are the values of design variables $Z = 1.65$, $h = 0.00173$ and the objective function value of 0.0075. The reliability of these results is confirmed by visual analysis (figure 5).

For 2nd problem the boundary of permissible values has the form close to a straight line (figure 6). Taking into account the descending direction of the volume function the minimum of the objective function is on the boundary of the search domain with the minimum values of hI (figure 6).

The results of solving 2nd problem using the developed optimization methodology are the values of design variables $hI = 0.0005$, $h = 0.00756$ when the value of the objective function is 1.07. This solution is consistent with a shell whose thickness decreases towards the centre. The validity of these results can be confirmed by the visual analysis of the diagram of the objective function changes (figure 6).

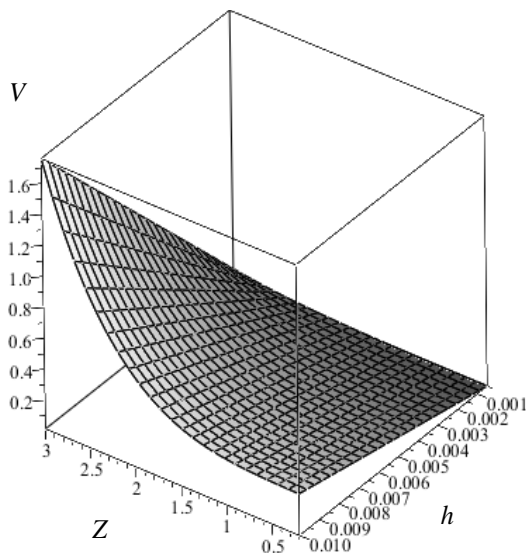


Figure 1. Dependence of shell volume change on parameters Z and h .

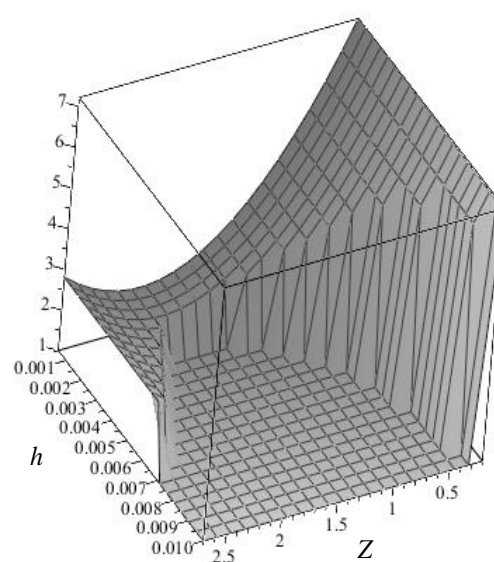


Figure 2. Dependence of penalty function for stability constraints on parameters Z and h .

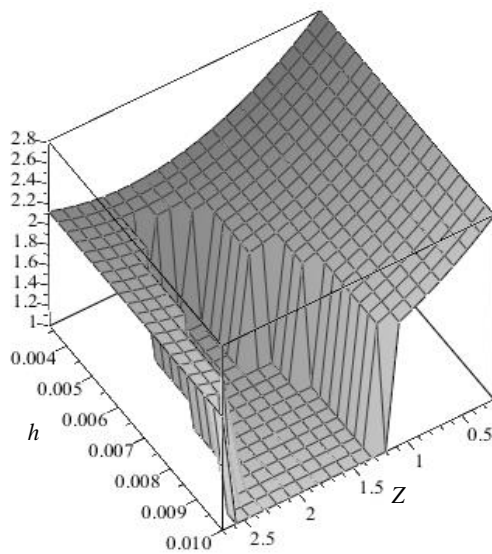


Figure 3. Dependence of penalty function for strength constraints on parameters Z и h .

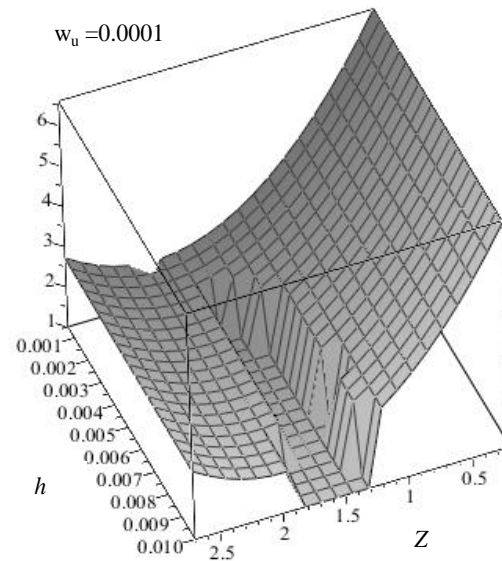


Figure 4. Dependence of penalty function for strain constraints on parameters Z и h .

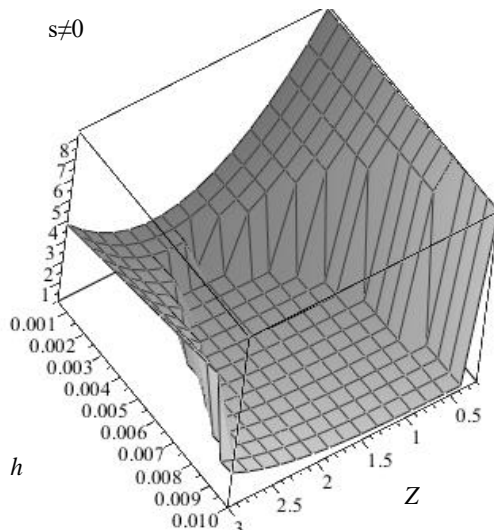


Figure 5. Dependence of 1st problem objective function on parameters Z and h .

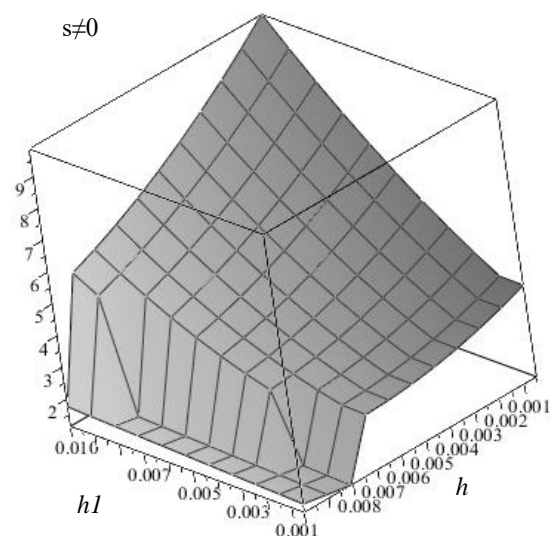


Figure 6. Dependence of 2nd problem objective function on parameters hl and h .

When studying the shape of the boundary of the permissible values domain with regards to stability, minor irregularities associated with the inaccuracy of critical load determination algorithm were found. They can cause difficulties when determining an optimal solution. Therefore, to obtain reliable results search for an optimal solution should be started from several points, and then the best solution should be selected.

5. Analysis of efficiency of optimization algorithms

A comparative analysis of the effectiveness of different optimization algorithms used to solve first problem have been carried out ([7-11]). Such methods as the grid (multidimensional continuous search) with the exclusion of domains, the steepest descent method with a variable step, a method based on a combination of gradient and random search [9 - 11] have been considered.

Grid search method with the exclusion of domains was used with the objective function (see equation 3) without iterative refinement with the constant penalty parameter $r_k = 1,000,000$ and parameter s , which is equal to the mean of function $V(\bar{x})$ beyond the boundaries of feasible solutions. Search domain is divided into 4 parts along each of the coordinates.

Grid search method with the exclusion of domains has proved to be a reliable one for searching an optimal solution; it also requires a small number of calculations (384 calculations of the objective function were done in the problem considered) when searching is carried out with a small number of design variables. However, with an increase in the number of design variables a significant increase in the number of calculations of the objective function should be expected which hinders the use of this method in such cases.

Application of the steepest descent method with a variable step and parameter $s=0$ (objective function without a barrier on the boundary of feasible solutions), has shown that in most cases it does not lead to finding an optimal solution. This is due to a ravine surface shape of the objective function (see equation 3). The algorithm usually 'loops' on the boundary of permissible values domain.

The method, based on a combination of gradient and random search, proved to be a reliable one when solving problems when in equation 3 $s=0$. To obtain sufficiently accurate results 412 calculations of the objective function were done. This value is higher than the one, which was done for the grid search method with the exclusion of domains. However, with an increase in the number of design variables, in contrast to the grid search method, there is no significant increase in the number of calculations of the objective function. Therefore, it can be recommended for solving problems with the use of the developed optimal design method with a large number of design variables.

6. Conclusions

Methodology of optimal design of geometrically nonlinear shells of revolution of minimum weight with strength, stability and strain constraints is developed. Stress and strain state of the shell is determined by Galerkin method in the mixed finite element definition.

Based on this methodology, algorithms for determining the optimal form and thickness distribution of a shell of minimum weight are obtained. The results of solving two test problems confirmed the possibility and accuracy of finding the optimal solution.

The analysis of the effectiveness of different optimization algorithms to solve the set problem is given. The selected algorithm based on a combination of gradient and random search, allows to effectively searching the optimal solution for a large number of design parameters.

7. Acknowledgement

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8. References

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