Kinematic Analysis and Performance Evaluation of Novel PRS Parallel Mechanism

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Abstract.　In this paper, a 3 DoF (Degree of Freedom) novel PRS (Prismatic-Revolute- Spherical) type parallel mechanisms has been designed and presented .The combination of striaght and arc type linkages for 3 DOF parallel mechanism is introduced for the first time. The performances of the mechanisms are evaluated based on the indices such as Minimum Singular Value (MSV), Condition Number (CN), Local Conditioning Index (LCI), Kinematic Configuration Index (KCI) and Global Conditioning Index (GCI). The overall reachable workspace of all mechanisms are presented. The kinematic measure, dexterity measure and workspace analysis for all the mechanism have been evaluated and compared.

1. Introduction

In modern world, Industrial robotic application plays a vital role inthe field of automation.For the past two decades parallel robotic architectures have seeked more attention, both theoretical and practical approaches. Numerous Configurations have been constituted and formulated and the same also fabricated its proto type and experimentally validated. [1, 2, 3, 5]In this study, a exhaustive list of different type of Parallel mechanisms were studied. A parallel mechanism (PM) typically consists of a moving platform that is connected to a fixed base by several limbs or legs. Due to its external heavy load can be shared by the actuators, PM tend to have a large load carrying capacity. In many applications, PM has been implemented such as airplane simulators, adjustable articulated trusses, mining machine, pointing devices, high speed machining center, and walking machines. One of the disadvantages of PMs is the difficulty of trajectory planning mainly due to singular configurations, in which the mechanism gains one or more degrees of freedom and therefore loses stiffness of the mobile platform completely. In Particularly, PRS configuration is clearly focused and analysed.[6]The kinematic analysis of such configuration also carried out with different actuator arrangement. The reachable workspace features, dexterity characterstics sucha kinematic manipulability and global dexterity also derived . [7] using homotophy continuation method also forward kinematic analysis also carried out.[12]In medical application,for CPR (Cardio Pulmanory Resuscitation) the same configuration used and its dynamic modell also analysed based on lagrangian formulation.[13] For the same configuration, the work volume also determined using fuzzy logic approach.[15, 14] Based on special decomposition of reaction force, a novel approach also identified for the dynamic analysis. Experimentally these 3 DOF PMswere verified and compared ith ADAMS software. A new index called Worst case Global Indices [18] (WGI) was introduced, to take into consideration the kinematic and force transmission performance of the parallel mechanism. This new index was compared with the existing indices such as Global Conditioning Index (GCI). These comparisons were carried out in delta robot which was used in ENT surgery application. Further exploring the wide

application of PMs a novel idea of spherical parallel wrist [19] was proposed. A detail kinematic analysis shows that actuator redundancy not only removes singularities but also improves the workspace by increasing the dexterity. Using parallel mechanism a new hip simulator mechanism [20] was proposed. The kinematic index is calculated to measure the force transmission ratio. The simulation results were compared and verified with the experimental works. To evaluate the performance of these mechanisms dexterity index, kinematic manipulability index and global dexterity index was derived. Another mechanisms 6URS [21] parallel mechanism, was proposed. A vast survey about the performance indices [22] which were used to evaluate the performance or characteristics of parallel mechanism was studied elaborately in kinematics, dynamics and in workspace aspects. A new 3 RRPRR [23] parallel mechanism was introduced to achieve purely translation motion. Singularity analysis is carried out from the derivation of Jacobian matrices. The concept of Jacobian matrix, manipulability index [24] and the condition no were discussed in the view of mechanisms performance.based on the literature survey,a novel PRS configuration is proposed. The kinematic analysis and performance measure based evaluation also carried out.

In this paper is organized in the following manner; the overall Design information including its construction & mobility information is presented in section II. The kinematic analysis & Jacobian Analysis are presented in section III and IV respectively,Section V includes information about the performance indices such as Minimum Singular Value (MSV), Condition Number CN, Local Conditioning Index (LCI) and Kinematic Configuration Index (KCI).It is then evaluated with a numerical case study & presented in section 6 as results & discussion. Conclusions are drawn in section VII followed by References
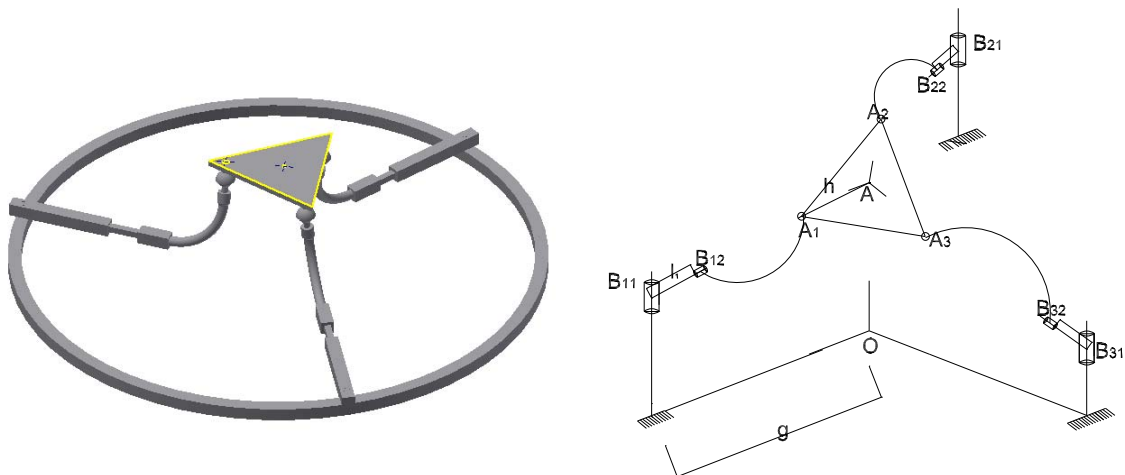
2. Proposed Mechanism

**Figure 1.** Proposed (Proposed (CAD & schematic) 3DOF PRS type Parallel Mechanism Configuration

The proposed novel RS type PMs consist of $n(=7)$ links, $j(=9)$ joints, where DoF of revolute joint and prismatic joint is one and for the spherical joint is three. Degree of freedom of this mechanism can be calculated using Grubbler Kutzbach criterion:

$$DoF = \lambda(n - j - 1) + \sum f_i \qquad (1)$$

where f_i is degree of freedom of individual joint. Thus, DoF = 6(7-9-1)+(3*3+3*1+3*1) = 3.

Table 1. Quantity of Joints and linkages used in all type of 6 DoF RS type configurations

S.No.	Components	Quantity
1	Number of links	7
2	Number of joints	9
3	Number of revolute/prismatic joints	3
4	Number of spherical joints	3
5	Number of actuating joints	3
6	Number of passive joints	3 spherical & 3 prismatic joints
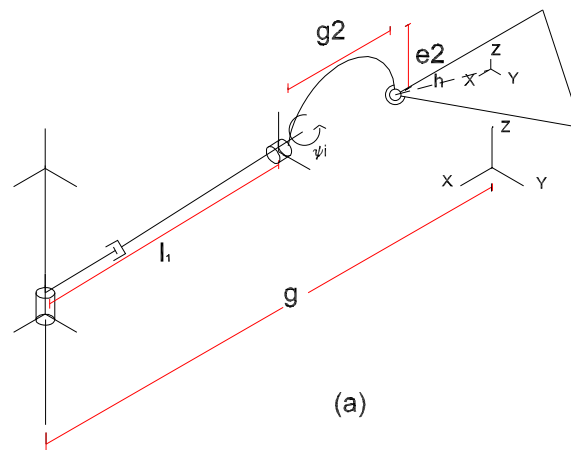
Figure 2. Schematic Representation of PRS
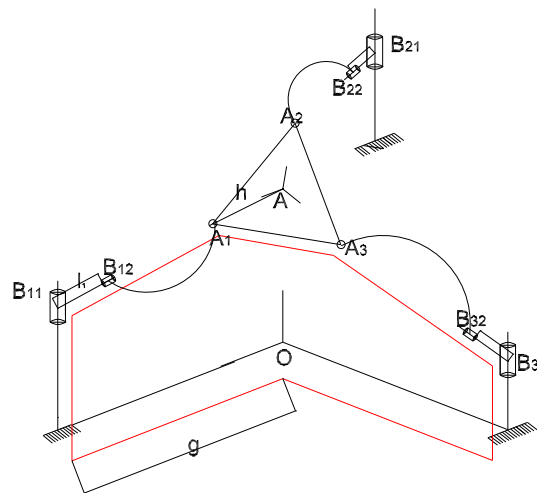
Figure 3. Vector loop closure formations in 3PRS parallel mechanisms

3. Kinematics

3.1. Loop closure formulation

From the below mentioned schematic representation , in PRS mechanisms constitute of passive Prismatic and spherical joint in each of its identical kinematic limb.using vector approach,the connectivity between the identical kinematic limb with its neighbour kinematic leg through the moving platform is called as

loop closure.In the figure it was represented in red lines. the mathematical expression for this loop closure is called as loop closure equation.

$$B_{A_i} = B_{B_{i1}} + B_{i1_{A_i}}$$

Based on the vector approach,

$$||B_{A_i} - B_{A_{i+1}}|| = \sqrt{3}h \text{ where i=1, 2, 3}$$

$$\left.\begin{array}{l} ||B_{A_1} - B_{A_2}|| = \sqrt{3}h \Rightarrow (B_{A_1} - B_{A_2})^T.(B_{A_1} - B_{A_2}) - 3h^2 = 0 \\ ||B_{A_2} - B_{A_3}|| = \sqrt{3}h \Rightarrow (B_{A_2} - B_{A_3})^T.(B_{A_2} - B_{A_3}) - 3h^2 = 0 \\ ||B_{A_3} - B_{A_1}|| = \sqrt{3}h \Rightarrow (B_{A_3} - B_{A_1})^T.(B_{A_3} - B_{A_1}) - 3h^2 = 0 \end{array}\right\} \qquad (2)$$

Based on the above mentioned expression 3 constraint equation were framed. these 3 equations consiats of all the joint variable.

$$\left.\begin{array}{l} \eta_1 = f(dl_1, \psi_1, dl_2, \psi_2) \\ \eta_2 = f(dl_2, \psi_2, dl_3, \psi_3) \\ \eta_3 = f(dl_3, \psi_3, dl_1, \psi_1) \end{array}\right\} \qquad (3)$$

In equations (4), (5) and (6) all active variables (ψ_1, ψ_1, ψ_1) are known. Hence,

$$\left.\begin{array}{l} \eta_1 = f(dl_1, dl_2) \\ \eta_2 = f(dl_2, dl_3) \\ \eta_3 = f(dl_3, dl_1) \end{array}\right\} \qquad (4)$$

To obtain the univariate polynomial expression, half-tangent angle formulae was implemented.

$$\text{i.e., } \cos\psi_i = \frac{1 - t_i^2}{1 + t_i^2} \text{ and } \sin\psi_i = \frac{2t_i}{1 + t_i^2} \text{ where, } t_i = tan\frac{\psi_i}{2}$$

3.2. Forward Kinematics
Hence,

$$\left.\begin{array}{l} \eta_1 = f(t_1, t_2) \\ \eta_2 = f(t_2, t_3) \\ \eta_3 = f(t_3, t_1) \end{array}\right\} \qquad (5)$$

Using Sylvester Dialytic elimination method in (5) can be converted in to 16 degree univariate polynomial expression of dl_3. By choosing the appropriate value for dl_3 from the 16 values we can find the suitable dl_1 and dl_2. Using this dl_1, dl_2 and dl_3, we can found the B_{A_1}, B_{A_2} and B_{A_3} respectively. Hence, we can can found the centroidal position, X, by the following expression.

$$A = \frac{B_{A_1} + B_{A_2} + B_{A_3}}{3} \qquad \Rightarrow \qquad X = [x, y, z] \qquad (6)$$

where,

$$x = f_1(\theta_i^*, \psi_i) \, ; \, y = f_2(\theta_i^*, \psi_i) \, ; \, z = f_3(\theta_i^*, \psi_i).$$

and orientation (B_{R_A}) by the following expression:

$$B_{R_A} = [\hat{x}, \hat{y}, \hat{z}]$$

where,

$$\hat{x} = \frac{B_{A_1} - B_{A_2}}{|B_{A_1} - B_{A_2}|},$$

$$\hat{y} = \left[\frac{B_{A_1} - B_{A_2}}{|B_{A_1} - B_{A_2}|}.\frac{B_{A_1} - B_{A_3}}{|B_{A_1} - B_{A_3}|}\right] \otimes \left[\frac{B_{A_1} - B_{A_2}}{|B_{A_1} - B_{A_2}|}\right] and \tag{7}$$

$$\hat{z} = \left[\frac{B_{A_1} - B_{A_2}}{|B_{A_1} - B_{A_2}|}.\frac{B_{A_1} - B_{A_3}}{|B_{A_1} - B_{A_3}|}\right]$$

The rotational matrix B_{R_A} can also be defined based on Rodriques formula as follows:

$$B_{R_A} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \tag{8}$$

Where, ω, is defined as follows:

$$\omega = \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \frac{1}{2}\sin(\hat{k}) \begin{bmatrix} r_{32} - r_{23} \\ r_{13} - r_{31} \\ r_{21} - r_{12} \end{bmatrix} \tag{9}$$

where,

$\alpha = f_4(\theta_i^*, \psi_i) \;;\; \beta = f_5(\theta_i^*, \psi_i) \;;\; \gamma = f_6(\theta_i^*, \psi_i)$
$\hat{k} = A\cos\left[\frac{1}{2}(r_{11} + r_{22} + r_{33} - 1)\right];$
in which $\theta_i^*, \psi_i^* = ((dl_i, \psi_i))$ for different configurations such as PRS configurations.

3.3. Inverse Kinematics

In Inverse Kinematics,the pose of the moving platform is given &the corresponding joint variable (passive and active variable) is to be determined and also the length of the leg travelled also calculated from this expressions.
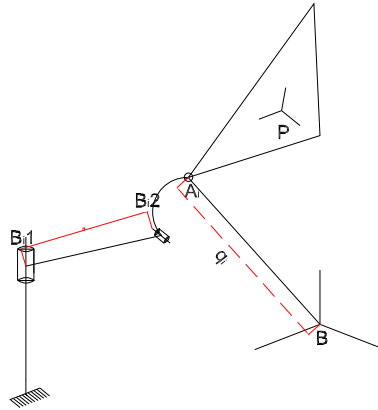

Figure 4. Schematic Representations for inverse kinematics

$$q_i = P + B_{R_A}.A_{A_i}$$

where,

Centroidal Position of the moving platform with respect to fixed orgin frame B, P = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$,

B_{R_A} is the rotational matrix of B to A in Euler's angle and
A_{A_i} is the distance between centroidal position of the moving platform to the spherical joint. Then the distance of the limb,

$$d_i^2 = (q_i - B_{B_{i1}})^T (q_i - B_{B_{i1}}) \tag{10}$$

i.e.,

$$\left.\begin{array}{l} d_1^2 = (q_1 - B_{B_{11}})^T (q_1 - B_{B_{11}}) \Rightarrow \zeta_1(x, y, z, \alpha, \beta, \gamma) \\ d_2^2 = (q_2 - B_{B_{21}})^T (q_2 - B_{B_{21}}) \Rightarrow \zeta_2(x, y, z, \alpha, \beta, \gamma) \\ d_3^2 = (q_3 - B_{B_{31}})^T (q_3 - B_{B_{31}}) \Rightarrow \zeta_3(x, y, z, \alpha, \beta, \gamma) \end{array}\right\} \tag{11}$$

4. Jacobian Matrix

The main non beneficial feature of a PMs is existing of singular configuration while perform in its workspace boundary.These can be identified using its Jacobian matrix derived from its corresponding kinematics.the relationship between the input joint rates and the moving platform velocity rate is eaxpressed as

$$[J_q]\,\dot{\Theta} = [J_X]\,\dot{X} \tag{12}$$

4.1. Forward kinematic Jacobian
From the loop closure equation,

$$\eta(dl_i, \psi_i) = 0 \qquad \Rightarrow \qquad \eta(\Theta) = 0 \tag{13}$$

Derivative of loop closure equations will give forward Jacobian matrix

$$[J_q]\,\dot{\Theta} = 0$$

where , $[J_q]$ is a 3×6 matrix. The determintant of $[J_q] = 0$ indicates the inverse singularity, whereas the motion of the platform cannot be accomplished or the platform loses its one or more degree of dreedom. Some of the Singular postures are highlighted as CAD Representation in Fig.5

4.2. Inverse kinematic Jacobian
The derivative of the limb length gives the inverse Jacobian matrix:

$$\zeta(x, y, z, \alpha, \beta, \gamma) = 0 \quad \Rightarrow \zeta(X) = 0 \tag{14}$$

$$[J_X]\,\dot{X} = 0$$

where , $[J_X]$ is a 3×6 matrix. The determinant of $[J_X] = 0$ indicates the forward singularity, at some special configurations whereas the instantaneous motion of the platform accomplished when all the joint are locked or in rest condition or the platform gain its one or more degree of freedom.

4.3. Overall Jacobian
The overall Jacobian was derived as follows.

$$\dot{X} = [J_X]^{-1}\,[J_q]\,\dot{\Theta} \qquad \Rightarrow [J]_{6\times6}\,\dot{\Theta} \tag{15}$$
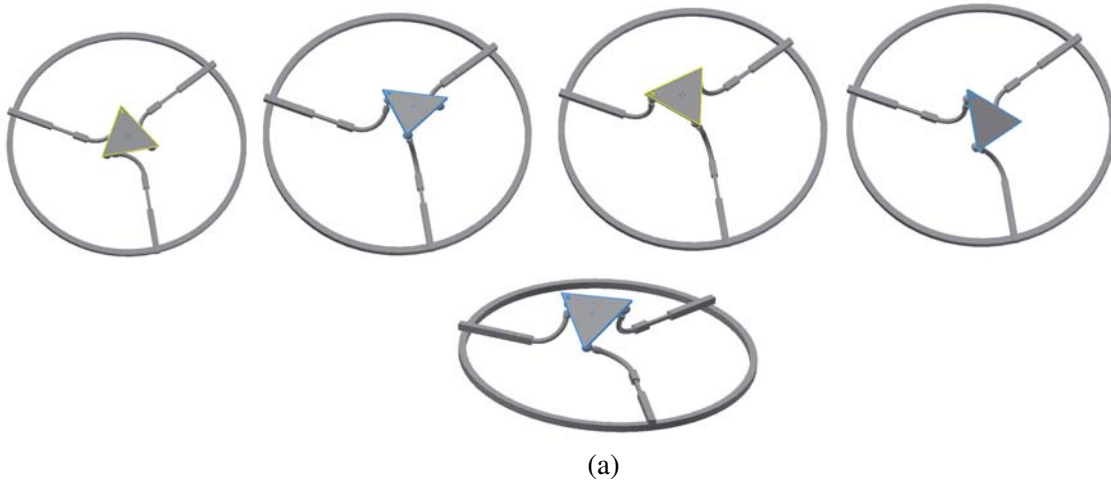
(a)

Figure 5. Few singular postures of (a) 3PRS

5. Performance Indices Based on Jacobian Matrix

5.1. Minimum Singular Value (MSV)

The Minimum Singular Value(MSV) is an important performance index for a mechanism. It is indicative of the minimum value that the transmission ratio attains, the maximum value of force that is transmitted, and the highest attainable accuracy. The direction represented by the MSV is the one in which it is most difficult for the platform to move when all other directions are ignored. For the mechanism in this paper, the MSV was obtained using the Singular Value Decomposition (SVD) theorem, which was applied to the Jacobian matrix. According to the SVD theorem, the Jacobian matrix could be split and represented as a product of three matrices.

$$J = U\Sigma V^T \tag{16}$$

where U is a mxm orthogonal matrix; V is a nxn orthogonal matrix; and Σ is a mxn diagonal matrix. The diagonal matrix Σ consists of elements $a_{i,j}$ such that $a_{i,j} = 0$ if $i \neq j$ and $a_{i,j} = \sigma_i$ if i =j.

$$\Sigma = \begin{bmatrix} \sigma_1 & & & & \\ & \sigma_2 & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & \cdot \\ & & & & & \sigma_m \end{bmatrix} \tag{17}$$

The elements of σ_i (scalars) are singular values of matrix Σ, such that:

$$\sigma_{min} = \sigma_1 \geq \sigma_2 \geq \ldots \sigma_m \tag{18}$$

Singular values of Jacobian (J) are defined as the non-negative square roots of non-zero Eigenvalues of the square matrices $(J.J^T)$ and$(J^T.J)$.

$$\sigma_{min} = min(\sigma_1, \sigma_2, \ldots, \sigma_m) \tag{19}$$

5.2. Condition Number (CN)

Yet another important measure of the kinematic performance of a mechanism is its Condition Number. The Condition Number, unlike the MSV, takes into account two directions of motion of the moving platform: the directions in which motion is offered the greatest and least resistance. In this paper,

the Condition Number is used as a measure of the distance between singularities, and of kinematic accuracy. If the Condition Number attains the value of unity, the mechanism is said to be isotropic. This is significant because an isotropic mechanism has no singular configuration. Mathematically, the Condition Number measures how independent the columns of the Jacobian matrix are. It is defined as

$$k = \frac{\sigma_{max}}{\sigma_{min}} \tag{20}$$

which is the ratio of the maximum singular value to the minimum singular value of the Jacobian. The computation of the Condition number is often complicated owing to the dependence of the singular values on the Eigenvalues, which cannot be analytically described easily. A relatively simpler form for computation is:

$$k = ||J||.||J^{-1}|| \tag{21}$$

where $||.||$ is the matrix norm.hence it is rectangular matrix ,J^{-1}used as considered to be pseudoinverse of Jacobian matrix.

5.3. Local Conditioning Index (LCI)
Since the minimum singular value of the Jacobian can be zero, the condition number has no upper bound, i.e. it can take values up to infinity. Computational difficulties arise when this happens, and to avoid this, another parameter, called LCI, is used. It is the reciprocal of the condition number. The LCI is thus a bounded performance measure, defined as:

$$LCI = \frac{1}{k} \in (0, 1) \tag{22}$$

5.4. Kinematic Configuration Index
The kinematic Configuration Index (KCI) is a performance measure that is related to the Condition Number, and by consequence, the LCI. It is posture independent, and is defined as:

$$KCI = \frac{1}{k} * 100 \in (0, 100\%) \tag{23}$$

where k is the condition number the mechanisms posture dependent. The maximum value that the CI can take is 100 %, and when this is the case, the mechanism is isotropic. Proximity of the CI to 0 % is indicative of the presence of singular configurations.

6. Numerical Simulation and Results
The architectural parameter of all the RS type novel and its equivalent lenkage mechanism such as 3PRS were as follows. g=2.00m; l1=0.75m; e1=0.40; g1= 0.40;h=0.45m
The input (actuator) joint variables in all the mechanisms were ψ_1=90° to 0°;ψ_2=90° to 0°;ψ_3=90° to 0°; Using above mentioned joint variables range and architectural parameters in the Proposed PRS type mechanisms the performance are carried out. The travel of moving platform centroidal position is taken as reference. The kinematic analysis and the performance evaluation is carried out in the following manner.

7. Results and discussions
Based on the numerical simulation,The obtained plot are discussed as, the pose characterstics of this configurations are progressively going upwards in Y and Z axis motion whereas in X axis motion has minimum level of proportional motional when compared with the other two axis. But in tilting capabilities, the α shows much variations compared with β &γ. From the overall reachable workspace,it shows that the moving platform will reaches it maximum of 0.17m in X and 0.1m in Y axis respectively
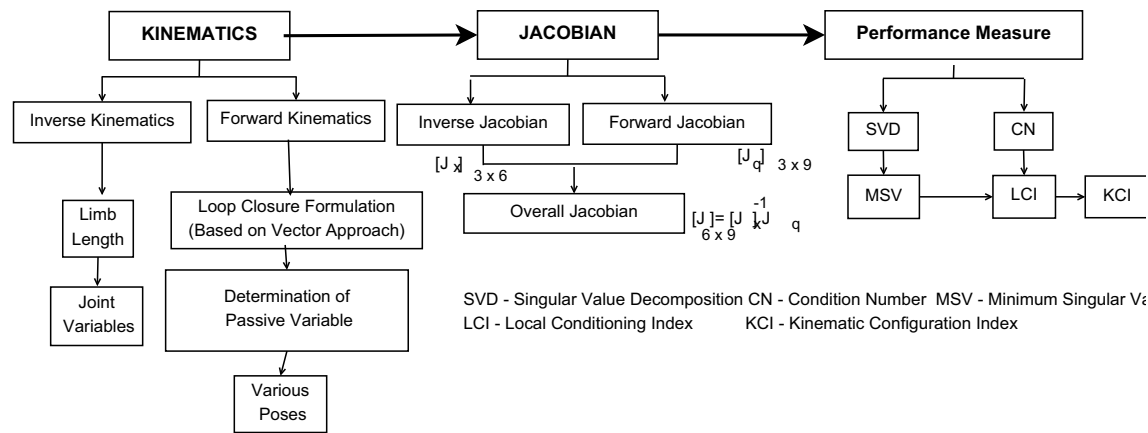
Figure 6. Overall Performance of 3PRS mechanism

and the maximum height of reach in Z axis is 0.4 m.From the kinematic and dexterity measure for this configurations it is clearly shown that the KCI attains up to maximum of 26%. The maximum and minimum parameters obtained during its kinematic and Indices based performance evalutions were shown in Table 2.

Table 2. Range of maximum and minimum parameter capabilities for 3PRS configurations

Sl.no	parameter	maximum	minimum
1.	X (m)	0.165	-0.3
2.	Y (m)	0.105	-0.300
3	Z (m)	0.4	0
4	$\alpha(°)$	43	-42
5	$\beta(°)$	15	-27
6	$\gamma(°)$	21	-10
7	GCI (%)	26	0

8. Conclusion

This paper, a 3 DoF (Degree of Freedom) novel PRS (Prismatic-Revolute- Spherical) type PMs has been designed and presented .The combination of striaght and arc type linkages for 3 DOF parallel mechanism is introduced for the first time. The performances of the mechanisms are evaluated based on the indices such as Minimum Singular Value (MSV), Condition Number (CN), Local Conditioning Index (LCI), Kinematic Configuration Index (KCI) and Global Conditioning Index (GCI). The overall reachable workspace of the mechanism are presented. The kinematic measure, dexterity measure and workspace analysis for the mechanism have been evaluated .

9. Appendix

9.1. Coordinates for circular link mechanism

$$B_{B_{11}} = \left\{ \begin{array}{c} g \\ 0 \\ 0 \end{array} \right\} ; B_{B_{21}} = \left\{ \begin{array}{c} 0.5g \\ \frac{\sqrt{3}g}{2} \\ 0 \end{array} \right\} ; B_{B_{31}} = \left\{ \begin{array}{c} -0.5g \\ \frac{\sqrt{3}g}{2} \\ 0 \end{array} \right\} ; B_{i1_{A_i}} = \left\{ \begin{array}{c} x \\ y \\ z \end{array} \right\} \tag{24}$$
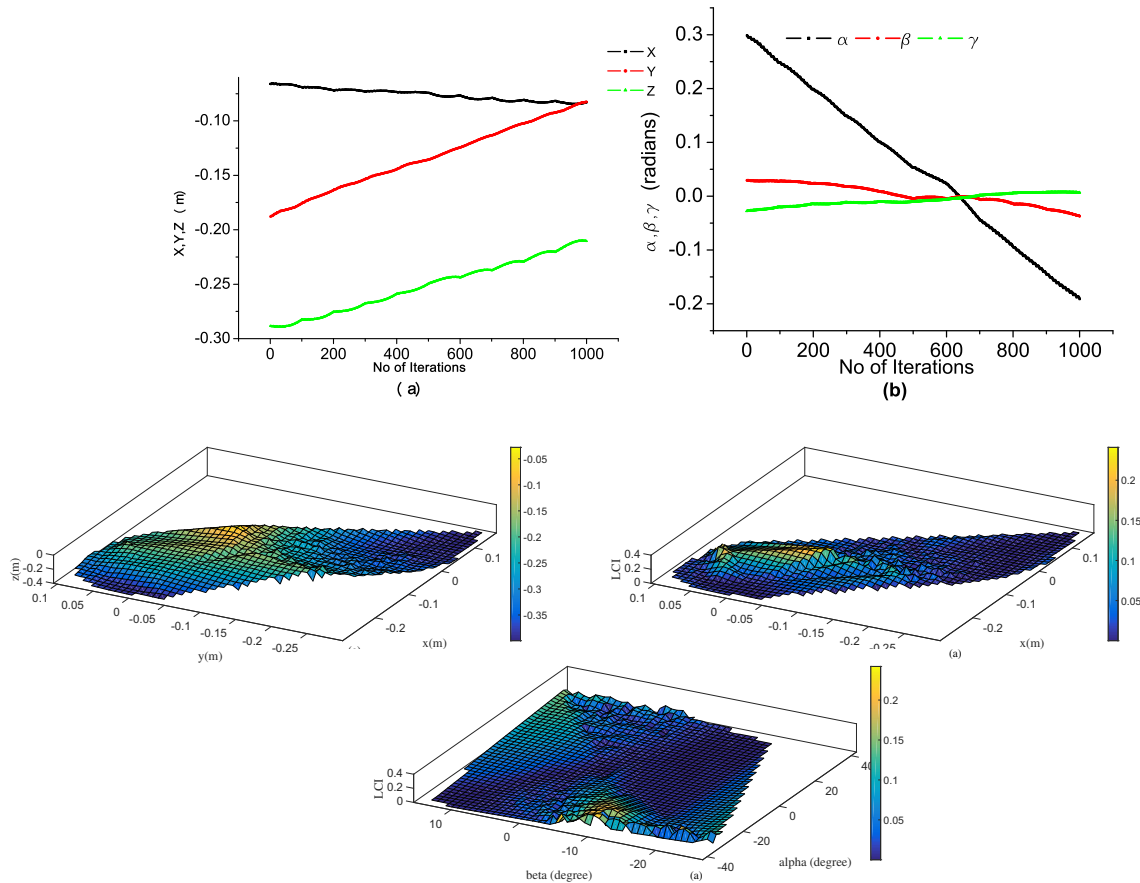
Figure 7. Graphical (a) Position and (b) Orientation Overall Reachable workspace, Kinematic and Dexterity measure of 3PRS Configuration

$$PRS = \begin{pmatrix} \mathrm{dl}_1 + \mathrm{g1} + \mathrm{l1} \\ -\mathrm{e1}\,\cos(\phi_\mathrm{i}) \\ \mathrm{e1}\,\sin(\phi_\mathrm{i}) \end{pmatrix} \tag{25}$$

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