

Convective heat transport in Viscoplastic material due to localized heating: An Experimental approach

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Abstract. Viscoplastic materials are found extensively both in natural and manmade form. In this work experimental investigation of natural convection in viscoplastic fluid with partially heated bottom wall and continuous cooling of top wall in a square cross section enclosure has been carried out. Carbopol Ultrez 20 gel of various concentrations has been used as sample viscoplastic fluid. Conduction and convection phase of heat transport are identified. The results are presented in terms of temperature distribution across the fluid for different gel concentrations and heat input. The average Nusselt numbers are also discussed for different conditions. Onset of convection is delayed and convection strength is weakened with increase in test fluid yield stress. Steady state temperature difference between hot and cold wall shows linear behaviour with heat input for conduction regime and non-linear behaviour in convection regime. Fluid temperature in enclosure shows sharp gradient closure to thermally active walls.

1. Introduction

During last several decades there has been a growing recognition of the fact that many substances of industrial significance do not conform to the Newtonian postulate of the linear relationship between stress (σ) and strain rate ($\dot{\gamma}$) in simple shear [1]. Both natural and man-made materials fall into this category. Materials of multi-phase nature (like foams, emulsions, dispersions, suspensions, slurries) and polymeric melts and solutions are some of the examples. Such materials are generalized as Non-Newtonian Fluids. Depending on stress-strain rate relationship they are further classified as Dilatant, thixotropic, viscoplastic etc. The viscoplastic fluid is the one which remains un-sheared until the applied stress exceeds a threshold limit termed as yield stress. Mathematically, Newtonian and viscoplastic fluids can be represented by following equations:

$$\tau = \eta \frac{du}{dy} \quad 1)$$

$$\tau = \tau_y + K \left(\frac{du}{dy} \right)^n \quad (2)$$

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The constant of proportionality η is the fluid property known as dynamic viscosity, n is flow behaviour index, K is the consistency factor, and τ_y is the minimum shear stress required to cause deformation in fluids. This is the reason it is also known as yield stress. For shear stress less than yield stress, the non-Newtonian fluid behaves like a solid. Such fluids could be natural like animal wastes, blood and saliva or man-made such as adhesives, paints, paper and pulp wastes etc. These fluids are frequently subjected to thermal gradient; therefore, it is imperative to investigate the thermal transport phenomenon occurring in these fluids. Convection in viscoplastic fluids have scientific significance owing to its widespread applications ranging from food, chemical and cosmetic industries to natural phenomena like flow of lava, mud and debris [1]. Food processing activities are often subjected to localized heating, which makes it an area to investigate upon.

Investigations pertaining to classical Rayleigh Benard convection (bottom wall heated in enclosure with cooling top wall) in Newtonian fluids are widespread in the literature [2-3]. Malkus et al. [4] found that Rayleigh Benard convection in Newtonian fluid is characterized by the formation of different convective patterns which can be explained from weak non-linear theory of convection. At the onset of convection, these patterns evolve from a static condition. Pitmann et al. [5] investigated about heat transfer by laminar natural convection between an electrically heated vertical plate for both Newtonian and non-Newtonian fluids.

Darbouli et al. [6] studied RB convection for Newtonian fluids and represented an archetype stability problem. The Rayleigh-Benard configuration corresponds to a buoyancy driven source of instability (natural convection). For a pure viscous fluid heated from below, the instability occurs when buoyancy effects are larger than the viscous and the thermal diffusion effects, characterized by the Rayleigh number, Ra.

There have been significant observations on convection in Non-Newtonian fluids. Kebiche et al. [7] imposed a vertical temperature gradient within a non-Newtonian fluid by heating it from below translates into a vertical gradient of the fluid density or buoyancy which, beyond a critical value of the temperature gradient, may overcome the viscous dissipation and trigger an upwards motion among the fluid elements. Zhang et al. [8] investigated RB convection in yield stress fluids based on inelastic Bingham model using stability theory. It was observed that yield stress affects the stability limit prior to onset of convection (static flow). However their investigation was confined to low Rayleigh number (Ra) and yield stress.

Vikhansky [9] investigated natural convection in Bingham fluid and found that convection in yield stress fluids at low yield stress endure for longer duration and weakens faster at higher yield stress. It was found that larger perturbation is required to instigate convection in viscoplastics due to containment of convective instability by yield stress. Same observation was maintained in experimental investigation by Balmforth and Rust [10]. They used aqueous gel of carbopol as model fluid with yield stress ranging from 0.07 to 10 Pa and corresponding concentrations ranging from 0.05% to 0.1%. Significant weakening of convection strength with rise in yield stress was reported.

The aqueous gel of Carbopol exhibit shear thinning behaviour ($n < 1$) and the plastic viscosity approaches towards infinity at diminishing shear rate ($< 10^{-5} \text{ s}^{-1}$), resulting in difficulty in measurement of viscosity [11]. Davaille et al. [12] experimentally investigated thermal instability due to localized heating in a rectangular enclosure filled with Carbopol gel. Although their experiment does not exactly confer to Rayleigh-Benard convection, it was observed that the convection patterns depend on relative strength of thermal stress and yield stress. Contrary to typical mushroom shape found in natural convection in Newtonian fluids, a rising finger shape plug flow in the enclosure is observed.

Hassan, et al. [13], performed the experimental analysis in a square cross enclosure made of transparent PMMA (Poly-methyl methacrylate). Bottom plate subjected to constant heat flux heating by a controlled resistive heater. R-B convection was achieved by circulating water bath or the top wall. Critical Rayleigh number for the onset of convection was proposed for various gel concentrations.

Turan et al. [14] numerically investigated the R–B convection in Bingham fluid for small values of Rayleigh number (Ra) and Prandtl number (Pr). They resolved the numerical discontinuity problem by using bi-viscosity model of Donovan and Tanner [15]. They observed the formation of convection rolls similar to Newtonian fluid. Albaalbaki and Khayat [16] carried out a theoretical study of convection in non-Newtonian fluid based on Carreau Bird rheological model.

A review of literature suggests that sizeable amount of work on localized heating has been carried out on Newtonian fluid, whereas only few have touch upon the convection in viscoplastic fluid with similar condition. Effect of heating zone length has been numerically studied by Aydin and Yang [17] for Newtonian fluid and by Hassan et al [18] for viscoplastic fluid. For viscoplastic fluid, the effect of Ra is found to be more significant at low yield stress. Also, the critical Ra for inception of convection increases significantly with yield stress of fluid. However, both the reports are computational and experimental investigation on viscoplastic convection due to localized heating is scarce. Therefore this work focuses on experimental investigation of natural convection in viscoplastic fluid in an enclosure with partially heated bottom wall and continuous cooling of top wall. Carbopol gel of various concentrations has been used as sample viscoplastic fluid. Conduction and convection phase of heat transport are identified. The results are presented in terms of temperature distribution across the fluid for different gel concentrations and heat inputs.

2. Sample fluid preparation and characterization

Detailed discussion on the yield stress fluid preparation can be found in [19]. Three samples of fluids of different concentrations are prepared.



Figure 1. Preparation of the sample fluid using overhead stirrer

Sample U1, U2 and U3 of concentrations 0.8 g/l, 1.0 g/l and 1.25 g/l respectively are prepared. The required amount of Carbopol Ultrez 20 gel powder is precisely measured and gradually added to distilled water through a sieve of fine mesh and stirred continuously using overhead stirrer as shown in Figure 1 at 1200 rpm for 20 minutes. It is then left to swell and dehydrate in a controlled environment for two hours. The optimum rheological properties of Carbopol aqueous gel appear when it is neutralized to a pH of 5-8. Therefore the gel is neutralized by adding 18% solution of NaOH. The solution is then stirred at 400 rpm for another 20 minutes to ensure homogeneous swell extensively and form a percolated gel, which is primarily responsible for optimum rheological performance. Finally, full hydration is ensured by keeping the gel in a sealed container for another forty eight hours. The density of hydrated gel at 298K has been found to be 998 kg/m^3 while pH value is maintained at 7.0 ± 0.2 .

Three test fluids have been prepared with Carbopol Ultrez 20 concentrations 0.8 g/l, 1.0 g/l and 1.25 g/l. The rheological properties of test fluids have been measured using Anton Paar MCR 302

rheometer with cup and cone configuration. The yield stress (τ_y), consistency factor (k) and power law index (n) have been measured and presented in Table 1.

Table 1. Rheological properties of Ultrez 20 Gel

Sample	Conc. (g/l)	pH	Yield Stress (Pa)	Consistency Factor (Pa-s ⁿ)	Power Law Index (n)
U1	0.8		3.263	1.281	0.6228
U2	1.0	7.0±0.2	3.390	1.742	0.5571
U3	1.25		6.845	4.658	0.5005

3. Experimental Setup

A complete visual of experimental setup can be seen in figure 2 where all the components of the experimental set-up can be seen. The schematic diagram of the experimental setup is shown in the figure 3. Test section consists of a square cross section enclosure of size 75mm x 75mm x 150mm, which is filled with test fluid. All four side walls are made up of 10 mm thick transparent PMMA (Polymethyl methacrylate) sheet for better visibility. Each side walls has four ports for inserting thermocouples at the middle, as shown in the figure 2. Top wall is made up of 1.0 mm thin glass sheet with water cooling arrangement while bottom wall is a composite wall with provision of controlled heating. Bottom wall is composed of two 3 mm thick copper plates of width 25mm with an electrical resistive heater sandwiched between them. An additional 10 mm thick polycarbonate material sheet forms the bottom most layer to avoid any heat loss to the surroundings.

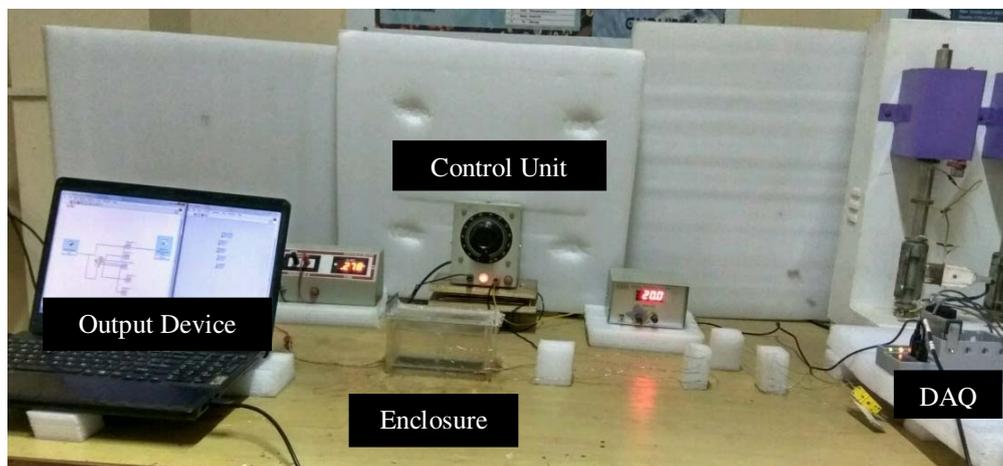


Figure 2. Visual of Complete Experimental Setup

A total of 11 thermocouples have been used. Two thermocouples are fixed on bottom heating plate and one on top cooling plate. The remaining eight thermocouples are used for measurement of temperature of the sample fluid as shown in figure. Each thermocouple is projected in to the fluid by 15 mm. The projected thermocouples may disturb the fluid motion due to convective current and it may produce perturbation in instability analysis. In order to reduce disturbance, very fine

thermocouples with 0.5 mm sheath diameter is used in the experimental setup. The data have been acquired using NI cDAQ-9174 with 16 channels and thermocouple module with 75 samples/sec capacity. Experiments have been carried out in a conditioned room maintained at a temperature of 25°C.

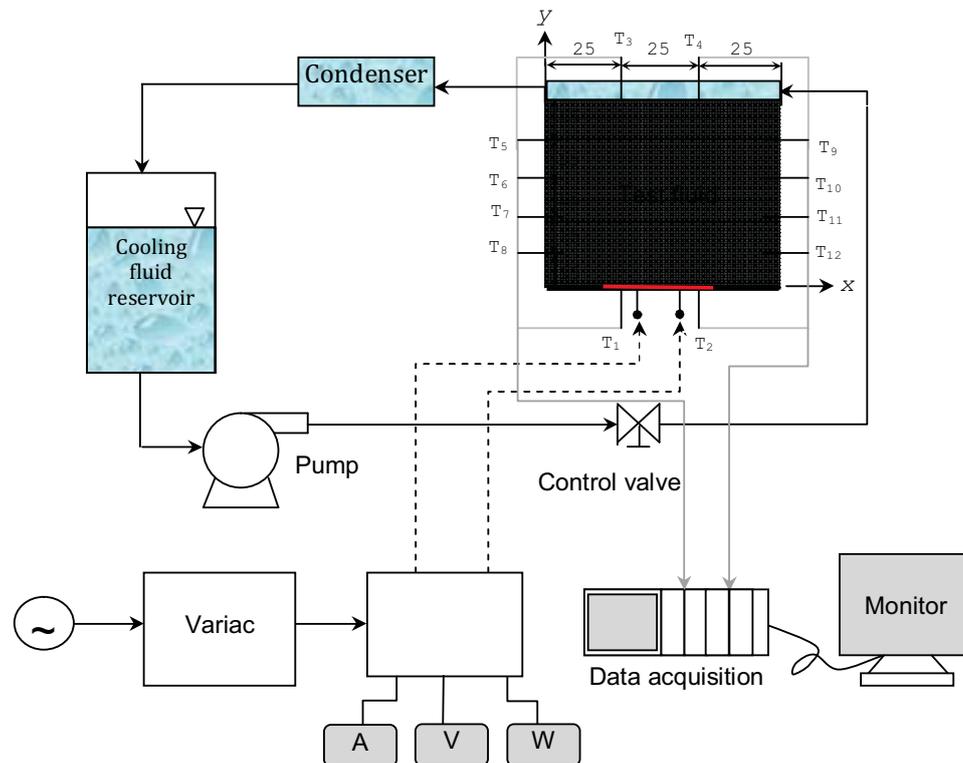


Figure 3. Schematic Sketch of the Experimental Setup

4. Results and Discussions

The sample fluid is charged into the enclosure slowly to avoid bubble formation. Further any trapped bubble is removed by slow shearing. Required heat input is ensured through Variac and control system. The heating continuous till a stable steady state is achieved which usually took 4000-6000 seconds. During the process temperature at all salient points is acquired through online data acquisition system.

After each experiment, the test setup is allowed to cool down to attain room temperature prior to next experiment. The same procedure is repeated for three heat inputs for each test fluid. The experimental data obtained from the localized heating of the viscoplastic fluid is analysed. Effect of test fluid rheology and heat input has been investigated for both transient and steady state condition. Experiments have been carried out for the three sample fluids each subjected to three levels of heat inputs worth 4.8 W, 5.2 W and 5.6 W. Low heat inputs ensures that the fluid is not subjected to boiling and remain in the sub cooled region. Prior to heat input, enclosed fluid is stagnant. Upon heating of the bottom wall, the thermal transport takes place solely due to conduction. As the time progresses, the temperature difference between the hot and cold wall ($\Delta T = T_h - T_c$) increases. The conduction phase of the heat transport results in linear rise in ΔT as visible in Figure 4. With further addition of heat, the thermal stress (τ_{th}) due to buoyancy forces increases leading to weakening of the yield stress (τ_y). When $\tau_{th} > \tau_y$, the convection is established.

Upon incipience of convection, linearity of the ΔT curve is lost. The time required for the establishment of connection is termed as diffusion time (t_d). As time progresses, the ΔT increases till it achieves a steady state. Since the inception of convection, the time required for achieving a steady state is termed as convection time (t_c). Yield stress of the test fluid plays an important role in thermal transport. Both the diffusion time and convection time is significantly dependant on the yield stress. For same level of heat input, the incipience of convection is delayed for fluid with higher yield stress.

At 5.6 W heat input, the t_d increases from 288 s for U1 to 538 s for U2 whereas it is 761 s for U3. For a rise in yield stress from 3.263 Pa to 6.845 Pa, there is 264 % increment in diffusion time t_d . This suggests that the effect of yield stress is more significant at higher levels.

From the comparison of plots for different fluids, it can be observed that for same heat inout, time for initiation of convection heat transfer increases with yield stress of the sample fluid. It can be seen that the difference between values of diffusion time is small compared to large increase in heat input. Thus, with the increase in yield stress, buoyancy effect becomes less sensitive for initiation of convection.

These observations are in line with the reported work [7]. However, magnitudes are slightly deviated due to difference in fluid rheology and size of the enclosure. Respective t_d and t_c for all the fluid samples at different heating conditions are tabulated in table 2.

Table 2. Diffusion and convective time for U1, U2 and U3 at different heat inputs

	Concentration (g/l)	Power (W)	t_d (s)	ΔT_{max} (°C)	t_c (s)	ΔT_{∞} (°C)
U1	0.8	4.8	514	9.4	3873	9.23
		5.2	367	9.61	3956	9.44
		5.6	288	12.08	4073	11.94
U2	1.0	4.8	588	9.04	4762	8.99
		5.6	538	10.14	5067	10.07
U3	1.25	5.2	812	11.05	5231	10.95
		5.6	761	11.16	5331	10.98

A closer look at the figure 4 reveals that for a particular fluid, the diffusion time reduces with rise in heat input. For test fluid U1 (0.8 g/l) at 4.8 W, the t_d is 514 s whereas at 5.2 W, it is 367 s only. However the convection time increases with the rise in heat input. For test fluid U2 (1.0 g/l) at 4.8 W heat input, the $t_c = 4762$ s and it increases to $t_c = 5067$ s for 5.6 W. This trend can be attributed to strengthening of convection with increase in heat input. With rise in heat input more time is required for the heat to be transported to deficient part of the enclosure. It leads to rise in the convection time. The temperature difference across the thermally active walls rises significantly leading to faster heat transport due to conduction prior to incipience of convection. Therefore the diffusion time reduces significantly. Similar trend is observed for all three sample test fluids.

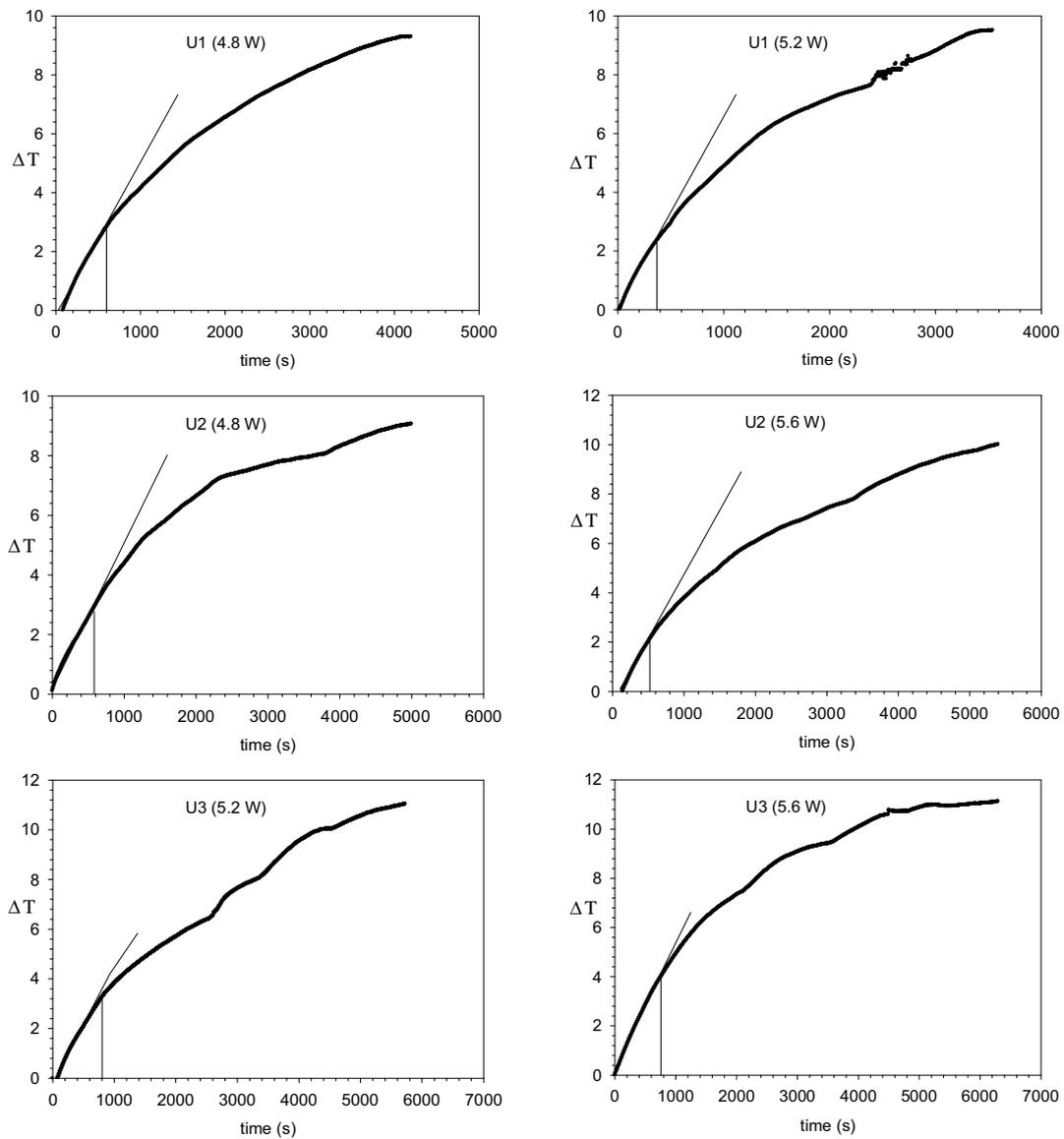


Figure 4. Variation of ΔT with respect to time

Dimensionless temperature distribution across the fluid in the enclosure is shown in figure 5. The dimensionless temperature (θ) is given as:

$$\theta = \frac{T - T_c}{T_h - T_c} \tag{3}$$

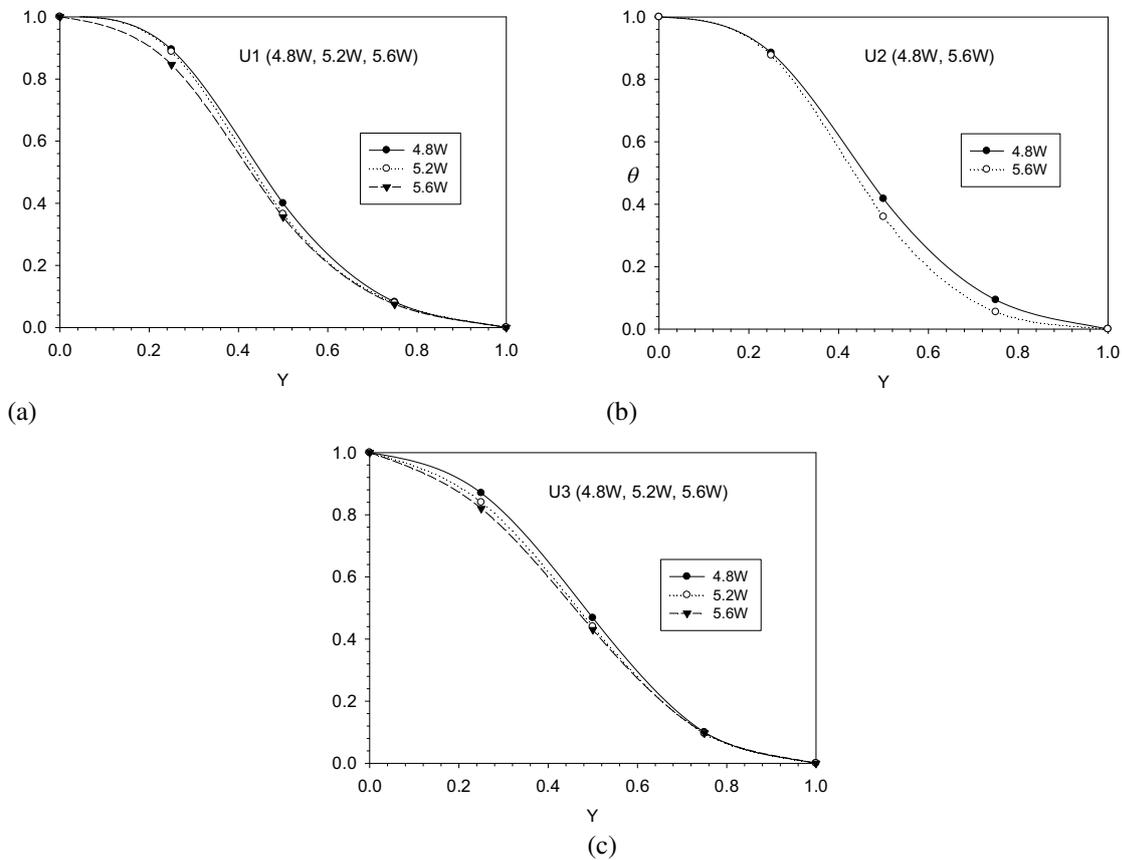


Figure 5. Dimensionless Temperature with position for different test fluids

The temperature distribution along the thermal gradient is plotted for test fluid U1, U2 and U3 in figure 5(a), (b) and (c) respectively. Analysing the steady state profiles, it is seen that the curve is steeper at the thermally active zones. The dimensionless temperature drops sharply closer to thermally active walls. It remains less steep in the central regions. With increase in heat input, non-linearity in temperature distribution increases. With increase in heat input, thermal stress overcomes the viscous stress, and set the enclosed fluid in motion, hence strengthening the convection. For same heat input, the curve tends towards linearity with increase in concentration of the test fluid, as the diffusion time is delayed as shown in Table 2. This clearly suggests the domination of conduction heat transfer with increase in the concentration of the fluid. The viscosity which is a temperature dependant entity will also be affected. This observation is of prime significance for food and pharmaceutical processing, where viscosity plays an important role.

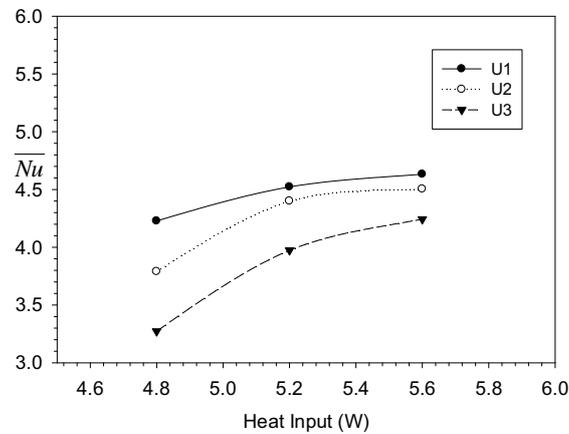


Figure 6. Average Nusselt Number with heat input for different test fluids

The variation of average Nusselt number (\overline{Nu}) for three test fluids at different heat input is depicted in figure 6. For a particular fluid \overline{Nu} increases with heat input. This result corroborates well with the previous reports both for Newtonian and non-Newtonian fluids. However for same heat input \overline{Nu} is highest for U1 and lowest for U3. Similar trend is observed in figure 7 where variation of \overline{Nu} is plotted for varying gel concentration. Rise in gel concentration results in increasing yield stress, which in turn depresses in the convection heat transport. Henceforth \overline{Nu} is decreasing with rise in gel concentration. The plot can be further extrapolated to obtain a condition where for a particular gel concentration the average Nusselt will approach to unity ($\overline{Nu} \rightarrow 1$), at which the convection ceases and conduction will be the sole mode of thermal transport.

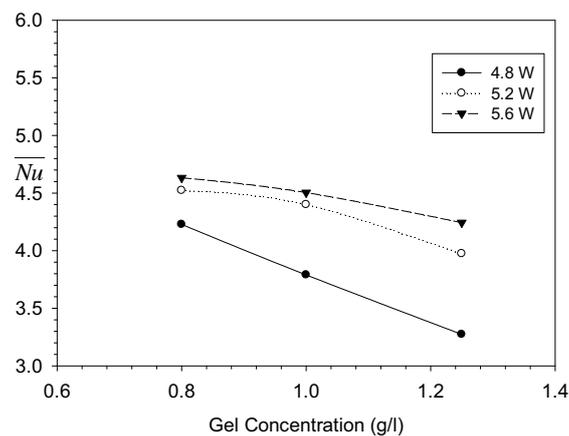


Figure 7. Average Nusselt Number with gel concentration at various heat input

5. Conclusions

In the present work an experimental approach has been entailed to investigate the convective heat transport in viscoplastic fluid due to localized heating.

Two transient regimes namely diffusion and convection are observed. Onset of convection is delayed and convective strength is weakened with increase in gel concentration and henceforth yield

stress. Steady state temperature difference between hot and cold wall shows linear behaviour with heat input for conduction regime and non-linear behaviour in convection regime. Fluid temperature shows sharp gradient closure to thermally active bottom and top walls. Large magnitude of heat input results non linearity in temperature distribution across the fluid. With the rise in yield stress, steady state temperature difference between hot and cold walls (ΔT) increases significantly.

For a particular fluid the average Nusselt number (\overline{Nu}) increases with heat input. However for same heat input \overline{Nu} is higher for low concentration gel. Suggestion has been appended to obtain a condition where for a particular gel concentration the average Nusselt will approach to unity ($\overline{Nu} \rightarrow 1$), at which the convection ceases and conduction will be the sole mode of thermal transport.

References

- [1] Chhabra R P and Richardson J F 2008 *Non-Newtonian Flow and Applied Rheology: Engineering Applications*, (Oxford, U.K.: Butterworth-Heinemann)
- [2] Ostrach S 1988, *J. Heat Trans.* **110** 1175
- [3] Bodenschatz E Pesch W and Ahlers G 2000 *Ann. Rev. Fluid Mech.* **32** 709
- [4] Malkus W V R and Veronis G 1958 *J. Fluid Mech.* **4** 225260
- [5] Pitmann J F T Richardson J F and Sheerard C P 1999 *International Journal of Heat and Mass Transfer* **42** 657-671
- [6] Darbouli M Metivier C Piau J. M Magnin A and Abdelali A 2013 *Phys. Fluids* **25** 023101
- [7] Kebiche Z Castelain C and Burghelca T 2014 *J. Non-Newt. Fluid Mech.* **203** 923
- [8] Zhang J Vola D and Frigaard I A 2006 *J. Fluid Mech.* **566** 389419
- [9] Vikhansky A 2009 *Phys. Fluids* **21** 103103
- [10] Balmforth N J and Rust A C 2009 *J. Non-Newt. Fluid Mech.* **158** 3645
- [11] Zhu H Kim Y D and Deekee D 2005 *J. Fluid Mech.* **129** 177181
- [12] Davaille A Gueslin B Massmeyer A and Giuseppe E D 2013 *J. Non-Newt. Fluid Mech.* **193** 144153
- [13] Hassan M A Pathak M and Khan M K 2015 *Journal of Non-Newtonian Fluid Mech.* **226** 32
- [14] Turan O Chakraborty N and Poole R J 2012 *J. Non-Newt. Fluid Mech.* **171-172** 8396
- [15] O'Donovan E J and Tanner R I 1984 *J. Non-Newt. Fluid Mech.* **15** 7583
- [16] Albaalbaki B and Khayat R E 2011, *J. Fluid Mech.* **668** 500550
- [17] Adyin O and Yang W 2000, *Int. J. Numer. Methods Heat Fluid Flow* **10** 518
- [18] Hassan M A Pathak M and Khan M K 2013 *ASME J. Heat Transfer*, **135** 122501
- [19] Hassan M A Pathak M and Khan M K 2015 *ASME J. Eng. Mat. Tech.* **137** 031002