

Optimal subparametric finite element approach for a Darcy-Brinkman fluid flow problem through a rectangular channel with one curved side

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Abstract. In this paper a sub-parametric finite element method is used to find solution of Darcy Brinkman flow through a rectangular channel with one curved side using curved triangular elements. Finding the numerical solution for the flow through a irregular geometry involves a lot of computation time. In order to solve the flow problem with a much effective computational process, a finite element approach with curved triangular elements and with less number of degrees of freedom is employed. The demonstration of the effectiveness of the method is the objective of the paper. Results obtained are in good agreement with previous works done.

1. Introduction

In the recent past, Darcy-Brinkman equation is used in various hydrodynamic studies including biomedical studies that found its use in modelling a thin fibrous surface layer coating blood vessels as it is a highly permeable, high porosity porous medium. The dynamics of porous medium are different from normal flows of streams and rivers. The pore matrix has its effect on the velocity distributions which is represented by its differential equation. Porosity depends on the pressure of the engineering system like a reservoir pressure. Porous medium influences various mechanical applications like thermal insulations, petroleum industry, heat exchangers and heat pipe technology. A lot of study has been undertaken in the field of forced convection for a porous media by Haji-Shiekh and Vafai [1], Nield and Kuznetsov [2], Hooman [3], Narashimhan and Lage [4]. Work has also been done in steady flow through porous media by Greencorn [5], where the effects of different geometries of porous media is related to capillary pressure. Finite element method is found to generate a good result when compared to the experimental method in various works. In this work, we have considered a flow through a rectangular channel with one curved side porous media and by the help of extensive computation using Mathematica 7.0 codes, a finite element analysis is done. The study of a steady, uni-dimensional, linear flow is considered in the present work. The use of curved triangular element is given in Ergatoudis *et. al.* [6], the matching of curved triangular elements with parabolic arcs are given in the works of Rathod *et. al.* [8], Nagaraja *et. al.* [9] and Kesavlu *et. al.* [11]. The description of the transformation is given by Nagaraja *et. al.* [9]. The numerical computation scheme adopted makes a faster calculation in comparison with the commercial software available. The present scheme is adopted to solve the flow through the given rectangular porous channel with a curved bounbry (figure 1).



2. Mathematical Formulation For Low Speed Flow

The Darcy–Brinkman momentum equation for the case of unidirectional (fully developed) steady state flow in the z -direction in a porous channel of irregular cross-section (figures.1,2) with velocity $u(x, y)$ is

$$\mu' \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \frac{\mu}{K} u = \frac{dp}{dz} \quad (1)$$

We now non-dimensionalize Eq. (1) using the following definitions.

$$X = \frac{x}{h}, \quad Y = \frac{y}{h}, \quad Z = \frac{z}{h}, \quad U = \frac{u}{u_R}, \quad P = \frac{ph}{\mu u_R} \quad (2)$$

Substituting Eq. (2) in the Eq. (1), we get

$$\Lambda^{-1} \left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) - \sigma^2 U = \frac{dP}{dZ}, \quad (3)$$

where

$$\Lambda = \frac{\mu}{\mu'} \text{ viscosity ratio (Brinkman number), and } \sigma^2 = \frac{h^2}{K} \text{ (porous parameter).}$$

Choosing $U^* = \frac{U}{\left(-\frac{dP}{dZ}\right)}$ and dropping the asterisks, Eq. (3) takes the form:

$$\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) - \Lambda \sigma^2 U = -\Lambda, \text{ in } \Omega \text{ (cross-section of the channel).} \quad (4)$$

Eq. (4) is the non-dimensional form of Eq. (1). Eq. (4) is to be solved subject to the boundary condition:

$$U = 0 \text{ on } \partial\Omega, \quad (5)$$

where $\partial\Omega$ is the boundary of the cross-section; see figure 2.

In order to solve the partial differential equation Eq. (4) subject to the boundary condition Eq. (5), the Galerkin weighted residual method is used ; see Bhatti [7]. The region Ω is divided into number of triangular elements and hence the boundary $\partial\Omega$ is also divided as shown in the figure 3.

In the present example, an experimentation is done with curved triangular elements with 15-noded quartic and 21-noded quintic; see Kesavulu [11]. The equations of the transformation are given in Nagaraja [9]. The Jacobian for the transformation is found in Kesavulu [11]. The linear degree of the Jacobian reduces the computational time. By applying Galerkin finite element procedure for differential equation and Gauss quadrature rule is applied for evaluating the finite element equations.

After taking the effect of all the elements into account and then imposing the boundary conditions, we get a linear system of algebraic equations, on solving, we get the unknown nodal point velocity profiles (see Kesavulu[11]).

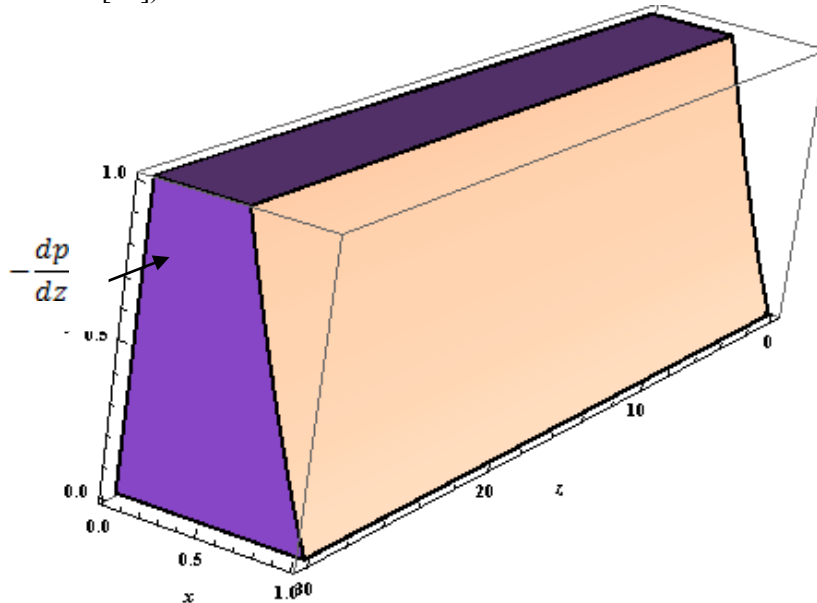


Figure 1. The flow channel with one curved side.

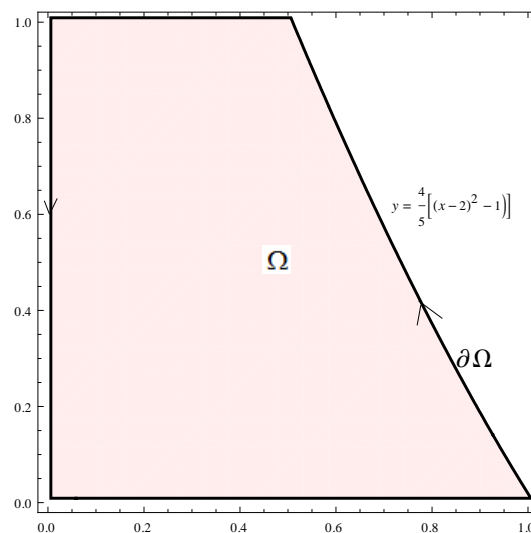


Figure 2. The cross section of the flow channel.

3. Results and Discussions

Now a days, faster computation with less computational time is the need of the hour in all the major analysis in mechanical industries. The method used in the analysis of the flow shows that it is very effective and accurate.. The exact boundary effects are found in the classical finite element methods with a lot of computational effort. Hence the straight triangular elements are replaced with curved triangular elements to get the exact boundary effects with lesser elements. Mathematica7.0 is used in the coding the designed algorithm. It is observed that, as we increase the order and number of elements, we get increased accuracy results, clearly establishing the effectiveness of the method used.

We can see the contour plots of the velocity distribution $U(X,Y)$ in a channel with porous materials which depicts the results in the present geometry.

In the figures. 4a, 4b, 4c, 4d, 4e shows the effect of change in Darcy number σ and Brinkman number Λ . It is observed that if we take the Darcy number as zero i.e. figure. 4a, it means when there is no Darcy friction the flow velocity is the highest.

Secondly, when the Brinkman number is kept constant and the Darcy number is varied, the results shows as the Darcy number increase the flow velocity decreases. Comparison can be seen in the results of contour plots of figure 4b with figure 4c and figure 4d with figure 4e.

Thirdly, when the Brinkman number was varied and the Darcy number is kept constant, the results shows, as the Brinkman number is increased the flow velocity increases, comparison can be seen in figure 4b with figure 4d and figure 4c with figure 4e.

It may be concluded that the results depicted by the figures are in tune with the observation of Givler and Altobelli [12].

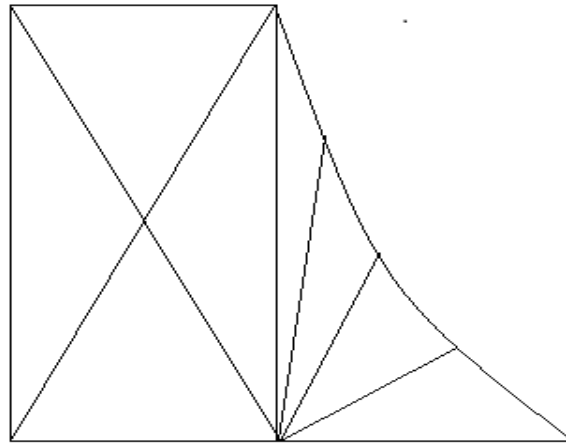


Figure 3. Discretization of domain into 8 elements

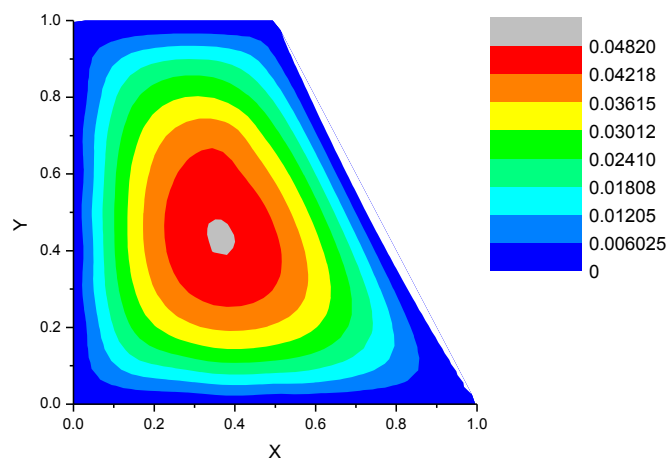


Figure 4a. Contour plot for quintic 8 elements with $\Lambda = 1$, $\sigma^2 = 0$.

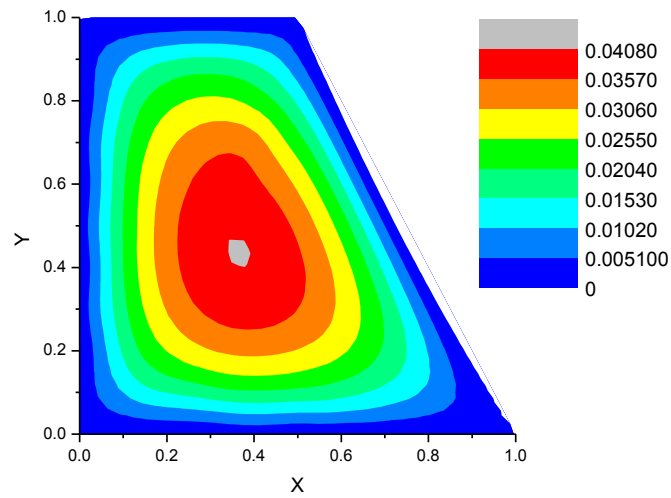


Figure 4b. Contour plot for quintic order 8 elements with $\Lambda = 1, \sigma^2 = 5$.

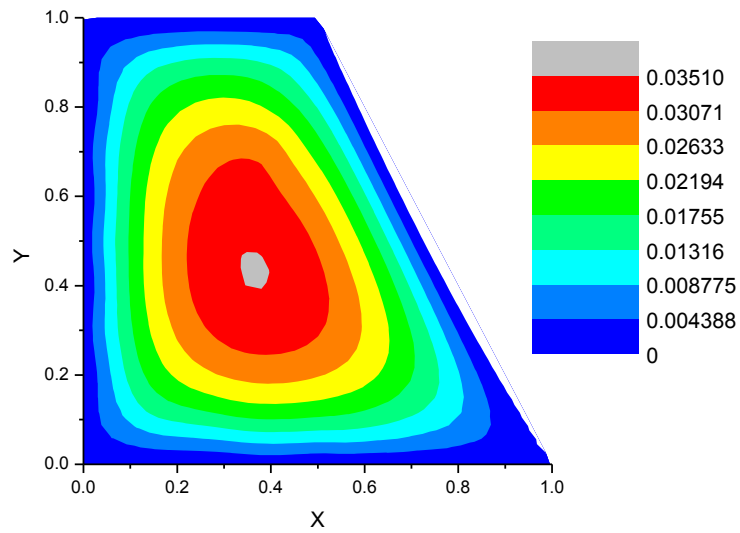


Figure 4c. Contour plot for quintic order with 8 elements with $\Lambda = 1, \sigma^2 = 10$.

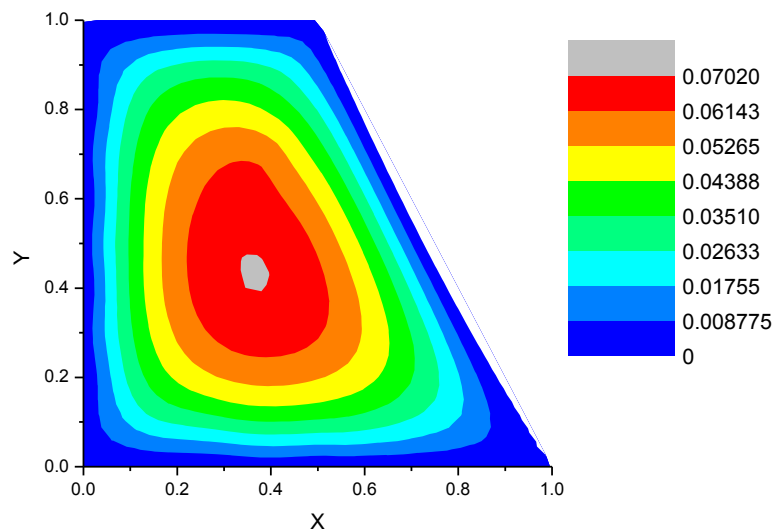


Figure 4d. Contour plot for quintic order with 8 elements with $\Lambda = 2$, $\sigma^2 = 5$.

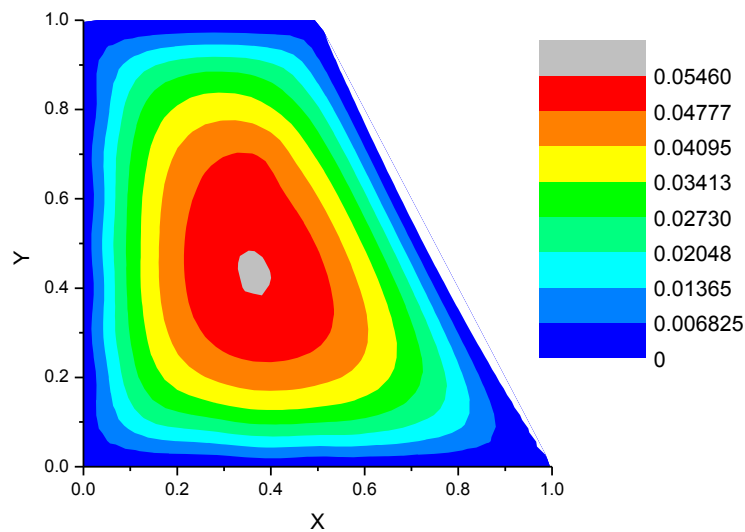


Figure 4e. Contour plot for quintic order with 8 elements with $\Lambda = 2$, $\sigma^2 = 10$.

4. Conclusions

- This method captures the effects for different irregular channels so it can be used for irregular geometries.
- As the friction increases the flow velocity reduces and as the pressure increases the flow velocity increases, this phenomenon is precisely shown using the unique numerical scheme.
- A good accuracy is achieved using higher order curved triangular elements and with lesser number of elements

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